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THISNESS AND VAGUENESS

0. This paper is about two puzzles, or two versions of a single puzzle, which deserve to be called paradoxes, and develops some apparatus in terms of which the apparently conflicting principles which generate the puzzles can be rendered consistent. However, the apparatus itself is somewhat controversial: the puzzles are modal ones, and the resolution to be advocated requires the adoption of a counterpart theoretic semantics of essentially the kind proposed by David Lewis,[1] which in turn requires qualified rejection of certain modal theses about identity which are valid in S5. Of these, we will label the strongest 'the necessity of identity' and write it as

(N) $\Box(\forall x)\Box(\forall y)\Box(x = y \rightarrow \Box(x = y))$.

But the interest of the apparatus to be described does not just lie in its power to dissolve paradox, or to provide an unfamiliar kind of putative counterexample to (N), for the puzzles themselves have a broader significance than a merely cautionary one about forms of reasoning. They have been used to cast doubt on the coherence of *de re* modality itself and also on the truth of a plausible philosophical doctrine about identity which appears to underpin some attractive *essentialist* theses about sets and their members, and about organisms and the cells from which they develop. Our resolution of the puzzles works, briefly, by showing how to generalize the doctrine and the essentialist theses. And in so generalizing, it is hoped, we learn something unexpected about the relation between the standard Kripke semantics for S5[2] and the counterpart-theoretic alternative.

1. The cast assembled, we now describe some of its members in detail. The principle which does most work in generating the paradoxes is a *tolerance* principle concerning what certain things could have been made from. Untutored intuition recommends the view that one and the same artifact designed according to a plan *P* could have been constructed according to *P* from a slightly different range of components from that from which it is actually constructed, at least if it

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is fairly complex and its parts are roughly equally contributory to its functioning. Combining the terminology of Robert Adams and Crispin Wright,[3] this view implies that the haecceity or *thisness* of an artifact is *tolerant to* at least some counterfactual change in original constitution. Of course, there is nothing magical about the *actual* constitution of an artifact in this respect; an ugly but more general formulation of tolerance, suppressing mention of P, is this:

(T) Necessarily any artifact could have originated from a slightly different collection of parts from any one collection from which it could have originated.

The plausible doctrine about identity, in this context *transworld* identity, with which (T) conflicts, says, in the terminology of Michael Dummett,[4] that there are no bare truths about transworld identity; rather, for each truth about the holding or failing to hold of the transworld identity relation, there must be something in virtue of which that truth obtains, something in which its obtaining consists. To see the plausibility of this principle and its relevance to the view, say, that the members of a set constitute an essence of the set,[5] consider one half of this view, according to which, in possible worlds discourse, if for instance α is a set in a world w with exactly the members x_1 and x_2 in w, then there is no world w' containing a set β with exactly the same members x_1 and x_2 in w', such that α is distinct from β. Someone who is operating with the iterative conception of set and who finds this claim irresistible is likely to do so because if α has the same members in w as β has in w' then there is no way in which there could be a numerical difference between α and β, for as the example is described, there is nothing in virtue of which such a difference could obtain.

As may be already obvious, the doctrine about transworld identity, which we shall refer to as the no-bare-facts doctrine, is inconsistent with the tolerance principle (T) both with respect to the first order logic of possible-worlds discourse and the S5 logic of the modal operators. That is, (T) can be used to establish both bare facts about identities and bare facts about nonidentities. The argument for the former has been presented by Chisholm,[6] so we call it Chisholm's Paradox. Let $\langle w_1 \cdots w_n \rangle$ be a sequence of worlds and let $\langle \alpha_1 \ldots \alpha_n \rangle$ be a sequence of artifacts such that each α_i exists in w_i, each α_i is constructed according to the same specifications and no α_i changes its parts through time (these last two conditions will be in force until further notice). Next, suppose that but for a very few components each α_i is made from the same parts as α_{i+1}, yet the members of the pairs (α_i, α_{i+1}) differ from each other in such a way that as i increases so the number of parts α_i has in common with α_1 decreases, until we reach α_n, which has no parts in common with α_1. This set-up is a model of certain possibilities allowed by the tolerance principle: w_2 may be taken to be a world which realizes the possibility that α_1 is made of such and such parts, those which make up α_2; that is, α_2 *is* α_1. But then w_3, which by (T) may be taken to realize the corresponding possibility for α_2, thereby realizes a possibility for α_1, and so on until we reach the conclusion that w_n realizes a possibility for α_1. But α_n is made of completely different parts from α_1, so this gives us our example of an identity which is a bare identity.[7]

We can also give an exposition of the paradox without appeal to possible worlds. For each w_i, let ϕ_i be a predicate which says with rigid designators what parts α_i is made of in w_i, and let us replace 'α_i' with 'α' and treat w_1 as the actual world. Then the following is a classically sound argument, for its conditional premises are true by the tolerance principle, the minor premise is true since $\phi_1(\alpha)$ is actually true *ex hypothesi*, and the only rule of inference employed is *modus ponens*:

$$\begin{array}{l} \Diamond\phi_1(\alpha) \\ \Diamond\phi_1(\alpha) \rightarrow \Diamond\phi_2(\alpha) \\ \quad\vdots \qquad\qquad \vdots \\ \underline{\Diamond\phi_{n-1}(\alpha) \rightarrow \Diamond\phi_n(\alpha)} \\ \qquad \Diamond\phi_n(\alpha) \end{array}$$

So α could have been constructed from parts none of which feature in its actual construction. This gives us bare identity and refutes any kind of essentialism about an artifact's parts. Moreover, Quine has appealed to Chisholm's Paradox to attack the strategy of making sense of *de re* modality in terms of transworld identity. He writes:

> . . . our cross-moment identification of bodies turned on continuity of displacement, distortion and chemical change. These considerations cannot be extended across worlds, because you can change anything to anything by easy stages through some connecting series of possible worlds.[8]

But the problem is not that we cannot think of any conditions on transworld identification: the question is rather whether conditions we find equally plausible are mutually consistent. So the solution of Chisholm's Paradox to be offered below constitutes a defense of *de re* modality against this criticism of Quine's.

The argument for bare facts about transworld differences, which we take from a paper by Nathan Salmon,[9] employs the same resources as the paradoxical argument above. There is intuitively such a thing as too much difference for transworld identity, and the difference between α_1 and α_n may for present purposes be taken to be an example. So α_1 and α_n are distinct. But the tolerance principle tells us that there is a sequence of worlds σ_1 like the first half of $\langle w_1 \ldots w_n \rangle$ and a sequence σ_2 like the second half (in the reverse order, beginning with w_n), each sequence terminating in a world just like a particular world, say w_k, from the middle of the original sequence, such that in the last world of σ_1, α_1 is just like α_k in w_k, and in the last world of σ_2, α_n is just like α_k in w_k. Since α_1 and α_n are distinct, so are these worlds, but there is no difference at all between the respective artifacts in them: there is nothing in virtue of which the nonidentity obtains across these two worlds. Moreover, because of the way the worlds were chosen, there is nothing which manifests their distinctness other than one's containing α_1 and the other α_n, itself an ungrounded distinction. Following Salmon, we call this the Four Worlds Paradox, the four worlds being w_1, w_n, the last world of σ_1 and the last world of σ_2.

