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Improving the computational efficiency of the ambiguity function algorithm

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ABSTRACT

Techniques are described in this paper for improving the Ambiguity Function Method (AFM) for differential GPS positioning using phase observations, (a) that take advantage of optimal dual-frequency observable combinations to improve the *reliability* of the AFM, and (b) that significantly shorten the computation time necessary for the AFM. The procedure can be used for kinematic positioning applications if a Kalman filter predicted position is accurate enough as an initial position for the suggested AFM searching procedure, or pseudo-kinematic mode using say a triple-difference solution as an initial position for static positioning if the baseline length is short (typically <5km).

INTRODUCTION

The Ambiguity Function Method (AFM) as a technique for the rapid determination of a GPS baseline using carrier phase data has both advantages and disadvantages. Because the AFM is insensitive to the presence of cycle slips in the carrier phase observations it has attracted much interest for its possible applications in GPS positioning. Counselman & Gourevitch (1981) first proposed the AFM technique, Remondi (1984) described the procedure appropriate for static GPS positioning, and subsequently pseudo-kinematic positioning (Remondi 1989; Remondi & Hilla 1993). Mader (1990; 1992) applied the AFM technique for GPS initialization, rapid static and kinematic positioning. Han (1994a; 1994b) extended the AFM in several respects: derived an expression for accuracy estimation, proved the equivalence of the AFM and the Least Squares ambiguity searching method, and described a strategy for kinematic positioning based on a combination of the AFM and a Kalman filter.

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There are two significant problems with using the AFM to determine GPS baselines. The first problem is the long computation time required to determine the optimal position because of the need for the searching step to be much less than the carrier phase wavelength (for example, 19cm for the L_1 observation, and 24cm for the L_2 observation). The second problem is that there may be several maxima points that the AFM algorithm must discriminate between within the search volume in order to identify the optimal position. If a high accuracy initial position can be obtained, the size of the search volume could be reduced, hence decreasing the number of possible positions to be tested (if the same step size is used). In this paper, the authors seek to determine what level of accuracy of the initial position is sufficient to obtain the sole maximum point for AFM. However the assumption is that this initial position is available from some other techniques, eg. Kalman filter predicted position for kinematic positioning applications, triple-difference solution in pseudo-kinematic mode for static positioning applications. On the other hand, if a phase observable with a longer wavelength is used, the step size can be increased, the number of possible positions to be tested decreases, and hence the search time can be shortened. Use of the wide-lane observable has been suggested by many investigators (Hofmann-Wellenhof et al. 1992; Cocard & Geiger 1992; Abidin 1993; Seeber 1993). In this paper, the authors seek to determine the maximum searching step size which will ensure the smallest number of grid points to be tested. In order to answer these two questions, the AFM algorithm is introduced and the accuracy estimate formulae are first presented. The characteristics of several dual-frequency carrier phase combinations are investigated and the *optimal* phase observables are then identified. The relationship between the searching step size and the distance between AF maxima is investigated. Finally, a rapid AF computation procedure is proposed.

THE BASIS OF THE AMBIGUITY FUNCTION METHOD

Mathematical details of the Ambiguity Function Method are found in, for example, Counselman & Gourevitch (1981) and Remondi (1984). The procedure may be described in terms of the following mathematical formulae:

$$AF(X) = \sum_{k=1}^{m}\sum_{l=1}^{n} AF_{kl}(X) \tag{1}$$

where

$$AF_{kl}(X) = \left|\sum_{j=1}^{m_{kl}} \exp(\vec{i}L_{kl}^{j}(X)\right| \tag{2}$$

$$L_{kl}^{j}(X) = \Delta\varphi_{obs}^{jl}(k) - \Delta\varphi_{calc}^{jl}(k,X) \tag{3}$$

$$\begin{aligned}\Delta\varphi_{calc}^{jl}(k,X) = {} & \frac{2\pi f_1}{c}(\rho_1^j(k) - \rho_2^j(k,X)) + \\ & +2\pi(f_1\delta_k - (f_{R_1}^k - f_{R_2}^k)\varsigma_k + N^{jl}) + \\ & +\frac{2\pi f_1}{c}(\dot{\rho}_1^j(k) - \dot{\rho}_2^j(k,X))\varsigma_k - \\ & -\frac{\pi f_1}{c}(\dot{\rho}_1^j(k) + \dot{\rho}_2^j(k,X))\delta_k \end{aligned} \tag{4}$$

and m is the number of epochs; n is the number of carrier frequencies; c is the velocity of microwaves; m_{kl} is the number of "l" frequency carrier phase observations at epoch k; X are the trial coordinates of station 2; $\vec{i} = \sqrt{-1}$; $\Delta\varphi_{obs}^{jl}(k)$ is the "l" frequency single-differenced observable for satellite j at epoch k between two stations; $\Delta\varphi_{calc}^{jl}(k,X)$ is the computed value of the "l" frequency single-differenced observable using the known coordinates of station 1, the trial coordinates of station 2 (the second term is not computed because it is a constant for all observations at epoch k and frequency "l", and does not effect the AF value); $\rho_1^j(k)$ and $\rho_2^j(k,X)$ are the distances from station 1 and station 2 to satellite j respectively; ζ_k and δ_k are the common and the relative time-tag errors (of the two receiver clock biases) at epoch k; f_{R_1} and f_{R_2} are the frequencies of two receivers; f_l is the carrier frequency for "l" signal; and N^{jl} is the single-differenced ambiguity for satellite j and frequency "l". Eqn (4) is the carrier phase observation model proposed by Remondi (1984).

The function defined by eqn (1) is the Ambiguity Function (AF). If there are no errors in the observations, the expectation of the AF should be the value $\sum_{k=1}^{m}\sum_{l=1}^{n} m_{kl}$.

