CROSS SECTIONS FOR THE PHOTODISINTEGRATION OF A DEUTERON AND NEUTRON CAPTURE BY A PROTON CALCULATED FOR THE CASE OF NONLOCAL POTENTIAL

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The processes of deuteron photodisintegration by a gamma-quantum and the radiative capture of a neutron by a proton with the emission of a gamma-quantum are considered. Interaction between nucleons is described by a nonlocal potential of the Yamaguchi type but allowing for repulsion due to the nucleon cores. In contrast to other potentials, the Schroedinger equation is solved exactly for the proposed potential. The potential is more exactly defined in comparison with the previously obtained values for the parameters; information on this potential is important in solving certain fundamental problems in nuclear theory. The effective cross sections for photodisintegration of a deuteron and for radiative capture are computed. Calculations show that the proposed potential makes it possible to describe the photodisintegration processes quite accurately for intermediate (up to 20 MeV) energies. The cross section computed for radiative capture is in somewhat better agreement with experiment than is the same cross section determined for other potentials.

To determine the total deuteron photodisintegration cross section it is necessary to compute the matrix elements of the transition of two nucleons from the bound state (deuteron) to the free state. To calculate the radiative capture of a neutron by a proton it is necessary to know the matrix elements of the reverse transition from the free to the bound state. For this purpose we must know the deuteron and nucleon wave functions in the scattering case. These functions are obtained by solving the Schroedinger equation with a nucleon-nucleon potential interaction whose exact form is unknown. Many of the forms proposed for the approximate description of this potential qualitatively allow for the fact that it consists of a comparatively short-range interacting part, corresponding to the meson cloud of the nucleon, and an even more short-range repulsive part, corresponding to the nucleon core, which possesses high rigidity with respect to high-energy nucleon-nucleon collisions. However, in addition to its clear participating in high-energy processes, the nucleon core plays an important role (although not as clear) in low-energy processes and in problems of the nucleon bound states. Thus, Weisskopf and his colleagues [1] showed that for justification of the cloud model it is necessary to take into account the presence of the repulsive nucleon core, while in the opposite case the nucleus must undergo "collapse". It proves to be essential to allow for the core when considering the problem of three and four nucleons [2]. In analyzing experimental data on the polarization of scattered nucleons, we should also consider the repulsion between nucleon cores, where the allowance for repulsion in accordance with the simple model of solid spheres surrounded by strong forces of attraction does not yield a satisfactory result; better agreement is obtained if there is a horizontal "area" connecting the two parts of the potential, between the attractive and repulsive parts of the empirical potential [2]. This forces us to assume that in the transition from the repulsive to the attractive part of the potential there is no sharp break such as is obtained upon introducing the core model in the case of a solid sphere, and that it is more accurate to consider repulsion as a very rapid but smoothly decreasing distance function. For the majority of potentials allowing for the qualitative features of nucleon interaction, the Schroedinger equation is solved only approximately. Yamaguchi [3] has proposed a specific form of nonlocal potential for which the Schroedinger equation is solved exactly. However, the repulsive core is not taken into account in this potential. In [4] the present authors consider the Yamaguchi type of potential but with an additional repulsive part to allow for the core; the corresponding Schroedinger equation is solved, and the parameters of the potential in question are determined. Here, we shall give the completed and adjusted* basic data of [4] and then use them to determine the photo-disintegration and radiative capture cross sections.

Consider a potential of the form

$$\langle p|V|p'\rangle = -\frac{\lambda_1}{M} g(p) g(p') + \frac{\lambda_2}{M} v(p) v(p') \quad (1)$$

in the momentum representation. This is a nonlocal potential with separable variables, where

$$g(p) = 1/(p^2+\mu^2), \quad v(p) = 1/(p^2+\nu^2). \quad (2)$$

The Schroedinger equation with such a potential has the form

$$(p^2-k^2)\psi(p) = \lambda_1 g(p)\int g(p')\psi(p')\,dp' - \lambda_2 v(p)\int v(p')\psi(p')\,dp' \quad (3)$$

in the momentum representation (k is the initial momentum, p and k are vectors). Taking $k^2 = -\alpha^2$,

*In [4], formulas for nucleon-nucleon scattering, through an oversight, were not copied exactly as they appeared in the calculations. The numerical values of the potential parameters computed according to the true formulas which were given earlier in [4] are correct. In place of the formulas for a and r_0 in [4], Eqs (15)–(18) of the present paper should be used.