For a modal operator object language formulation of the paradox, we introduce the descriptive name 'β', whose reference is fixed by the description 'the artifact which would have resulted if...', completing the description by filling in the details of α_n's construction in w_n, and we agree that β would not be α.[10] We can then give two arguments like Chisholm's Paradox, one which concludes $\Diamond\phi_k(\alpha)$ and the other $\Diamond\phi_k(\beta)$. Recalling that we are at the moment holding specifications constant, the truth of these two statements together with that of $\Box(\alpha \neq \beta)$, delivered by the necessity of identity, is inconsistent with the requirement that there must be something in virtue of which transworld differences obtain where they do.

2. The two modal paradoxes are Sorites paradoxes, that is, paradoxes of vagueness. This is especially easy to see in the case of Chisholm's Paradox, which is exactly like familiar Sorites paradoxes such as the Paradox of the Tall Man. Corresponding to the tolerance principle (T) for transworld heirlines we have a tolerance principle for height classification: someone only marginally (say, one centimeter) shorter (taller) than a tall (short) man is himself tall (short). To be absolutely

precise, one tolerance principle concerns the application conditions of a single predicate, ξ *is tall*, whereas the other tolerance principle is really a family of principles, one for each object x constructed out of parts, and the role of the various men is played by the various sets of components which could make up an artifact.[11] If α is an artifact, then the predicate whose application conditions are tolerant is a predicate expressing the thisness or haecceity of α, for which we may simply use the predicate $\xi = \alpha$.

The analogue to the Four Worlds Paradox is obtained by starting with a man five feet in height and applying the tolerance principle to conclude that a man five foot six is short, and then by taking a six foot tall man and concluding from the tolerance principle that a man of five foot six is tall. Here we have a bare difference in height classification: there is no difference between such men in which their difference in height status consists: in particular, there is no difference in their height. This kind of bare difference is indisputably ludicrous.

The classification of the modal paradoxes as Sorites paradoxes makes it desirable that the method of resolving them be an instance of a general strategy for resolving Sorites paradoxes. This immediately eliminates some proposed solutions. For example, it has been suggested that the introduction of an accessibility relation on worlds which would prevent, say, the last world in the sequence for the Chisholm Paradox from being accessible to the first world in the sequence, resolves the paradoxes.[12] But this resolution is quite *ad hoc*, and does not speak at all to the underlying source of the paradox, unless there is a sense in which tall men are not accessible to short men. Moreover, the accessibility solution applied to the Four Worlds Paradox decrees that the last world of σ_1 is accessible to w_1, but the last world of σ_2 is not. It therefore requires us to distinguish between these worlds, a distinction which has the same problematic status as the one between their contained artifacts. So someone motivated to seek a solution to the Four Worlds Paradox because he does not wish to draw distinctions which mark no differences could not be content with such a treatment of it.

Another nonsolution of the modal paradoxes involves giving up the tolerance principle for predicates of the form $\xi = t$, since tolerance principles for the predicates which turn up in the familiar Sorites paradoxes are *true*. Michael Dummett and Crispin Wright have convincingly argued that tolerance in the application conditions of such predicates is a consequence of the point of using them, which is to effect classifications of objects just on the basis of how they look. To give up the tolerance principles would be to eliminate predicates with such a use from our language, since 'sharpening' such predicates would change their use radically. Predicates which are applied just on the basis of how things look cannot have strictly delimited ranges of application, because, as Wright puts it, "if the conditions under which a predicate applies are to be generally memorable, [that predicate] cannot be unseated by changes too slight to be remembered".[13] In other words, sharp observational predicates would be *unlearnable* if the phenomena to which they apply form a sensible continuum, as do colors, sizes, and so on. Another of Wright's examples involves predicates for stages of human life, such as 'infant', 'adolescent', etc. One who is an infant at time *t* is still an infant a few seconds later, but then no one ever reaches adulthood. Here the explanation of the tolerance is that with different stage classifications go explanatory distinctions and difference of moral and social status which a sufficiently small degree of development is too slight to support. Hence, in Wright's irresistible illustration, if we are forced to draw a sharp line, as we are in the matter of electoral qualifications, we do so "with a sense of artificiality and absurdity". And although it can hardly be used as an uncontested example, the predicate 'person' or 'bearer of a right to life' is surely another case, definitely applying to teenagers and definitely not applying to embryos, and tolerant because small degrees of biological and psychological development cannot constitute the difference between a case in which they do and a case in which they do not apply, while large degrees of development do constitute such differences.[14]

The distinction between what is possible and what is impossible for an object is as large a distinction as that between the tall and the short, one primary color and another, or persons and nonpersons, and therefore cannot turn upon a small degree of change in the respect relevant to making the difference. To apply this idea, of course, we need prior agreement about what counts as a small degree of change: for sets, any degree of change in membership is too large. But this distinction between sets and artifacts is hardly incomprehensible. The ground of the distinction lies in the trans-temporal identity conditions of items of these categories, since we allow that the same artifact can undergo replenishment of parts through time, provided not too much is changed too quickly. We are able to allow this since a single artifact is a unified locus of functional organization and its specifications and purpose contribute enough to its individuality for the latter to survive change of parts (and concomitantly for smallish slowish changes in specifications). To see that it would be absurd to allow replenishment of parts through *time* but not difference in original constitution across *worlds*, consider a sequence of worlds in which the time at which a particular part of a given artifact is replaced by a certain new part is moved further and further back until we have a world in which the artificer is choosing which of the two parts to put in place in his original construction (we hold constant the stretch of time occupied by the lifespan of the artifact). It might be held that until the construction of the artifact is completed it does not exist, but it would be as well to say that it goes in and out of existence as a part is taken off and then replaced by a new part; indeed, this *has* to be said if bare differences across time are not to arise. So we conclude that the tolerance principles underlying the modal paradoxes are as inviolable as those underlying any Sorites paradox, and turn to the problem of extending the most reasonable solution of the standard paradoxes to these modal ones.[15]

3. How should we resolve standard Sorites paradoxes? It is very plausible that such paradoxes arise from the application of a semantic apparatus appropriate only for sharp predicates to languages or portions of natural language rife with vague ones. This view is in conscious opposition to the idea that vagueness arises from deficiency of meaning[16] or is a source of incoherence[17]; rather, vague concepts are held to be legitimate and unproblematic as they stand, so long as we associate only the appropriate semantics with them.