The AFM for determining the optimal solution $\hat{X}$ requires that a search be made to find the location within a search volume where the value of the function $AF(X)$ is a maximum. The standard deviation of unit weight is defined below, and details of the derivation are given in Han (1994a):

$$m_0 = \sqrt{2(\sum_{k=1}^{m}\sum_{l=1}^{n} m_{kl} - AF(\hat{X}))/(\sum_{k=1}^{m}\sum_{l=1}^{n}(m_{kl}-1)-3)} \tag{5}$$

The co-factor of $\hat{X}$ has the following expression:

$$Q_X = (\sum_{k=1}^{m}\sum_{l=1}^{n}(D_{kl}B_{kl})^T P_{kl}(D_{kl}B_{kl}))^{-1} \tag{6}$$

where

$$B_{kl} = (B_{kl}^1 \cdots B_{kl}^j \cdots B_{kl}^{m_{kl}})^T \tag{7}$$

$$D_{kl} = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ \cdots & & \cdots & & \cdots & & \cdots \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}_{(m_{kl}-1)\times m_{kl}} \tag{8}$$

$$P_{kl} = (D_{kl}D_{kl}^T)^{-1} \tag{9}$$

and B_{kl}^j is the difference between the unit vectors from station 1 and station 2 to satellite j. The single-differenced phase observations are assumed to be independent. The variance-covariance of the baseline is:

$$D_X = m_0^2 \cdot Q_X \tag{10}$$

DUAL-FREQUENCY CARRIER PHASE OBSERVABLES

There are many linear combinations of the L_1 and L_2 carrier phase observations φ_1 and φ_2 that can, in principle, be used. The so-called "ionosphere-free" observable and the "wide-lane" φ_1-φ_2 observable, are used widely for special applications such as kinematic positioning and ultra-precise long baseline solutions. This section identifies those characteristics that can be used to systematically investigate possible dual-frequency combinations. Ultimately, the purpose of this study is to find the best dual-frequency combination (or combinations) that can be used in conjunction with the AFM procedure.

The basic observation equation for the L_1 and L_2 phase measurements can be written as:

$$L_1: \varphi_{1[cy]} = \rho / \lambda_1 - N_1 \quad (11)$$

$$L_2: \varphi_{2[cy]} = \rho / \lambda_2 - N_2 \quad (12)$$

where ρ is the distance from station to satellite; N_1 and N_2 are the integer ambiguities for φ_1 and φ_2 respectively. λ_1 (=c/f_1) and λ_2 (=c/f_2) are the wavelengths of the L_1 and L_2 carrier waves (f_1=1575.42MHz, f_2=1227.60 MHz). The equation for a linear combination of two phase observations can be expressed as (Hofmann-Wellenhof et al. 1992; Seeber 1993):

$$\varphi_{[cy]} = i \cdot \varphi_{1[cy]} + j \cdot \varphi_{2[cy]} = (i/\lambda_1 + j/\lambda_2) \cdot \rho - (i \cdot N_1 + j \cdot N_2) \quad (13)$$

where i and j are arbitrary integer values. The effective wavelength, frequency and integer ambiguity of the linear combination can be written as:

$$\lambda = \frac{1}{(i/\lambda_1 + j/\lambda_2)} = \frac{c}{(i \cdot f_1 + j \cdot f_2)} \quad (14)$$

$$f = i \cdot f_1 + j \cdot f_2 \quad (15)$$

$$N = i \cdot N_1 + j \cdot N_2 \quad (16)$$

If the wavelength of L_2 carrier phase observation is half that of λ_2, denoted by λ_{2c}, the following relation holds:

$$L_2: \varphi_{2c[cy]} = \rho / \lambda_{2c} - N_{2c} \quad (17)$$

and

$$\varphi_{2[cy]} = \frac{1}{2}\varphi_{2c[cy]} = \rho / \lambda_2 - \frac{1}{2} N_{2c} \quad (18)$$

$$N = i \cdot N_1 + \frac{j}{2} \cdot N_{2c} \quad (19)$$

therefore j must be an even integer. This will be a criterium to be applied to judge which dual-frequency combinations are better.

1. Ionospheric Delay

The ionospheric delay on φ_1 and φ_2 can be approximated by:

$$I_{1[m]} = -E / f_1^2 \quad (20)$$

$$I_{2[m]} = -E / f_2^2 \quad (21)$$

where E is a function of the Total Electron Content (expressed as the number of free electrons per square metre). Using eqn (13), the following equations can be derived:

$$I_{[m]} / \lambda = i \cdot I_{1[m]} / \lambda_1 + j \cdot I_{2[m]} / \lambda_2 \quad (22)$$

$$I_{[m]} = K \cdot I_{1[m]} \quad (23)$$

where

$$K = \frac{4620 \cdot i + 5929 \cdot j}{4620 \cdot i + 3600 \cdot j} \quad (24)$$

2. Random Errors

If we assume that the standard deviations of the random errors on both frequencies are equal to M_0(cy), expressed in units of cycles of the corresponding wavelength, the standard deviation M(cy) of the linear combination is (eqn (13)):

$$M_{[cy]} = \sqrt{(i^2 + j^2)} \cdot M_{0[cy]} \quad (25)$$

$$M_{[m]} = M_{[cy]} \cdot \lambda \quad (26)$$

These formulae clearly show that the random error, expressed in cycles of the effective wavelength, is always greater than the noise on either L_1 or L_2.

3. Characterisation of Dual-Frequency Observables

We introduce the following parameters for characterising dual-frequency observables (Abidin 1993; Seeber 1993):

(a) Wavelength Parameter:

$$WL = \lambda / \lambda_1 = \frac{77}{77 \cdot i + 60 \cdot j} \quad (27)$$

(b) Ionospheric Parameter (refer to eqn(23)):

$$IS = I_{[m]} / I_{1[m]} = K \quad (28)$$

If we choose $IS = 0$, the values of i = 77 and j = -60 from eqn (24) will produce an ionosphere-free observable:

$$\varphi_3 = 77 \cdot \varphi_1 - 60 \cdot \varphi_2 \tag{29}$$

For half wavelength carrier phase observations, the same φ will be obtained as the value of j (= -60) is even (see eqn (19)).