where α^2/M is the deuteron binding energy, we immediately obtain an expression for the deuteron wave function (allowing only for the s-state, consisting of 96% of the total state):

$$\psi(p) = N\,\frac{g(p) - \eta v(p)}{\alpha^2 + p^2},\ \eta = \frac{\nu + \alpha}{\mu + \alpha},\ N = \text{const.} \qquad (4)$$

Allowing for (2) and changing to a coordinate representation, we obtain the deuteron wave function as a function of the relative distance between nucleons:

$$\Psi(r) = (\pi/2)^{1/2} N\left\{\frac{1}{\mu^2 - \alpha^2}\,\frac{e^{-\alpha r} - e^{-\mu r}}{r} - \right.$$

$$\left. - \frac{\eta}{\nu^2 - \alpha^2}\,\frac{e^{-\alpha r} - e^{-\nu r}}{r}\right\}. \qquad (5)$$

This function resembles the Hulthen wave function but with a small addition introduced by the repulsive part of the potential in (1). In accordance with the sense of the introduced repulsive part of the potential, we have

$$\Psi(0) = 0. \qquad (6)$$

$\Psi(r)$ may be rewritten in the form

$$\Psi(r) = \frac{N_g}{r}\{e^{-\alpha r} - \gamma_1 e^{-\mu r} + \gamma_2 e^{-\nu r}\},$$

$$\gamma_1 = \frac{\nu - \alpha}{\nu - \mu},\ \gamma_2 = \frac{\mu - \alpha}{\nu - \mu}. \qquad (7)$$

Turning to the scattering case, we seek a wave function with initial momentum k in the form

$$\psi_k(p) = \delta(p - k) - \frac{f(k, p)}{2\pi^2(k^2 - p^2 + i\varepsilon)}. \qquad (8)$$

In the scattering case $\Psi_k(p)$ satisfies the following integral equation obtained from the Schroedinger equation [5]:

$$\psi_k(p) = \varphi_k(p) +$$

$$+ \frac{M}{k^2 - p^2 + i\varepsilon}\int \langle p|V|p'\rangle\,\psi_k(p')\,dp', \qquad (9)$$

$$\varphi_k(p) = \delta(p - k), \qquad (10)$$

$$\langle p|V|p'\rangle = \int \varphi_p(q)\langle q|V|q'\rangle\,\varphi_{p'}(q')\,dq\,dq'. \qquad (11)$$

We take [5] as the scattering operator determined as follows:

$$\langle p|T|p'\rangle = \int \varphi_p^*(q)\langle q|V|q'\rangle\,\psi_{p'}(q')\,dq\,dq', \qquad (12)$$

which, in accordance with (9), clearly satisfies the equation

$$\langle p|T|k\rangle = \langle p|V|k\rangle + \qquad (13)$$

$$+ M\int \frac{\langle p|V|p'\rangle\langle p'|T|k\rangle}{k^2 - p'^2 + i\varepsilon}\,dp'.$$

Allowing for (1) we give $\langle p|T|k\rangle$ in the form [5]

$$M\langle p|T|k\rangle = Ag(p)\,g(k) + Bg(p)\,v(k) +$$

$$+ B'g(k)\,v(p) + Cv(p)\,v(k). \qquad (14)$$

Substituting (14) into (13) and allowing for (1), (9), and (8), we obtain after certain manipulations

$$f(k, p) = -2\pi^2 M\langle p|T|k\rangle =$$

$$= 2\pi^2\left\{\lambda_1 g^2(k) - \lambda_2 v^2(k) - \right.$$

$$- 2\pi^2\lambda_1\lambda_2 g^2(k)\,v^2(k)\,\frac{(\mu - \nu)^2}{2(\mu + \nu)}\,\frac{k^2 - \mu\nu}{\mu\nu} \times$$

$$\times\left\{-ik\cdot 2\pi^2\left[\lambda_1 g^2(k) - \lambda_2 v^2(k) - \right.\right.$$

$$\left. - 2\pi^2\lambda_1\lambda_2 g^2(k)\,v^2(k)\,\frac{(\mu - \nu)^2}{2(\mu + \nu)}\,\frac{k^2 - \mu\nu}{\mu\nu}\right] +$$