The crucial notion of this semantics is that of the *degree* to which an object falls under a concept or the degree to which a predicate applies to an object, and there is a familiar tradition of semantics for vagueness using this notion, which finds its perhaps most comprehensive and sophisticated expression in J. A. Goguen's logic for inexact concepts.[18] We have already used the notion of degree in a number of places above; indeed, in connection with vagueness it is almost impossible to avoid its use. The basic concepts of degree-theoretic semantics are straightforwardly legitimized by the use of vague predicates in the comparative form, for if of two red color patches, one can be redder than another, then the first is red to a greater degree than the other, and so satisfies the predicate 'is red' to a greater degree than the other, and so the judgment that the first is red has a higher degree of truth than the judgment that the second is. It is hard to find a well-motivated objection to any of these transitions, although it must be borne in mind that the resulting notion of the degree to which an object is red is nonobservational, unlike the question of its color.[19]

The suggestion is, then, that the familiar two-valued semantics be modified by including between its two values of absolute truth and absolute falsehood a range of intermediate degrees of truth, each of which is a possible semantic value for a sentence containing a vague expression. Since ordinary cases of vagueness often arise out of sensible continua, it seems reasonable to allow the degrees of truth to form a continuum; to begin with, then, the closed interval $[0, 1]$ on the real line is a useful model of the set of degrees of truth, with 0 playing the role of absolute falsity and 1 the role of absolute truth. A model for a countable propositional language will therefore consist of an assignment of exactly one degree of truth from $[0, 1]$ to each sentence letter, and the truth-value of any sentence can be computed as soon as we generalize the truth-tables for the connectives to the new degree-theoretic framework.

Noting that in two-valued logic a disjunction takes the better of the two values of its disjuncts, and a conjunction the worse of its conjuncts, we obtain the following clauses for degree-theoretic semantics:

(i) $\mathrm{Val}[A \vee B] = \mathrm{Max}\{\mathrm{Val}[A], \mathrm{Val}[B]\}$.

(ii) $\mathrm{Val}[A \,\&\, B] = \mathrm{Min}\{\mathrm{Val}[A], \mathrm{Val}[B]\}$.

In two-valued logic, the value of a negation is the complement in the two-membered set $\{0, 1\}$ of the degree to which the negated sentence falls beneath absolute truth. So for negation, we put

(iii) $\mathrm{Val}[\sim A] = 1 - \mathrm{Val}[A]$.

Clauses (i), (ii) and (iii) together give us the usual interdefinabilities of '&' and '$\vee$' for the truth-value interval $[0, 1]$. But the natural clause for '$\rightarrow$' does not preserve its classical definability by the other connectives. Intuitively, we want the conditional to be material in a generalized sense, that is, it should be true if the consequent is at least as true as the antecedent, but we also want it to take values in the other cases which *reflect the gap* in degree of truth between antecedent and consequent. If the antecedent is only marginally more true than the consequent, the conditional should be only marginally less than wholly true, while if the antecedent is much more true than the consequent, the conditional should be considerably less than wholly true, with the limiting case being that of classical falsehood. The simplest clause which bestows these features on '$\rightarrow$' is

(iv) $\mathrm{Val}[A \rightarrow B] = 1 - (\mathrm{Val}[A] - \mathrm{Val}[B])$ if $\mathrm{Val}[A] > \mathrm{Val}[B]$, otherwise $= 1$.

For this system of propositional logic we define a formula to be valid iff its value is 1 on any assignment of degrees of truth to its sentential letters, and we say that an argument is valid iff there is no assignment such that the value of the conclusion falls below the lowest-valued premise, if there is one, or the greatest lower bound (g.l.b.) of the premise values, if these can be ordered into an infinite descending sequence.

Clauses (i)–(iv) suffice for a resolution of standard Sorites paradoxes. Let $\langle a_1 \ldots a_n \rangle$ be a sequence of men of increasing height such that the statement that a_1 is short is wholly true and the statement that a_n is short is wholly false, although there is only a marginal difference in height between adjacent men in the sequence. The tolerance of 'is short' implies, with respect to the two-valued framework, that each conditional of the form a_i *is short* $\rightarrow a_{i+1}$ *is short* is true. Hence the following argument is classically sound:

$$\begin{array}{l} a_1 \text{ is short} \\ a_1 \text{ is short} \rightarrow a_2 \text{ is short} \\ \underline{a_{n-1} \text{ is short} \rightarrow a_n \text{ is short}} \\ \qquad a_n \text{ is short} \end{array}$$

But this is inconsistent with the fact that a_n is, say, six foot six. However, on the degree-theoretic framework, we see the argument in a different light. The problem is that *modus ponens* is an unreliable rule of inference in this framework, in a way that &-elimination, for instance, is not: if $\mathrm{Val}[A \rightarrow B] = 1$ then applications of *modus ponens* are unproblematic, but in our argument none of the conditionals is wholly true. In each, the degree of truth of the antecedent is marginally higher than the degree of truth of the consequent because each a_i is marginally shorter than the corresponding a_{i+1} (note, again, that even the comparative facts about degrees of truth need not be accessible to simple looking and seeing, since a marginal difference in height need not be observationally detectable). By clause (iv), therefore, each conditional is very slightly less than wholly true, and *modus ponens* is being used to detach consequents whose degrees of truth are dropping steadily towards 0. The paradoxical argument is therefore a concrete illustration of a possibility implicit in the semantics, that from an absolute truth we may reason through a chain of conditionals each of which is almost wholly true to a complete falsehood.

This is an elegant and appealing diagnosis of paradoxes of vagueness; it is because the conditionals are almost wholly true that the argument seems to us to be irresistible, and so, besides being neutralized, the paradox's suasive force is explained. The only serious objections to this approach to vagueness involve what Fine has called 'penumbral connections', which, if they obtain, are inconsistent with the fact that on clauses (i) to (iv) the degree of truth of a compound formula is a uniform function of the degrees of truth of its components. Fine has given a putative counterexample to degree-functionality:[20] 'is pink' and 'is red' are contraries (a penumbral connection), hence 'α is pink and α is red' is false. But if α is exactly poised between pink and red, then each conjunct and so the whole conjunction has a middle degree of truth according to degree-theoretic semantics, which in this special case, Fine concludes, gives the wrong result. However, this objection is unconvincing. To say that 'is pink' and 'is red' are contraries is to say, in Fine's usage, that nothing can be both; but the degree-theoretic framework allows us to make a more discriminating judgment. Nothing can be wholly pink and wholly red, of course, but a thing can be red to a certain degree and pink to a *concomitant* (herein is the penumbral connection) degree. If one man says 'α is red' and another 'α is pink' and neither is judged to have uttered something wholly false, why should this fate befall the first man if he anticipates and utters the second man's thought, using '&' to avoid an unnatural break in his speech? A reply of this kind can also be made to someone who holds that 'α is red and α is not red' should be wholly false, or that 'α is red or α is not red' should be wholly true. One reason (not Fine's) for ascribing complete truth or falsity in these cases should certainly be utterly rejected: vagueness arises from an accessible and real feature of the objects, and *not* because there is a range of actually inaccessible facts which our sensory apparatus is insufficiently finely tuned to discriminate.