(c) Random Error Parameters:

$$RE_1 = M[cy] / M_1[cy] = \sqrt{i^2 + j^2} \tag{30}$$

and

$$RE_2 = M[m] / M_1[m] = \sqrt{i^2 + j^2} \cdot \frac{77}{77 \cdot i + 60 \cdot j} \tag{31}$$

indicating random error in relation to the L_1 or L_2 measurements ($RE_1 = 1$).

IMPROVING THE COMPUTATIONAL EFFICIENCY OF THE AFM

There are several strategies that can be adopted for increasing the computational speed of the AFM. One of the most effective strategies is to simply reduce the number of mathematical operations that are necessary to find the position that maximises the value of the AF. There are basically two components: (a) the size of the searching volume, and (b) the step size of the grid of candidate positions that must be tested.

In order to make the searching step size longer and the distance between the AF maxima points greater (see comments with regard to Figures 1 to 4), we can use long wavelength dual-frequency combinations. However, the longer the wavelength, the higher the noise. Let us choose the following specifications for the dual-frequency combinations we wish to consider further (eqns (27, 28 and 30)):

$$WL > 1.284 \text{ or } \lambda > \lambda_2$$

$$IS < 20$$

$$RE_1 \le 5$$

The linear combinations that satisfy the above criteria are those with values of i and j of (-3,4), (-2,3), (-1,2), (1,-1), (2,-2), (-3,3). The combination of (-2,3) and (-3,3) are no better than the others and have not been selected. Furthermore, as j is odd it cannot be used for half wavelength L_2 phase observations. The following are therefore the dual-frequency combinations of interest for AFM studies:

$$\varphi_{1,-1} = \varphi_1 - \varphi_2 \tag{32}$$

$$\varphi_{-3,4} = -3 \cdot \varphi_1 + 4 \cdot \varphi_2 \tag{33}$$

$$\varphi_{-1,2} = -\varphi_1 + 2 \cdot \varphi_2 \tag{34}$$

$$\varphi_{2,-2} = 2 \cdot \varphi_1 - 2 \cdot \varphi_2 \tag{35}$$

If we were not concerned with the parameters IS and RE_1, the longest wavelength combination (14.6526m) can be obtained for i = -7 and j = 9, which make $77 \cdot i + 60 \cdot j = 1$ in eqn (27):

$$\varphi_{-7,9} = -7 \cdot \varphi_1 + 9 \cdot \varphi_2 \tag{36}$$

Each of these combinations has different characteristics (wavelength factor, noise, or suitable for half wavelength L_2 phase observations, etc.) that may make them suitable for different applications. Pertinent details of these linear combinations (including the ionosphere-free combination) are given in Table 1.

Table 1. Characteristics of Certain Dual-Frequency Combinations (full wavelength L_2)

i	j	f (MHz)	λ (m)	WL	IS	RE_1	RE_2
1	0	1575.42	0.1903	1.0000	1.0000	1.0000	1.0000
0	1	1227.60	0.2442	1.2833	1.6469	1.0000	1.2833
77	-60	47651.34	0.0063	0.0331	0.0000	97.6166	3.2273
1	-1	347.82	0.8619	4.5294	-1.2833	1.4142	6.4056
-1	2	879.78	0.3408	1.7907	2.8054	2.2361	4.0041
2	-2	695.64	0.4310	2.2647	-1.2833	2.8284	6.4056
-3	4	184.14	1.6281	8.5556	18.2518	5.0000	42.7778
-7	9	20.46	14.6526	77.0000	350.3500	11.4018	877.9350

1. Comments on the Dual-Frequency Observables

Let us successively fix two components of the known baseline vector X, and change the third component in the region of $\pm 3\lambda$, and compute the values of the AF (eqn (1)) for step sizes of 0.1λ. First we consider the x-component, holding the y- and z-components fixed, then repeat by considering the y-component holding the x- and z-components fixed, etc. The results are shown in Figures 1 to 4. Note the different horizontal scale used based on units of λ, not metres. We can draw the following conclusions:

(1) These combinations produce AF values that have well defined maxima. The maximum value is related to the accuracy of the combined phase observations. The more accurate the combination (the lower RE_1 value in Table 1), the larger the maximum. The ionosphere-free combination cannot be used for AFM.

(2) The maxima are similar to each other for all these combinations. The form of the AF curve is related to the satellites' and stations' geometry, and appears not to be strongly dependent on the dual-frequency phase combination considered.

(3) If we use the dual-frequency observations as independent observations (eqn (1) where n=2 and Figure 2), the maximum is 12 and the other local maxima are almost less than 50% of the expectation. Obviously, the AF pattern is not the same as for the other dual-frequency combinations (where i and j are both non-zero).

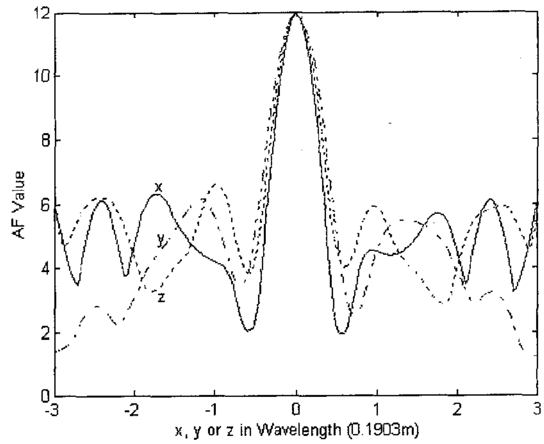

Fig. 2. Maxima points of AFM (L1/L2)