$$+ 1 + 2\pi^2\left[\lambda_1 g^2(k)\,\frac{k^2 - \mu^2}{2\mu} - \lambda_2 v^2(k)\,\frac{k^2 - \nu^2}{2\nu} - \right.$$

$$- 2\pi^2\lambda_1\lambda_2 g^2(k)\,v^2(k)\left(\frac{(k^2 - \mu^2)(k^2 - \nu^2)}{4\mu\nu} - \right.$$

$$\left.\left.\left.\left. - \frac{(k^2 - \mu\nu)^2}{(\mu + \nu)^2}\right)\right]\right\}^{-1}, \qquad (15)$$

and

$$k\,\mathrm{ctg}\,\delta_0 = \left\{\frac{1}{2\pi^2} + \lambda_1 g^2(k)\,\frac{k^2 - \mu^2}{2\mu} - \lambda_2 v^2(k)\,\frac{k^2 - \nu^2}{2\nu} - \right.$$

$$- 2\pi^2\lambda_1\lambda_2 g^2(k)\,v^2(k)\left[\frac{(k^2 - \mu^2)(k^2 - \nu^2)}{4\mu\nu} - \right.$$

$$\left.\left. - \left(\frac{k^2 - \mu\nu}{\mu + \nu}\right)^2\right]\right\}\left\{\lambda_1 g^2(k) - \lambda_2 v^2(k) - \right.$$

$$\left. - 2\pi^2\lambda_1\lambda_2 g^2(k)\,v^2(k)\,\frac{(\mu - \nu)^2}{2(\mu + \nu)}\,\frac{k^2 - \mu\nu}{\mu\nu}\right\}^{-1}. \qquad (16)$$

Hence we obtain the parameters of the effective radius theory a and r_0,

$$k\,\mathrm{ctg}\,\delta_0 = -\frac{1}{a} + \frac{1}{2}r_0 k^2 + \ldots \qquad (17)$$

$$a = -\frac{m}{l},\ r_0 = 2\left(\frac{n}{m} - \frac{lp}{m^2}\right), \qquad (17^1)$$

where

$$2l = \left\{\frac{1}{\pi^2} - \frac{\lambda_1}{\mu^3} + \frac{\lambda_2}{\nu^3} - \pi^2\,\frac{\lambda_1\lambda_2}{\mu^3\nu^3}\left(\frac{\nu - \mu}{\nu + \mu}\right)^2\right\}(\mu\nu)^4,$$

$$m = \left\{\frac{\lambda_1}{\mu^4} - \frac{\lambda_2}{\nu^4} + \pi^2\lambda_1\lambda_2\,\frac{(\mu - \nu)^2}{\mu + \nu}\,\frac{1}{(\mu\nu)^4}\right\}(\mu\nu)^4, \qquad (18)$$

$$n = \left\{ 2\,\frac{\nu^2+\mu^2}{\pi^2} + \frac{\lambda_1}{\mu^3}\,(\nu^2-2\mu^2) - \frac{\lambda_2}{\nu^3}\,(\mu^2-2\nu^2) + \right.$$

$$\left. + 4\pi^2\,\frac{\lambda_1\lambda_2}{\mu^2\nu^2}\left(\frac{\nu^2+\mu^2}{4\mu\nu} - \frac{2\mu\nu}{(\nu+\mu)^2}\right)\right\}\frac{\mu^2\nu^2}{2},$$

$$p = 2\left\{\lambda_1\nu^2 - \lambda_2\mu^2 - \lambda_1\lambda_2\,\frac{\pi^2}{2\mu\nu}\,\frac{(\nu-\mu)^2}{\nu+\mu}\right\}. \quad (18)$$

(cont'd)

Then the quantities μ, ν, λ_1, λ_2 are completely determined, since α^2, a, r_0 and condition $\Psi(r)|_{r=0} = 0$ due to the core are known; it follows that

$$\lambda_{1t} = \frac{\mu_t}{\pi^2}\,\frac{\nu_t+\mu_t}{\nu_t-\mu_t}\,(\mu_t+\alpha)^2,$$

$$\lambda_{2t} = \frac{\nu_t}{\pi^2}\,\frac{\nu_t+\mu_t}{\nu_t-\mu_t}\,(\nu_t+\alpha)^2. \quad (19)$$