If the conditional premises of a Sorites argument are not wholly true, what of the tolerance principles which justify those premises? Such principles are universal quantifications to the effect that if one thing is related thus and so to another the second has a certain property if the first has it. So to be precise about the truth-values of these principles, we have to extend the degree-theoretic semantics to predicate calculus, which will also have the advantage of enabling us to see how the degree of truth of an atomic sentence such as 'a is short' arises out of the semantic properties of its constituents. Our intuition was that a predicate like 'is short' is satisfied by different objects to different degrees, so the extension of a vague predicate of one place, like 'is short', should be a set of objects 'given' along with the degrees to which each of its members satisfies the predicate. Following Goguen, we think of such an extension as a function from a set of objects X into the set of degrees of truth J; such functions are sometimes called 'fuzzy sets', since they can be regarded as giving information about degree of membership of sets, for instance, the set of short things. Note that on this approach, vagueness resides entirely in concepts. The objects in X are perfectly determinate and the fuzzy sets themselves also have exact identity conditions: two such sets are the same iff the same things are members of each to the same degree.

More generally, if F is an n-place atomic predicate then we assign to F a function Φ_F from a set X of n-tuples of objects drawn from a domain D into a set J of degrees of truth; X is called the *universe* of F. Then for atomic sentences we have

(v) $\mathrm{Val}[F(t_1 \ldots t_n)\langle a_1 \ldots a_n\rangle] = \Phi_F(a_1^* \ldots a_n^*)$ where a_i^* is a_i if t_i is a variable and is the referent of t_i if t_i is an individual constant.

The clauses for the existential and universal quantifiers generalize the classical valuations with respect to the set of truth-values $\{0, 1\}$ and are written:

(vi) $\mathrm{Val}[(\exists x)\phi(x, y_1 \ldots y_n)\,\langle a, a_1 \ldots a_n\rangle] =$
$\text{l.u.b.}\ \{\mathrm{Val}[\phi(x, y_1 \ldots y_n)\,\langle b, a_1 \ldots a_n\rangle]\colon b \in D\}$

and

(vii) $\mathrm{Val}[(\forall x)\phi(x, y_1 \ldots y_n)\langle a, a_1 \ldots a_n\rangle] =$
$\text{g.l.b.}\ \{\mathrm{Val}[\phi(x, y_1 \ldots y_n)\langle b, a_1 \ldots a_n\rangle]\colon b \in D\}$.

Returning now to the tolerance principles, we see that none of these are wholly true; for instance, a version of the principle for the Tall Man Paradox is

$$(\forall x)(\forall y)(\mathrm{Short}(x)\ \&\ y \text{ is one centimeter taller than } x \rightarrow \mathrm{Short}(y))$$

and the conditional matrix is either wholly true (when an assignment fails to satisfy the second conjunct of the antecedent) or slightly less than wholly true because 'Short(y)' is satisfied to a slightly lesser degree than 'Short(x)'. However, if D contains all possible men then we can construct paradoxes with tolerance principles mentioning arbitrarily small differences in height and therefore with a degree of truth arbitrarily close to 1. So our earlier remark that tolerance principles are true requires qualification when we move out of the two-valued framework: they are merely almost wholly true. But this in itself is sufficient to show that it would be absurd to deny them.

4. We turn now to the task of extending the degree-theoretic solution of the Sorites paradoxes to the modal paradoxes. There are two obstacles in the way of such an extension, one technical and one philosophical. The technical obstacle is that when we compare artifacts across worlds, we may wish to assess degrees of similarity in more than one respect. In our presentation of the modal paradoxes, we held the specifications of artifacts constant and allowed only changes in original constitution. But this is an artificial restriction; variation in design and perhaps also in function should be allowed as well. Suppose we now consider two artifacts a and b in a world w and ask to what degree they satisfy $\xi = \alpha$ at w, where α is some actual artifact. Perhaps a is close in design to α but not in constitution, and conversely for b. So with each of a and b we can associate a pair of numbers, measuring degree of similarity to α in each of two respects. But there does not seem to be any reason why

these two numbers have to be resolvable into a single number giving overall degree of similarity to α so that a and b can be compared in this respect. But if only pairs of numbers are available, a and b may be *incomparable* in respect of overall similarity, and in such a case, $[0, 1]$ would not be an adequate model of the set of degrees.

However, this technical difficulty can be overcome. Although we have considered only linearly-ordered degrees of truth, the considerations which motivated the connective clauses in §3 permit generalizations of the set of degrees of truth, such as have been algebraically investigated by Goguen.[21] Goguen concludes that for a logic of vagueness, the minimum acceptable structural requirement on the set of degrees is that it have the isomorphism type of a closed lattice-ordered semi-group (*closg*) in which the lattice maximum is identity for the group operation $*$. But in the definitions to come, it suffices for the reader to keep in mind two examples of closgs, $[0, 1]$ as above in its natural order and $[0, 1] \times [0, 1]$, or perhaps more generally $[0, 1] \times \cdots \times [0, 1]$ (k times) with the order defined component-wise, i.e.,

$$\langle a_1 \ldots a_k \rangle \leq \langle b_1 \ldots b_k \rangle \text{ iff } a_i \leq b_i \text{ for each } i,\ 1 \leq i \leq k.$$

That was the technical difficulty in the envisaged extension of degree theory to modality. The philosophical difficulty concerns the coherence of the notion of the degree to which an object satisfies such a predicate as $\xi = \alpha$ at a world. In the standard semantics for S5, transworld heirlines of objects are given by transworld identities: the only object which satisfies $\xi = \alpha$ at a world is α. So if there can be degrees of satisfaction of $\xi = \alpha$ at a world w then it looks as if there must be degrees of being identical to α at w. Yet the notion of degrees of identity is incoherent. We saw how the idea that a predicate F is a predicate of degree arises out of the admissibility of the comparative form 'ξ is more F than ζ', but it would be quite hopeless to try to make literal sense of 'ξ is more identical to α than is ζ', and we will not waste space in the attempt.

Instead, we need to replace standard S5 semantics with some other sort, in which transworld heirlines are given not by identities and nonidentities across worlds, but by some other transworld relation which it does make sense to regard as a relation of degree. The prescient reader will have anticipated that counterpart theory is about to appear on the stage, for as originally explained by Lewis, the counterpart relation is a crossworld relation in a model in which the individuals in the domains of the worlds need not have transworld being (if they do, this is a nonrepresentational feature of the model) and, more importantly, the extension of the relation is fixed by considerations of similarity across worlds. Since there is no problem at all about degrees of similarity, degrees of counterparthood are equally straightforward. Nevertheless, it can be anticipated that the proposal to use counterpart theory will meet with some resistance, so we should consider a few imaginable objections before turning to the final resolution of the paradoxes. We divide these objections into four groups: (A) Problems about semantic evidence at the intuitive level; (B) Problems about the logic of existence in counterpart theory; (C) Problems about the logic of identity; and (D) Problems about earlier formulations of ours which used transworld identity.