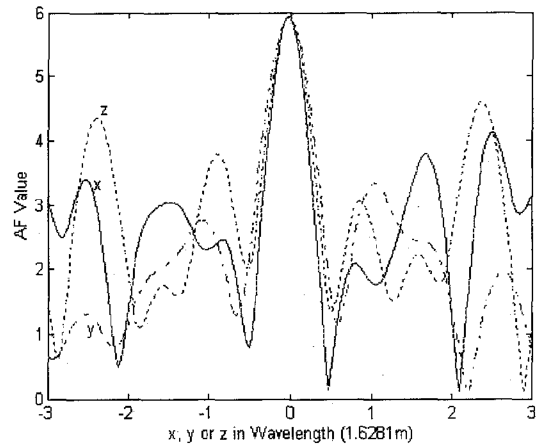

Fig. 3. Maxima points of AFM (i=-3,j=4)

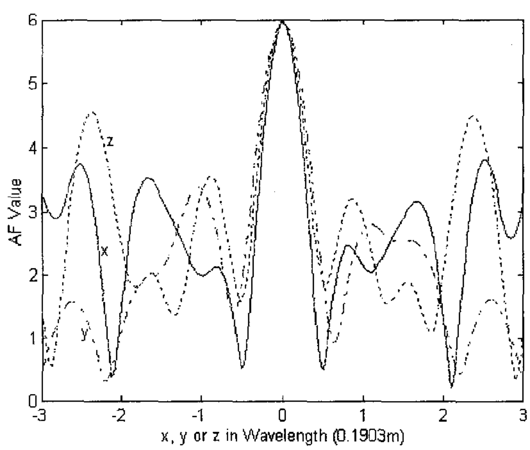

Fig. 1. Maxima points of AFM (L1)

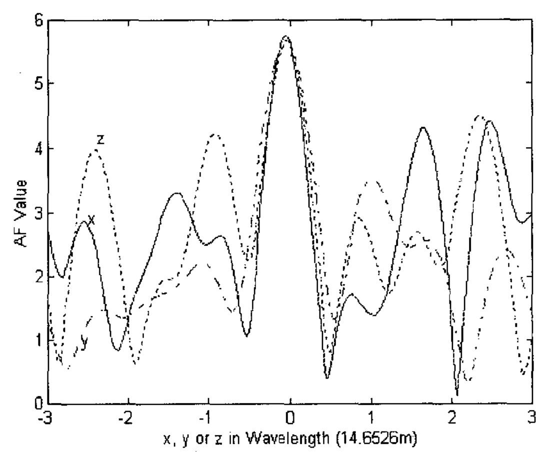

Fig. 4. Maxima points of AFM (i=-7,j=9)

2. Determination of the Maximum Searching Step Size

If the searching step size is too large it is possible to miss the AF's maximum, and hence the search procedure fails. On the other hand, if the searching step size is too small, the computation time will be long (perhaps unacceptably so). Figure 5 shows the one-dimensional case of the searching procedure. If we select 95% of the expectation as a criterium, the distance d in Figure 5 should be the longest searching step size. If the searching step size is longer than d, it is possible to miss the maximum point. If the searching step size is shorter than d, more computations will be needed.

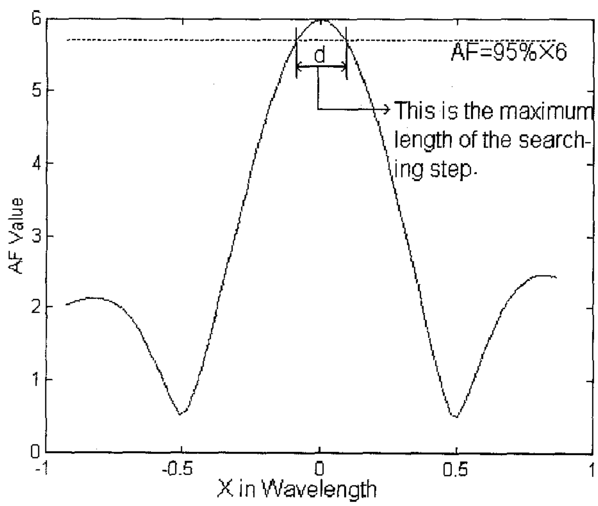

Fig. 5. One-dimensional Case in the Search for the Maximum Point

The task in this section is to determine the appropriate searching step size for each dual-frequency combination. The suggested procedure is as follows. The first step is to use the particular dual-frequency observable to search for the AF maximum within a volume centred at an approximate position and estimate the optimal position. The second step is to define several testing regions around the AF maximum (column 1 in Table 2) considering this position as the initial value. These testing regions are in fact possible step sizes for AF searching. A much finer testing step (0.1 of the testing region and having 11^3=1331 testing points) is used to evaluate the AF. If the AF is less than 95% of the AF expectation, then the point is rejected. If the AF is equal to or more than 95% of the AF expectation, it is accepted. If the testing region (or search step size) is too large, then there will be an unacceptably large number of rejected points. The step size for which less than 2% of the total tested points are rejected is considered the reasonable one to use with this phase combination. It must be emphasised that we are looking for the *maximum* step size appropriate for the phase combination. Hence choosing a step size where there are no rejected points will imply too small a step size and hence too long a computation procedure for the AFM. As shown in Table 2, the testing region of ±1.0 cm can be considered the optimal one. Therefore, the best searching step size should be 2cm for the φ_1 observation. Suggested searching step sizes for the other combinations are given in Table 3. This result has been verified for different sets of data (different number of satellites, different baseline length, etc.), hence the present results are indicative of the procedure and conclusions that can be drawn.