Using the experimental values of α^2, a_i, and r_{0t} [2] (t is a triplet, s a singlet), we obtain

$$1.45\cdot10^{13}(\text{cm})^{-1} \leqslant \mu_t \leqslant 1.48\cdot10^{13}\,(\text{cm})^{-1},$$

$$3\cdot10^{14}(\text{cm})^{-1} \leqslant \nu_t \leqslant \infty.$$

It is clear that the values of the parameters μ and ν previously determined [4] lie within the assigned limits. It turns out that if values for μ and ν are chosen within the limits indicated, the values of various quantities computed from μ and ν agree with experiment within the limits of experimental error. For more accurate values of μ and ν, we should turn either to higher energies or to more refined tests. In accordance with [4], we take

$$\mu_t = 1.453\cdot10^{13}\,(\text{cm})^{-1}, \quad \nu_s = 87.3\cdot10^{13}\,(\text{cm})^{-1}.$$

The singlet value of ν_s is ν_t and that of μ_s is μ_t; we also let $\mu_s = 1.56 \cdot 10^{13}$ cm^{-1}. Knowing the parameters indicated, we can write the wave functions explicitly both for the bound state and for scattering and, consequently, we can determine the photodisintegration and radiative capture cross sections.

Let us turn to calculating the deuteron photodisintegration process when a moving photon (gamma-quantum), by transmitting its energy to a deuteron, causes the latter to split into a proton and a neutron. The total cross section for this process has the form [2]

$$\sigma = \frac{\pi}{3}\,\frac{e^2}{hc}\,\frac{h\nu N_g^2}{k}\left\{\frac{\alpha^2}{\varepsilon}\,[I_{sp}]^2 + \frac{(\mu_p-\mu_n)^2}{Mc^2}\,[K_{ss}]^2\right\}. \quad (20)$$

Here $h\nu$ is the photon energy, $\varepsilon = \alpha^2/M$ is the deuteron binding energy, μ_p and μ_n are the magnetic moments of the proton p and neutron n, I_{sp} and K_{ss} are integrals of the wave functions

$$I_{sp} = \int_0^\infty r u_g(r)\,u_s(r)\,dr, \quad K_{ss} = \int_0^\infty r u_g(r)\,v_0(r)\,dr, \quad (21)$$

$u_g(r) = (r/N_g)\Psi_d(r)$, Ψ_d is the deuteron wave function N_g a normalization constant, $u_s(r)$ is the wave function of the triplet s-state and is asymptotic as $\sin(kr + \pi/2 + \delta_1)$, $v_0(r)$ is the singlet wave function of the s-state. Determining $u_g(r)$ from (6), N_g from (7) and $u_s(r)$ from [2] (Eq. (39.8)) with $\delta_1 = 0$, since we only allow for interaction in the s-state, and taking $v_0(r)$ in accordance with [2] but allowing for repulsion, we have

$$u_g(r) = e^{-\alpha r} - \gamma_1 e^{-\mu r} + \gamma_2 e^{-\nu r}, \quad (22)$$

$$u_s(r)\,\frac{\sin kr}{kr} - \cos kr, \quad (23)$$

$$v_0(r) = \sin kr\cos\delta_{0s} +$$

$$+ \cos kr\sin\delta_{0s}\,(1-\gamma_{1s}e^{-\mu r} + \gamma_{2s}e^{-\nu r}),$$

$$\gamma_{1s} = \frac{\nu_s a_s - 1}{a_s(\nu_s-\mu_s)}, \quad \gamma_{2s} = \gamma_{1s} - 1. \quad (24)$$

Substituting these quantities into (21), we have

$$I_{sp} = 2k^2\,[\rho^2(\alpha) - \gamma_{1t}\rho^2(\mu) + \gamma_{2t}\rho^2(\nu)],$$

$$K_{ss} = \sin\delta_{0s}\,\{k\,\text{ctg}\,\delta_{0s}\,[\rho(\alpha) - \gamma_{1t}\rho(\mu) + \gamma_{2t}\rho(\nu)] +$$

$$+ \alpha\rho(\alpha) - \gamma_{1t}\mu\rho(\mu) + \gamma_{2t}\nu\rho(\nu) -$$

$$- \gamma_{1s}\,[(\mu_s+\alpha)\,\rho(\mu_s+\alpha) - \gamma_{1t}(\mu_s+\mu_t)\,\rho(\mu_s+\mu_t) +$$