(A) According to Kripke, there is an intuition that counterpart-theoretic readings of modal sentences misrepresent their content. He claims that according to the counterpart theorist, if we say 'Humphrey might have won the election' then

> we are not talking about something that might have happened to *Humphrey* but to someone else, a 'counterpart'. Probably, however, Humphrey could not care less whether someone *else*, no matter how much resembling him, would have been victorious in another possible world.[22]

But as Allen Hazen has forcefully pointed out,[23] this objection is quite unfair. The counterpart theorist is proposing a semantic interpretation in the metalanguage L_c of counterpart theory of the sentence 'Humphrey could have won the election', which belongs to the modal object language L_m. His interpretation, roughly 'some counterpart of Humphrey wins in some world' does not say that someone *other* than Humphrey *would have* won in some *world*, since this claim is expressed in a mixture of L_m and L_c vocabulary (the counterpart theorist can use heavy emphasis too!) and is, strictly speaking, not even well-formed. For Humphrey to care that he could have won is just what it is, according to the counterpart theorist, for Humphrey to care that some counterpart of his wins in some world. Intuitions about the meanings of L_m sentences do not carry over to sentences of the theoretical language with its terms of art; rather, a semantic theory is justified by its fruitfulness and capacity to organize and account for the data, and it is the thesis of this paper that counterpart theory does this best with respect to our feelings about the paradoxical arguments in their modal operator formulations.

(B) The logic of existence is separated from the logic of identity because, even though an existence predicate $E(\xi)$ can be defined from identity and quantification, it is not at all clear that it ought to be.[24] Moreover, the problem about existence in counterpart theory is of a different nature from the one about identity. As is well known, the translation scheme which Lewis proposes for taking L_m sentences into L_c sentences translates

(B) $(\forall x)\Box(\exists y)(x = y)$

i.e., 'everything necessarily exists', into a theorem of counterpart theory; equivalently, (B) is valid on the counterpart-theoretic model theory associated with the translation scheme.[25] So it looks as if counterpart theory cannot make adequate sense of contingent existence.

However, the difficulty here is relatively minor, for we can devise a better translation scheme on which (B) is translated into a falsehood of counterpart theory. In the Kripke semantics for S5, an object x_1 which exists at a world w_1 can satisfy atomic formulae at a world w_2 at which it does not exist; to reflect this feature of the semantics in counterpart theory we have to allow an object x_1 existing at w_1 to have a counterpart *at* w_2 which may satisfy atomic formulae there even if it does not exist at w_2 (the simplest strategy is to stipulate that each object is its own counterpart at worlds where it has no existing counterpart, and for this it is best to assume that all individuals are worldbound, in the sense of existing at only one world). These remarks employ the locution of one thing's being a counterpart of another *at a world*, which suggests that we need a three-place counterpart relation C, using which we can write 'x_2 is a counterpart of x_1 at w_2' as $Cx_2x_1w_2$; and we do not say that if $\langle a, b, w\rangle \in C$ then $a \in D_w$. Without further ado,[26] we present a translation scheme *Trans* which is correct in the following sense: if (i) the language L_c is two-valued (ii) the counterpart relation is an equivalence relation in its individual variables and (iii) each object has exactly one counterpart at each world, then each formula of L_m which is valid with respect to Kripke semantics (with contingent existence, strong necessity and actualist quantifiers) is translated by some theorem of counterpart theory and is valid according to the model theory associated with *Trans*. Here we assume L_c to be a language which has as lexicon an $n+1$-place predicate for each n-place predicate of L_m (except that '=' is not altered), no modal operators and a three-place counterpart predicate.

If $\phi \in \text{Form}(L_m)$ then $\text{Trans}(\phi) = \text{Rel}(\phi, w^*)$, 'Rel' for relativization and 'w^*' for the actual world. The clauses of interest in the recursive definition of $Rel(\phi, w)$ for arbitrary w are:

(I) If $F(t_1 \ldots t_n)$ is atomic, $\text{Rel}(F(t_1 \ldots t_n), w) = F(t_1 \ldots t_n, w)$ except that we do not add a world variable to '='.

(II) $\text{Rel}(\forall vA, w) = (\forall v)(E(v, w) \rightarrow [\text{Rel}(A, w)])$.

(III) $\text{Rel}(\exists vA, w) = (\exists v)(E(v, w)\ \&\ [\text{Rel}(A, w)])$.

(IV) $\text{Rel}(\Diamond A, w) = (\exists w_j)(\exists v_1) \ldots (\exists v_n)(Cv_1t_1w_j\ \& \ldots \&\ Cv_nt_nw_j\ \&\ [\text{Rel}(A(t_i/v_i), w_j)])$.

(V) $\text{Rel}(\Box A, w) = (\forall w_j)(\forall v_1) \ldots (\forall v_n)(Cv_1t_1w_j\ \& \ldots \&\ Cv_nt_nw_j \rightarrow [\text{Rel}(t_i/v_i), w_j)])$.

In (IV) and (V), the world and counterpart variables are peculiar to L_c; $t_1 \ldots t_n$ are terms in A not within the scope of any modal operator in A, and if there are no such terms in A then $A(t_i/v_i) = A$ and *Rel* introduces no expressions involving the counterpart predicate. According to this scheme, Trans(B) is

$$(\forall x)(E(x, w^*) \rightarrow (\forall w)(\forall y)(Cyxw \rightarrow (\exists z)\ (E(z, w)\ \&\ y = z))$$

which is false for the right reason: y can be a counterpart of x at w without existing at w. The invalidity of (B) persists through all the generalizations of counterparthood we are going on to permit, so the problem about the logic of existence is solved.

(C) It would be inconsistent with the motivation of this paper just to make any old stipulation about the counterpart relation merely so as to ensure a 'nice' logic. We are going to say that things can be counterparts of things to various degrees, and we want these degrees to be fixed by degrees of resemblance in the relevant respects. Suppose, then, that α is an artifact in w_1 designed according to a plan P and that β and γ are artifacts in w_2 with the same specifications, each of which is made from parts of which roughly half turn up in α in w_1. Other things more or less equal, β and γ should be counterparts of α to more or less the same degree, approximately .5, but this means that a consequence of (N),

$$(\mathrm{N}_1) \qquad a = b \rightarrow \Box(a = b)$$

will not be valid in counterpart-theoretic semantics with degrees. (N_1) translates as

$$(\mathrm{N}_2) \qquad a = b \rightarrow (\forall w)(\forall x)(\forall y)(Cxaw \;\&\; Cybw \rightarrow x = y)$$

but if 'a' and 'b' both refer to α then the consequent of this conditional has a lower degree of truth than its antecedent, since $Cxaw$ & $Cybw$ has degree of truth roughly .5 while $x = y$ is wholly false, when w_2, β and γ are taken for w, x and y. By the clauses in §3, the quantified conditional will also have the degree of truth of roughly .5, the lowest degree its instances can take in these simple situations, which gives (N_2) degree .5 as well. It should be emphasized that this result does not come about because of any vague identities: vagueness resides wholly in the counterpart predicate. Nevertheless, the result itself may seem objectionable.