Table 2. Determination of the Searching Step Size for φ_1

Testing Region (cm)	Testing Step (mm)	No. of Testing Points	No. of Accepted points	No. of Rejected Points
±1.2	2.4	1331	1204	127
±1.1	2.2	1331	1274	57
±1.0	2.0	1331	1313	18
±0.9	1.8	1331	1330	1
±0.8	1.6	1331	1331	0

Table 3. Searching Step Size for Various Dual-Frequency Combinations

i	j	Region (cm)	Testing Step(mm)	No. of Accepted	No. of Rejected	Suggested Step (cm)
1	0	±1.00	2.00	1313	18	2.00
0	1	±1.25	2.50	1322	9	2.50
1	-1	±4.30	8.60	1329	2	8.60
-3	4	±8.00	16.00	1319	12	16.00
-7	9	±36.00	72.00	1329	2	72.00
φ_1/φ_2		±1.00	2.00	1330	1	2.00

We can draw the following conclusions:

(1) The longest searching step size of a combination is of the order of 0.1λ. Hence, the longer the effective wavelength, the greater the searching step size. However, in the case of the observable $\varphi_{-7,9}$, the searching step size according to Table 3 should be about 0.05λ.

(2) When the L_1 and L_2 observations are processed separately (eqn(1), n=2), the φ_1/φ_2 case, the searching step size is the same as for the L_1 observation alone.

(3) The searching step size of $\varphi_{-3,4}$ is eight times that of the φ_1/φ_2 observations. If we use $\varphi_{-3,4}$ in place of the φ_1/φ_2 observations, the searching time will be reduced to $1/8^3$ for the same region.

3. Determination of the Maximum Search Region

If the ambiguity function value is more than the 95% of its expectation, the point can be considered to be a candidate for the optimal position. If there are more than one candidates (or AF maxima) in the search region, we cannot identify unambiguously the optimal position and the search procedure cannot be terminated successfully. In this case, we should use more observations (to evaluate eqn (1)) or define a smaller search region (though this may not be always possible).

The task in this section is to determine how large the search region should be to contain the sole maximum point. From this maximum region, we hence know what level of accuracy of the initial position is sufficient to contain the sole maximum point for the AFM. The method to determine the maximum search region is to determine the minimum distance among the local maxima whose AF values are larger than 95% of the expectation. If the search region in each coordinate component is less than this minimum distance and the optimal position is located in this region, the sole maximum point will be located in this region.

As an example, let us to determine the appropriate search region for *one-epoch* six-satellite observations (for a 1.5km baseline). We choose a large searching region ($\pm 2\lambda$) and using the suggested searching step size, identify all candidates for the optimal position (those that have local maximum AF values). The positions and the distances between all candidates are listed in Tables 4 to 7 for φ_1, $\varphi_{1,-1}$, $\varphi_{-3,4}$, φ_1/φ_2. In these tables, the first column is the candidate point number; X, Y, Z columns indicate the coordinate corrections from the initial position; the AF column gives the ambiguity function values; Distances columns give the distances between any two candidate points. Table 8 gives the results for another set of data that was investigated (one-epoch, five-satellites, 4.1km baseline) for $\varphi_{-3,4}$, and Table 9 gives the results for the φ_1/φ_2 combination.

Table 4. φ_1 (± 0.40 m, step=2cm, λ=0.1903 metre, 1.5km baseline)

	X	Y	Z	AF	Distances (cycle)			
	(cycle)	(cycle)	(cycle)		1	2	3	4
1	-1.366	0.946	-1.051	5.939				
2	-1.156	0.946	1.261	5.835	2.322			
3	0.000	0.000	0.000	5.976	1.966	1.955		
4	0.946	1.682	1.787	5.792	3.734	2.288	2.630	
5	1.366	-0.946	1.051	5.808	3.933	3.160	1.966	2.761

Table 5. φ_1 - φ_2 (± 1.75m, step=8cm, λ=0.8619 metre, 1.5km baseline)

	X	Y	Z	AF	Distances (cycle)					
	(cycle)	(cycle)	(cycle)		1	2	3	4	5	6
1	-1.381	1.033	-1.009	5.871						
2	-1.195	0.940	1.311	5.828	2.330					
3	-1.009	-1.566	-1.752	5.741	2.728	3.962				
4	0.012	0.012	0.012	5.991	2.006	2.001	2.577			
5	0.940	1.682	1.775	5.704	3.682	2.307	5.176	2.601		
6	1.125	-0.917	-1.288	5.726	3.187	3.948	2.279	1.947	4.021	
7	1.404	-1.009	1.033	5.879	4.012	3.260	3.727	2.006	2.831	2.339

Table 6. -3φ_1+4φ_2 (± 3.25m, step=16cm, λ=1.6281metre, 1.5km baseline)

	X	Y	Z	AF	Distances (cycle)			
	(cycle)	(cycle)	(cycle)		1	2	3	4
1	-1.407	0.952	-1.013	5.897				
2	-0.031	-0.031	-0.031	5.930	1.956			
3	0.952	1.640	1.738	5.843	3.689	2.624		
4	1.149	-0.915	-1.308	5.888	3.178	1.951	3.981	
5	1.443	-1.112	0.952	5.775	4.031	2.075	2.904	2.288

Table 7. φ_1/φ_2 (±3.25 m, step=2cm, 1.5km baseline)

	X (metre)	Y (metre)	Z (metre)	AF	Distances (metre) 1	2	3	4
1	-3.190	1.510	2.750	11.421				
2	0.010	-0.010	-0.010	11.904	4.491			
3	0.270	2.390	-3.210	11.558	6.947	4.008		
4	1.110	-0.710	1.070	11.437	5.123	1.693	5.351	
5	2.150	0.410	2.370	11.438	5.465	3.228	6.212	2.006

Table 8. $-3\varphi_1+4\varphi_2$ (±2.00m, step=16cm, λ=1.6281metre, 4.1km baseline)

	X (cycle)	Y (cycle)	Z (cycle)	AF	Distances (cycle) 1	2	3
1	-0.246	-0.541	-0.934	4.949			
2	-0.049	0.246	0.147	4.976	1.351		
3	0.049	1.032	1.228	4.964	2.690	1.340	
4	0.737	-1.228	-1.228	4.799	1.235	2.164	3.409