$$+ \gamma_{2t}(\mu_s+\nu_t)\rho(\mu_s+\nu_t)] + \gamma_{2s}\,[(\nu_s+\alpha)\,\rho(\nu_s+\alpha) -$$

$$- \gamma_{1t}(\nu_s+\mu_t)\,\rho(\nu_s+\mu_t) + \gamma_2(\nu_s+\nu_s)\,\rho(\nu_s+\nu_t)]\},$$

$$\rho(\alpha) = \frac{1}{k^2+\alpha^2}, \quad (25)$$

$k\,\text{ctg}\,\delta_0$ is given by (16).

Substituting all the quantities into (21) and performing the necessary computations, we obtain the following values for the total photodisintegration cross section σ_{theor} for various values of the photon energy (see the table, where for comparison experimental values for similar photon energies are also

E_γ (MeV)	σ_{theor} 10^{-28} (cm)^2	σ_{exp} 10^{-28} (cm)^2	E (MeV)	σ_{theor} 10^{-28} (cm)^2	σ_{exp} 10^{-28} (cm)^2
2.504	10.08	10.6±1.1	7.39±0.15		18.4±1.5
2.504	10.08	11.9±0.8	7.4	18.3	
4.46	24.34		12.5±0.21		10.4±1
4.45±0.04		24.3±1.7	12.5	10.39	
17.8	6.497		20	5.51	5.1±0.4
17.6		7.1±1.5			

given [2]). For intermediate values of photon energy, the agreement of the theoretical values for the cross section with the experimental ones is no worse than that given above.

Thus, we may assume that the proposed potential (1) makes it possible to describe the photodisintegration process quite accurately at intermediate energies. This case is now being studied for higher energies.

To determine the cross section of radiative capture of a neutron by a proton as a result of which a deuteron is formed and a photon carrying the excess energy is emitted, we use the formula [2] (ignoring the very small contribution of the tensor forces)

$$\sigma = \pi \frac{e^2}{\hbar c}\left(\frac{\varepsilon}{Mc^2}\right)^2 \sqrt{\frac{\varepsilon}{2E}}(\mu_p - \mu_n)^2 \alpha N_g^2 a_s^2 \left[\int_0^\infty u_g(r) u_s(r) dr\right]^2. \quad (26)$$

Here $\varepsilon = \alpha^2/M$ is the deuteron binding energy, M the nucleon mass, μ_p and μ_n the magnetic moments of the proton and neutron, respectively, a_s the single scattering length, and N_g a normalized constant of the deuteron wave function. $u_s(r)$ is given by

$$u_s(r) = 1 - \frac{r}{a_s} - \gamma_{1s} e^{-\mu_s r} + \gamma_{2s} e^{-\nu_s r}.$$

Substituting all these for μ and ν into (26) and performing the necessary calculations, we obtain for $\mu_t = 1.453 \cdot 10^{13}\text{cm}^{-1}$

$$\mu_s = 1.56 \cdot 10^{13} (\text{cm})^{-1}, \quad \nu_s = \nu_t = 87.3 \cdot 10^{13}\ \text{cm}^{-1},$$

$$\sigma_{theor} = 0.326 \cdot 10^{-24}\,\text{cm}^2,$$

while for

$$\mu_s = \mu_t = 1.453 \cdot 10^{13}\text{cm}^{-1} \quad \sigma_{theor} = 0.3222 \cdot 10^{-24}\,\text{cm}^2.$$

At the same time, the experimental value of the capture cross section is

$$\sigma_{exp} = (0.329 \pm 0.006) \cdot 10^{-24}\,\text{cm}^2,$$

while the theoretical value given in [2] is

$$\sigma_{theor} = (0.3137 \pm 0.010) \cdot 10^{-24}\,\text{cm}^2.$$

The best value for σ computed by Yamaguchi is $0.321 \cdot 10^{-24}\ \text{cm}^2$. Comparing the above radiative capture cross sections, we may conclude that the cross section computed in this paper is in somewhat better agreement with experiment than are the others. This evidently means that the proposed potential has certain advantages over other forms of the phenomenological potential of nucleon interaction.

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