In fact, it is not at all clear that there is anything objectionable about it. The validity of (N_1) and (N) itself in Kripke semantics is just a consequence of the fact that in that system transworld heirlines in a model are fixed by real crossworld identities in the model. This may indeed be the most appropriate way of proceeding when the situations specified in modal language for which we wish to give extensional models involve entities only of the kinds which turn up in typical examples put forward to engage our intuitions on behalf of (N_1), for example, planets and people. Characteristic of entities of this kind is the fact that the notion of part has no very natural application to them, and artificial applications do not seem to yield parts related essentially to the whole. The Kripke framework is also adequate for entities whose identity consists in nothing more than the identities of the entities naturally regarded as their parts, for instance sets and mereological sums. But artifacts are more complex, as the tolerance principles testify, so it is reasonable to expect that the Kripke framework will require generalization to accommodate our intuitions, and the counterpart-theoretic framework seems to afford the most appropriate generalization. It follows from this that it would be a mistake to think of these two frameworks as in *competition* with one another. And it is of the essence of generalization that principles which hold of what is being generalized (e.g. (N)) do not continue to hold. Let α and β be clocks of identical manufacture on opposite walls of a room. Will anyone claim to have a clear intuition that someone who conceives of a single clock manufactured half out of α's parts and half out of β's and who therefore says 'These two clocks could have been one' says something *wholly false*, or a clear intuition that to say that he has said something with some degree of truth is absurd?

(D) Since we are resolving the modal paradoxes by moving to a more complex semantics for the language L_m in which they are stated, we have to reinterpret earlier formulations which used notions from the simpler semantics. For instance, in the extensional renderings of the paradoxes, we presumed that transworld identity conditions for parts of artifacts were unproblematic relative to the identity conditions of the artifacts themselves; in analogous L_c-renderings, we would make a similar assumption that in other worlds it is unproblematic which artifact parts are the counterparts of artifact parts in a given world. In addition, the paradoxes will not now be said to establish something counterintuitive about transworld identity; rather, they show that the counterpart relation, if it is not a relation of degree, must hold between entities which have nothing in common in virtue of which it could hold (Chisholm's Paradox) or fails to hold in a situation where it seems it must (the Four Worlds Paradox). Finally, another role identity has played was to facilitate an argument from replacability of parts through time to replacability of parts across worlds. But this argument does not depend upon the use of transtemporal identity to interpret *de re* sentences with tense operators; analogous arguments to the modal one can be formulated in tensed discourse, for instance the Ship of Theseus cases, and a counterpart relation might well be appealed to here also.[27]

5. We now give a brief but rigorous formal description of counterpart theory with degrees. The language L_c contains two sorts of terms, including the constant 'w^*' of sort 1, all the constants of L_m (which are of sort 2 in L_c) and for each n-place predicate of L_m an $n+1$-place predicate whose last place is reserved for a term of sort 1. So the existence predicate $E(\xi)$ of L_m is correlated with $E(\xi, \zeta)$ of L_c, a predicate of sort $\langle 2, 1\rangle$. In addition, L_c contains a three-place predicate $C(\xi_1, \xi_2, \zeta)$ of category $\langle 2, 2, 1\rangle$ which is read 'ξ_1 is a counterpart of ξ_2 at ζ'. The two-sorted language is for ease of readability, while the three-place counterpart relation is needed to obtain a correct logic of existence.

A *degree-model* M for L_c is a two-sorted 9-tuple

$$\langle W, D, J, Q, R, I, H, w^*, v\rangle,$$

where W is a set of entities of the first sort, and D is a set of individuals. J is a closg (see §4) whose elements are k-tuples of real numbers from the interval [0, 1], for fixed k, with the lattice ordering defined componentwise and the group operation $*$ given by

(i) $\langle a_1 \ldots a_k\rangle * \langle b_1 \ldots b_k\rangle = \langle (a_1 \times b_1) \ldots (a_k \times b_k)\rangle$.

Q is a distinguished function from $D \times W$ into the subset $\{0, 1\}$ of J, where $\underline{0}$ is the k-tuple $\langle 0 \ldots 0\rangle$ and $\underline{1}$ is the k-tuple $\langle 1 \ldots 1\rangle$, the lattice minimum and maximum respectively. Q interprets $E(\xi, \zeta)$ and is subject to the constraint

(ii) $\forall w, w' \in W, (w \neq w' \rightarrow (Q(x, w) = \underline{1} \rightarrow Q(x, w') = \underline{0}))$.

So we are going to restrict ourselves to worldbound individuals, things which exist in at most one world.

R is a function from $D \times D \times W$, the universe of the counterpart relation, into J, and meets a number of conditions. First, reflexivity in individual variables:

(iii) $\forall x \in D, \forall w \in W, (Q(x, w) = \underline{1} \rightarrow R(x, x, w) = 1)$.

But symmetry of individual variables with respect to degrees is plausible only when design is held constant; if complex artifacts can be counterparts of simple ones, proportion of parts in common may not be the same. And obviously, no version of transitivity with respect to degrees is desirable. But we do impose two other conditions:

(iv) $\forall x \in D, \forall w \in W, [\forall y \in D(Q(y, w) = \underline{1} \rightarrow R(y, x, w) = \underline{0})] \rightarrow [\forall y \in D(R(y, x, w) = \underline{1} \leftrightarrow y = x)]$.

This says that any object with no existing counterpart at w is its own sole counterpart there, and is the condition which enables Trans(B) to be false. Note that it does not reintroduce transworld identity. Finally, we will insist that counterparthood be properly a crossworld relation when it holds between distinct things:

(v) $\forall x, y \in D, \forall w \in W(Q(x, w) = Q(y, w) = \underline{1} \rightarrow (R(x, y, w) > \underline{0} \leftrightarrow x = y))$.

I interprets the identity symbol of L_c, which is of category $\langle 2, 2\rangle$ and is a function from $D \times D$ into $\{\underset{\sim}{0}, \underset{\sim}{1}\}$ such that $I(a, b) = \underset{\sim}{1}$ iff $a = b$. H is a set of characteristic functions f^i_{n+1}, one for each $n + 1$-place non-logical predicate F^i of L_c. Each such function has domain $D^n \times W$ and range $\{\underset{\sim}{0}, \underset{\sim}{1}\}$. w^* is a designated member of W and lastly, v is a function which assigns members of D to constants of sort 2 under the constraint that for all x

(vi) $v(c) = x$ only if $Q(x, w^*) = \underset{\sim}{1}$.[28]

The connective clauses of §3 need to be modified slightly for the more general truth-value set J, which, for instance, may not be complemented, since not all closg's are Boolean algebras. To interpret negation and implication, Goguen defines the functions Neg and Imp thus:

(vii) $\mathrm{Neg}(\langle a_1 \ldots . a_k\rangle) = \langle (1 - a_1) \ldots . (1 - a_k)\rangle$;
(viii) $\mathrm{Imp}(a, b) = \text{l.u.b. } \{x : x^*a \leq b\}$.