Table 9. φ_1/φ_2 (±1.50m, step=2cm, 4.1km baseline)

	X (metre)	Y (metre)	Z (metre)	AF	Distances (metre) 1	2	3
1	-0.600	1.460	-0.580	9.611			
2	0.000	0.000	0.000	9.808	1.682		
3	0.100	0.600	0.840	9.834	1.802	1.037	
4	0.740	-1.280	-1.500	9.578	3.186	2.106	3.069

We can draw the following conclusions:

(1) For φ_1, φ_2 and their combined observations, the minimum distance between the candidates is almost twice their wavelength for one-epoch, six-satellite observations (Tables 4, 5 and 6) and is almost 1.2 times their wavelength for the one-epoch, five-satellite observations (Table 8). Therefore the search region should be of the order of $\pm\lambda$ for six satellites and of the order of $\pm0.6\lambda$ for five satellites.

(2) Using φ_1/φ_2 observations separately, the minimum distance between the candidates is about 1.7 metres for one-epoch, six-satellite observations (Table 7) and is about 1.0 metres for the one-epoch, five-satellite observations (Table 9). Therefore, the search region should be of the order of ±0.85 metres for six satellites and ±0.50 metres for five satellites.

(3) If we use the combination $\varphi_{-3,4}$, the search region (±1.6 metres for six satellites, ±1.0 metres for five satellites) is about twice that for the φ_1/φ_2 (0.85 metres for six satellites and 0.5 for five satellites).

(4) The combination $\varphi_{-7,9}$ has much more noise than $\varphi_{-3,4}$. It is therefore difficult to use in AFM procedures unless an efficient technique is found to account for the ionospheric delay and other biases.

(5) Even when a session of several minutes static data are used, the maximum search region is almost the same as in the case of one-epoch data.

4. A Fast Computational Procedure for the AFM

Based on the above analysis, the search region can be ±1.6 metres for six satellites or ±1.0 metres for five satellites if the combined observable $\varphi_{-3,4}$ is used, and the corresponding searching step size is 16 cm. However, due to the comparatively low accuracy of $\varphi_{-3,4}$, only a low accuracy baseline solution can be obtained. The searching results using the $\varphi_{-3,4}$ combination are given in Row 3 of Table 10 using the initial position given by Row 2 and the suggested step size. Using these results as the new initial position, and setting the search region as three times the standard deviation of the position components, the results of using the φ_1-φ_2 combination are given in Row 4.

Table 10. Results of a Fast Computational Procedure for AFM (1.5 km baseline)

Obs. Type	Region	Step (cm)	Baseline (m) and Standard Deviation (cm)						Time (sec)
			X	m_X	Y	m_Y	Z	m_Z	
Initial			-712.006		431.502		-883.648		
$-3\varphi_1+4\varphi_2$	±1.6 metres	16.0	-712.806	11.10	431.822	16.03	-884.768	10.89	3.90
$\varphi_1-\varphi_2$	$\pm 3(m_{X_{-3,4}}, m_{Y_{-3,4}}, m_{Z_{-3,4}})$	4.0	-712.899	2.27	431.982	3.27	-884.655	2.22	3.02
φ_1/φ_2	$\pm 3(m_{X_{1,-1}}, m_{Y_{1,-1}}, m_{Z_{1,-1}})$	1.0	-712.907	0.43	432.004	0.63	-884.652	0.42	2.63
φ_1/φ_2	$\pm 3(m_{X_{1/2}}, m_{Y_{1/2}}, m_{Z_{1/2}})$	0.2	-712.906	0.35	432.001	0.50	-884.647	0.34	2.14

Table 11. Results of the Standard AFM Procedure

Region (cm)	Step (cm)	Baseline (m)			AF	Time (sec.)
		X	Y	Z		
		-712.006	431.502	-883.648		
±160.0	2.0	-712.906	432.001	-884.648	11.964	2814.22
		-711.786	431.262	-883.588	11.415	
		-710.766	432.422	-882.268	11.408	

Table 12. Results of a Fast Computational Procedure for AFM (4.1km baseline)

Obs. Type	Region	Step (cm)	Baseline (m) and Standard Deviation (cm)						Time (sec)
			X	m_X	Y	m_Y	Z	m_Z	
Initial			-421.000		-2371.000		3369.000		
$-3\varphi_1+4\varphi_2$	±1.0 metres	16.0	-421.840	8.57	-2371.600	14.17	3369.920	21.16	0.77
$\varphi_1-\varphi_2$	$\pm 3(m_{X_{-3,4}}, m_{Y_{-3,4}}, m_{Z_{-3,4}})$	8.0	-421.620	2.35	-2372.020	3.89	3369.580	5.80	0.44
φ_1/φ_2	$\pm 3(m_{X_{1,-1}}, m_{Y_{1,-1}}, m_{Z_{1,-1}})$	1.5	-421.631	0.62	-2371.942	1.02	3369.616	1.53	2.20
φ_1/φ_2	$\pm 3(m_{X_{1/2}}, m_{Y_{1/2}}, m_{Z_{1/2}})$	0.4	-421.626	0.59	-2371.953	0.97	3369.606	1.45	2.14

Finally, using the results of the combination of $\varphi_1-\varphi_2$ to define the search region, the results of using φ_1/φ_2 are given in Row 5, using 1.0 cm as the step size, and in Row 6, using 0.2 cm as the step size. The computing times using an IBM compatible 486 Personal Computer (33MHz clock rate) are given in the final column. The total computation time is 11.69 seconds. If we use the ellipsoidal search region defined by

$$(X-X_0)^T D_{X_0}^{-1}(X-X_0) \le \chi^2_{3,1-\alpha} \tag{37}$$

the searching points can be decreased further, but it requires more computational effort. In eqn (37), X_0 is the previous estimated position with variance-covariance matrix D_{X_0}.