For disjunction and conjunction, we take join and meet defined componentwise in J by Max and Min, while the quantifier clauses stay the same as in §3, as do the definitions of validity for formulae and sequents. This completes the account of degree-theoretic model theory for counterpart theory.

It remains only to explain how the new semantics defuses the paradoxes. Chisholm's Paradox is straightforwardly dealt with, for when we consider its modal-operator formulation in §1, we see that its conditional premises have L_c-translations of the form

$$(\exists w_1)(\exists x)(Cx\alpha w_1 \ \& \ \phi_i(x, w_1)) \rightarrow (\exists w_2)(\exists y)(Cy\alpha w_2 \ \& \ \phi_{i+1}(y, w_2))$$

and that none of these conditionals is wholly true.[29] In each, the consequent is slightly less true than the antecedent because anything with constitution ϕ_{i+1} is slightly less similar to α than anything with constitution ϕ_i, and so is a counterpart of α at a world, if at all, to a slightly lesser degree than something with constitution ϕ_i. Our clauses for the connectives ensure that this small gap in degree of counterparthood translates itself up through the structure of the formula to yield an expression slightly less than wholly true. Thus our resolution of Chisholm's Paradox is absolutely parallel to that of the Paradox of the Tall Man.

The Four Worlds Paradox is dealt with similarly: we reduce it to a three-world truism. That is, we say that the last world of σ_1 and the last world of σ_2, if they are supposed to differ only with respect to the identity of their contained artifacts, are in fact the same world, and their contained artifacts the same artifact. This artifact is a counterpart at its world of both α_1 and α_n, to the same degree, even though these latter are not counterparts of each other at all. In modal language, the Four Worlds Paradox is in fact a model of a situation in which it is half true to say that α and β could have been one. The version of the necessity of identity used in deriving the paradox,

$$(\mathrm{N}_3) \qquad (\forall x)\Box(\forall y)(x = y \rightarrow \Box(x = y))$$

is of course not valid degree-theoretically. As we can see by considering its L_c translation,

$$(\forall x)(E(x, w^*) \rightarrow (\forall w)(\forall y)(Cyxw \rightarrow (\forall z)\ (E(z, w) \rightarrow (y = z \rightarrow (\forall w')\ (\forall s)\ (\forall t)\ (Csyw' \ \&\ Ctzw' \rightarrow s = t))))$$

the validity of (N_3) would require the counterpart relation to be transitive in its individual variables in the following sense: if y is a counterpart of x at w to degree m and z a counterpart of y at w' to degree n then z is a counterpart of x at w' to a degree $\geq \min(m, n)$. But the relevance of relations of degree to paradoxes of vagueness is precisely that they fail to be transitive. So by rejecting (N_3), we block the paradoxical argument; but this move is not *ad hoc*. Rather, it is a consequence of our view that the modal paradoxes are Sorites paradoxes, and of our applying a general technique for dealing with such paradoxes to the modal case.

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NOTES AND REFERENCES

* The paper you are perhaps about to read has been a popular conversation piece of mine for some while now and has been revised many times. I would like to thank all those who have commented on it for the clarifications and improvements they have forced, and in this respect wish to mention by name George Boolos, David Bostock, Hugh Chandler, Lloyd Humberstone, Esa Saarinen, Nathan Salmon, David Wiggins, Crispin Wright and especially Mark Sainsbury for his 'Reply' at a meeting of the Oxford Philosophical Society. Conversations with David Kaplan, in the dark, on Port Meadow, and with Christopher Peacocke, were particularly helpful.

I was prompted to work out the details of my views on thisness and vagueness by incisive questions from the late Gareth Evans about a confused doctrine I once held as

an ancestor of my present position. It is with sadness that I dedicate this paper to his memory.

[1] Lewis [1968]. I will be presupposing acquaintance with this paper.

[2] By 'standard' I mean S5 with contingent existence, actualist quantifiers and strong necessity. See Fine [1978] pp. 127–31.

[3] See Adams [1979] and Wright [1975].

[4] See Dummett [1973] p. 89.

[5] I use 'essence' in the sense of Plantinga [1974] p. 70. A property P is an essence of x iff x has P in every world in which x exists and no other object has P in any world.

[6] See Chisholm [1968].

[7] Bare, that is, relative to the simplifying assumptions of the example. A stronger case is obtained by allowing the specifications to change little by little as well. I assume time and place of origination and identity of artificer are contingent, and would defend these assumptions by arguments of the kind to be found in Forbes [1981].

[8] Quine [1976] p. 861. I do not agree that identity through time is more secure than transworld identity, for where tolerance principles are plausible in the modal case, analogous principles give rise to problematic temporal cases: recall the Ship of Theseus, or the repairs to Michelangelo's *Pieta* (how far can you go?). Those who think that in temporal cases where there is branching, the line of continuity is always the one which traces identity, should consider the following entertaining example, due to David Kaplan. Suppose a museum in California hires a philosopher to go to Greece, obtain the Ship of Theseus, dismantle it, pack it and dispatch it back to the museum. Suppose also that the philosopher follows these instructions, but that as he removes each plank from the ship he replaces it with a new plank of the same shape, so that when the original planks are all crated, he still has a ship in dry dock. The museum receives the planks, reassembles the ship and is about to exhibit it when it receives a phone call from the philosopher, who announces that *he* has the real ship, and demands large sums of money for keeping quiet about the fraud the museum is about to perpetrate. Who has the ship the museum really wants?

[9] Salmon [1979]. Our no-bare-truths doctrine is the source of the plausibility of Salmon's principle (P2′), op. cit. p. 716, which he identifies as underlying a certain argument for essentialism about origin.

[10] For a general account of descriptive names, see Evans [1979]. I do not know if Evans would have approved of the use of modal vocabulary in putatively reference-fixing descriptions, even ones as carefully chosen as the one in the text.

[11] In Anil Gupta's recent book [1980] half a chapter is devoted to the modal paradoxes. Gupta argues (p. 103) that they are not Sorites paradoxes since the latter can be blocked by sharpening the tolerance of vague predicates in a way that the former cannot. He points out in substantiation of this claim that there is no paradox of baldness if we just stipulate that a man is not bald iff he has 10^6 hairs or more on his head, whereas paradox remains even if we stipulate that an artifact retains its identity only if we change at most, say, one part across worlds. In reply to this, I agree that *this* stipulation does not block the paradoxical arguments, but object that Gupta has failed to compare like with like. The numerical stipulation for baldness effects a classification of states of the head into bald and nonbald on which no state is both, and a fair comparison with the modal case should involve a stipulation with the same effect.