Another test was carried out to compare this computational procedure with the standard AFM procedure. Using the same initial position as given in Row 2 of Table 11, the same search region (±1.6 metres) and the maximum suggested step size (2.0cm), the results of using the dual-frequency observations φ_1/φ_2 are given in Row 3. Three candidates are identified and shown in Row 3, however the correct one cannot be found. Furthermore, such a procedure requires a much longer computation time to perform the search (2814.22 seconds) due to the need to evaluate and test the AF at 4173281 points. The software computing speed is almost the same as given by Lachapelle et al (1992), where a test of 125000 points required several minutes of computation time.

Based on the above analysis, a significantly faster computational procedure has been obtained. The procedure is, in summary:

Step 1: The initial position is obtained and within a volume of ±1.6 metres (for six satellites) or ±1.0 metres (for five satellites) it is assumed that the true position is located. Using $\varphi_{-3,4}$, carry out a search of the position that maximises the AF. In practice, accurate initial position is available from some other techniques, eg. Kalman filter predicted position for kinematic positioning applications (Han 1994b), triple-difference solution in pseudo-kinematic mode for rapid static positioning applications (Remondi 1989; Remondi & Hilla 1993).

Step 2: Based on the results of Step 1, the search region for φ_1-φ_2 can be defined and a more precise position, typically with a standard deviation of 2-8 centimetres, can be determined.

Step 3: Based on the results of Step 2, use φ_1/φ_2 observations for precise searching.

In order to get the correct position for Step 2, the position determined by $\varphi_{-3,4}$ should be located $\pm\lambda$ (0.86 metre) of the true position, for six satellites, or $\pm 0.6\lambda$ (0.52 metre), for five satellites. Because the longer the baseline, the larger the effect of the noise and ionospheric bias in $\varphi_{-3,4}$, the length of a baseline should be restricted. Figures 6 and 7 illustrate the coordinate differences between the coordinates determined using $\varphi_{-3,4}$ and precise coordinates at epoch 1, 5, 15, ..., 50 for the 1.5 km baseline dataset and the 4.1 km baseline dataset respectively. The biases for the 4.1 km baseline are much larger than that for the 1.5 km baseline and are almost 0.5 metre. Therefore, the length of a baseline should be restricted to 4 - 5 kilometres if five satellites are tracked.

Using this procedure, the results for the 4.1 km baseline test at epoch 1 are given in Table 12. If we use the standard AFM approach with φ_1/φ_2 in the same search region, there are two candidates which have almost the same AF values and the correct one cannot be identified. Furthermore, the computation time is 616.37 seconds.

Using this procedure, different epoch data of 1.5 km baseline, at epoch 1, 5, 10, 15, ..., 50 with the same initial position given by Row 1 in Table 10 or 11 have been processed. The optimal positions can be obtained and the results of the differences between the coordinates at different epochs and their mean values are illustrated in Figure 8. We can see that the differences are less than 5mm and the computation time for each epoch is almost 12 seconds. For the other 4.1 km baseline dataset, the results are given in Figure 9. The differences are less than 10mm and the computation time at each epoch is also less than 10 seconds.

With only 10 seconds computation time, the Ambiguity Function Method is very attractive for applications in GPS rapid static positioning and kinematic positioning.

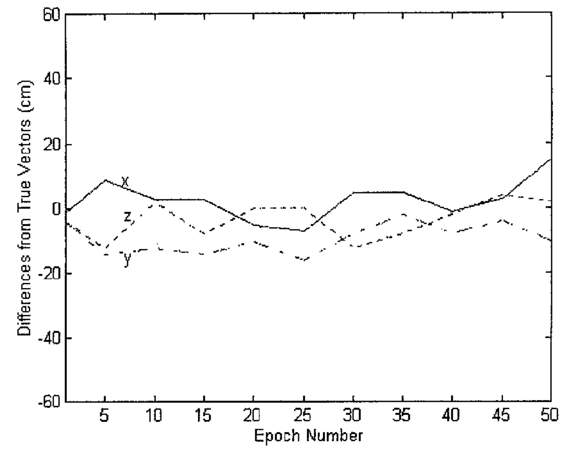

Fig. 6. Vectors of phase combination (-3,4) for different epochs (1.5km baseline test)

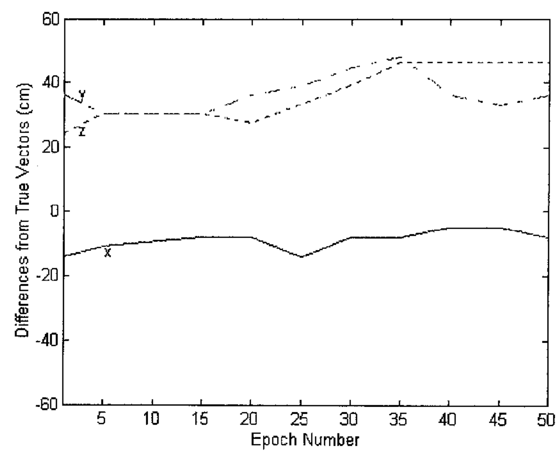

Fig. 7. Vectors of phase combination (-3,4) for different epochs (4.1km baseline test)

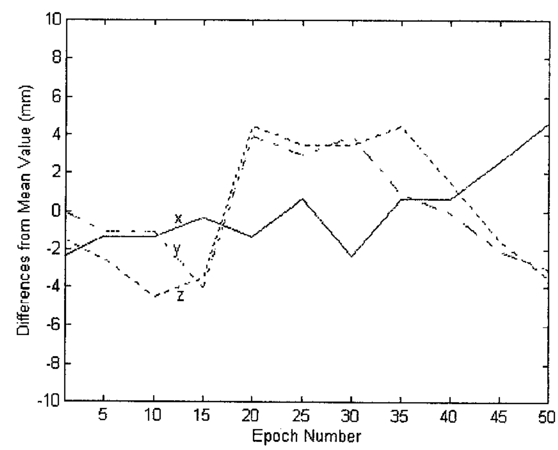

Fig. 8. Results of suggested AF procedure for different epochs (1.5km baseline test)

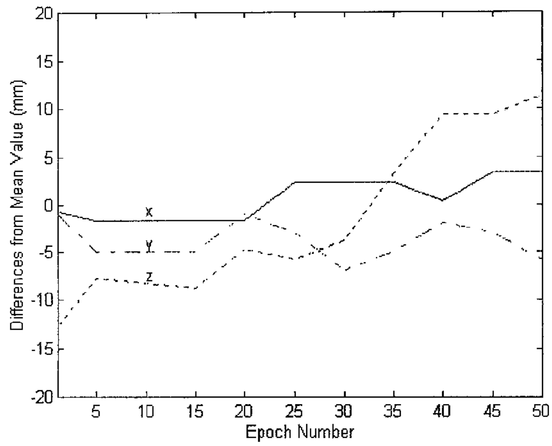

Fig. 9. Results of suggested AF procedure for different epochs (4.1km baseline test)

CONCLUSIONS

Based on the investigations carried out so far, the following conclusions are drawn:

1. Several dual-frequency phase observables can be used in the AFM, in addition to φ_1 and φ_2. Although they (especially $\varphi_{1,-1}$ and $\varphi_{-3,4}$) may have more noise, they can be used to reduce the size of the search region.

2. All observables have the same geometric configuration. This means that they have the same distribution of maxima and the distances between these maxima are proportional to their wavelength.

3. The searching step sizes should be less than one tenth of the observable's wavelength and the search region should be within $\pm\lambda$ for six satellites or $\pm 0.6\lambda$ for five satellites. φ_1/φ_2 can be given a larger search region, but the searching step size should be less than 2 cm, for a rejection level of 95% of expectation.

4. The suggested searching procedure based on the $\varphi_{-3,4}$ observable permits the search region to be twice that of the φ_1/φ_2 and the searching step size to be eight times that of φ_1/φ_2. Therefore, it improves the reliability of the AFM and significantly reduces the computation time necessary for AFM.

5. Based on the tests carried out using one-epoch observations, the suggested searching procedure is suitable for GPS kinematic positioning. In practice, accurate initial position is available from some other techniques, eg. Kalman filter predicted position for kinematic positioning applications (Han 1994b). Of course, the suggested procedure is also suitable for pseudo-kinematic positioning where the initial position can be obtained from the triple-difference solution (good to about a metre (Remondi & Hilla 1993)).

6. The challenge in the further use of this procedure is to somehow reduce the level of biases and errors in the dual-frequency combinations. This limitation however merely restricts the applicability of this method, for the time being, to the determination of relatively short baselines (<5km). Intensive research is needed to overcome this restriction.

REFERENCES

Abidin HZ (1993) Computational and Geometrical Aspects of 'On The Fly' Ambiguity Resolution. Technical Report No. 164, Department of Surveying Engineering, University of New Brunswick, Fredericton, N.B., Canada.

Cocard M, Geiger A (1992) Systematic Search for all Possible Widelanes. In: Proceedings of The Sixth International Geodetic Symposium on Satellite Positioning, March 17-20, Columbus, Ohio, USA, pp 312-318.

Counselman CC, Gourevitch SA (1981) Miniature Interferometer Terminals for Earth Surveying: Ambiguity and Multipath with the Global Positioning System. IEEE Transactions on Geoscience and Remote Sensing, Vol. GE-19, No. 4, pp 244-252.

Han SW (1994a) Equivalence between the Ambiguity Function Method and the Least Squares Method with GPS Carrier Phase Observables and The Improved Ambiguity Function Method. ACTA GEODAETICA et CARTOGRAPHICA SINICA, Vol. 23, No. 4, pp 282-288.

Han SW (1994b) Ambiguity Function Method with Constrained Conditions and its Application in GPS Kinematic Positioning. Journal of Wuhan Technical University of Surveying and Mapping, Vol. 19, No. 1, pp 7-14.

Hofmann-Wellenhof B, Lichtenegger H, Collins J (1992) GPS Theory and Practice. Springer-Verlag Wien New York.

Lachapelle G, Cannon ME, Erickson C, Falkenberg W (1992) High Precision C/A Code Technology for Rapid Static DGPS Surveys. In: Proceedings of The Sixth International Geodetic Symposium on Satellite Positioning, March 17-20, Columbus, Ohio, USA, pp 165-173.

Mader GL (1990) Ambiguity Function Techniques for GPS Phase Initialization and Kinematic Solutions. In: Proceedings of Second International Symposium on Precise Positioning with the Global Positioning System, Ottawa, Canada, September 3-7, pp 1233-1247.

Mader GL (1992) Rapid Static and Kinematic Global Positioning System Solutions Using the Ambiguity Function Technique. Journal of Geophysical Research, Vol. 97, No. B3, pp 3271-3283.

Remondi BW (1984) Using the Global Positioning System (GPS) Phase Observable for Relative Geodesy: Modeling, Processing and Results. Doctoral Dissertation, Center for Space Research, University of Texas at Austin.

Remondi BW (1989) Pseudo-kinematic GPS Results Using the Ambiguity Function Method. Journal of The Institute of Navigation, Vol. 38, No. 1, pp 17-36.

Remondi BW, Hilla SA (1993) Pseudo-kinematic Surveying Based Upon Full-wavelength Dual-frequency GPS Observations. NOAA Technical Memorandum NOS NGS-56, November.

Seeber G (1993) Satellite Geodesy Foundations, Methods, and Applications. Walter de Gruyter.