Gupta's does not; we need a stipulation which says exactly which sets of parts can make up which artifacts, with no set being included in the haecceity of more than one artifact. This restores the parallel with baldness by blocking the argument and by being equally as unattractive as the stipulation about the number of hairs.

[12] See Chandler [1976]. In response to an earlier version of this paper, Salmon suggested (in correspondence) the introduction of a vague accesibility relation. My view would be that this locates the vagueness in the wrong place, since it arises out of a comparison of objects across worlds and only derivately from the comparison of worlds.

[13] Wright [1975] p. 337. See also Dummett [1975].

[14] Wertheimer [1971] tries to defend the conservative position about abortion from this suggestion. He writes (p. 81): "The conservative points ... to the similarities between each set of successive stages of fetal development ... if this were the whole conservative argument ... it would be open to the liberal's *reductio* ... which says that if you go back as far as the zygote, the sperm and egg must also be persons. But in fact the conservative can stop at the zygote; fertilization does seem to be a nonarbitrary point marking the inception of a particular object." But if we go back to the zygote, we have already gone too far. Writers on abortion often miss the point that it is no mere matter of intuition whether or not a zygote is a person, or human being. A zygote cannot be identical to any person, or human being, since it is a cell which undergoes mitotic division to give rise to a pair of daughter cells and thereby ceases to exist itself. Wertheimer goes on to say: "It needs to be stressed here that we are talking about life and death on a colossal scale ... so the situation contrasts sharply with that in which a society selects a date on which to confer certain legal rights." But there is no contrast in any respect which tends to show that 'person' is not a predicate of degree like 'adult'. An insane tyrant could turn the possession of *any* property which is in fact fuzzy into a matter of life and death. Where great importance ataches to how we draw a line which we are forced to draw by certain practical or moral pressures, the best we can do is to see to it that the line is drawn in such a way, compatible with easing the pressures, that no case which clearly should be on one side is on the other, or even close to the edge.

[15] Christopher Peacocke pointed out to me that the line of reasoning here does not yield the same objection to sharpening haecceity predicates as was made to the proposal to sharpen observational predicates, that such sharpening would nullify the point of having the predicates in the language. The conclusion he draws from this is that not every predicate of degree is one to which Wright-like considerations apply.

[16] This is the viewpoint of Fine [1975].

[17] This is the conclusion drawn by Dummett [1975], and according to Wright also, is a possible moral of the paradoxes.

[18] See Goguen [1969].

[19] See Peacocke [1981] p. 125.

[20] Fine [1975] p. 26.

[21] See Goguen, op. cit. p. 354.

[22] Kripke [1972] p. 334 footnote 13. But in footnote 18 Kripke says that counterpart theory could perhaps solve a problem he describes which seems to be much like Chisholm's paradox. A more sophisticated version of the objection in Note 13 is obtained if we add to L_m a certain sort of 'actually' operator devised by Peacocke,

which enables us to write sentences with the orthodox possible worlds truth-conditions 'a in w is identical to b in w'' and 'a in w is a counterpart of b in w'', where 'is a counterpart of' is a two-place relation in L_m. But it is very unclear that any native-speaker intuitions could settle the propriety of a translation of augmented L_m into L_c in which *de re* modalities are still interpreted by the L_c counterpart relation.

[23] Hazen [1979] pp. 320–25.

[24] Somewhat briefly, my reason for saying this is that in the entity-invoking extensional semantics for L_m which is closest to our intuitive conceptions, possibility semantics, one wants the information contained in the assignments of partial extensions and partial counterextensions to predicates of L_m which determine particular possibilities to manifest itself completely in the *atomic* sentences made determinately true or determinately false by a given assignment. Possibility semantics for propositional systems are given in Humberstone [1981] and for quantified S5 in Forbes [1981a].

[25] Lewis discusses this difficulty at p. 119.

[26] See Forbes [1982] for further discussion of this translation scheme and motivation of the details.

[27] So I am saying that *really* there are no such things as continuants, and what we think of as single things are in fact sequences of time-slices bound together by a counterpart relation? Not at all. The focus on semantical definitions in this paper is perhaps unfortunate, since we are really trying to show only how to argue correctly with modal (tense) operators. I take an instrumentalist (formalist) attitude to the sentences of the extensional languages in which the semantics of the intensional languages are given. See Forbes [1981a].

[28] The restriction of reference to actual objects is merely a convenience; it is not difficult to accommodate such descriptive names as 'β'.

[29] Some find it natural to formulate the conditional premises of the argument as counterfactuals of the form

$$\phi_i(\alpha)\Box\!\!\rightarrow \Diamond\phi_{i+1}(\alpha)$$

and although this makes the analogy with the Tall Man more remote, essentially the same points hold. First, to handle translation of counterfactuals into L_c, we need to define a new operation Rel* thus:

> if there are terms $t_1 \ldots t_n$ in A outside the scope of modal operators in A, then
>
> $$\text{Rel}^*(A, w) = (\exists w')(\exists v_1)\ldots(\exists v_n)(Cv_1t_1w' \;\&\ldots\&\; Cv_nt_nw' \;\&\; \text{Rel}(A(v_i/t_i), w)),$$
> $$= \text{Rel}(A, w) \text{ if there are no such terms.}$$

We now expand L_c by adding the three-place relation symbol $S\zeta_1\zeta_2\zeta_3$ for comparative similarity, of sort $\langle 1, 1, 1\rangle$, and a degree model is concomitantly understood to be a 10-tuple whose new component is a function $S: W^3 \rightarrow \{0, 1\}$. The Lewis-Stalnaker analysis of '$\Box\!\!\rightarrow$' then motivates the following:

$$\text{Rel}(A\Box\!\!\rightarrow B, w) = (\exists w_1)(\text{Rel}^*(A \;\&\; B, w_1) \;\&\; (\forall w_2)(\text{Rel}^*(A \;\&\; {\sim}B, w_2 \rightarrow Sw_1ww_2)).$$

Curiously, this renders every counterfactual in the quasi-Sorites argument wholly false, excepting only $\phi_1(\alpha)\Box\!\!\rightarrow\Diamond\phi_2(\alpha)$ (reader's exercise: why?). The problem lies with Goguen's clause for Imp, which makes every conditional with a wholly false consequent itself wholly false, provided just that the antecedent has some degree of truth, regardless of how little. However, for the purposes of handling our argument, I see no objection to replacing Imp with a generalized version of clause (iv) in §3. Then this counterfactual version of Chisholm's Paradox will not lead us to the paradoxical conclusion, since we will resist the inference from "possibly P' and 'if it had been that P it would have been possible that Q' to 'it could have been that Q' for the familiar reason, without appeal to accessibility.

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