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Transient conjugated forced convection in a parallel plate duct with convection from the ambient and periodic variation of inlet temperature

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Abstract. This work shows an approximate analytical solution for the heat transfer problem of transient laminar forced convection valid for the thermal entrance region of a parallel plate duct, with walls interacting with an ambient medium outside the duct. The inlet temperature varies periodically with time and duct wall thermal capacity is considered finite. Periodic solutions, obtained by using Laplace transform, are presented for the duct wall temperature, fluid bulk temperature and wall heat flux as a function of the axial position and of the involved parameters.

Nomenclature

A = amplitude
a^* = dimensionless parameter defined by equation (3)
b^* = dimensionless parameter defined by equation (9b)
c_p = fluid specific heat at constant pressure
c_w = wall specific heat
c^* = defined by equation (9a)
h = heat transfer coefficient
$i = \sqrt{-1}$
Im = imaginary part of
k = thermal conductivity of fluid
L = half-spacing between plates
l = wall thickness
Nu_0 = outside Nusselt number
n = number of terms in the series
Re = real part of
s = Laplace transform parameter
$T(x, z, t)$ = fluid temperature
T_0 = cycle mean temperature
T_∞ = reference temperature
t = time
U = mean velocity
u = flow velocity
$W(r)$ = function related to error function
X = dimensionless normal coordinate
x = normal coordinate
Z = dimensionless axial coordinate
z = axial coordinate

Greek symbols

α = thermal diffusivity of fluid
ΔT_0 = amplitude of inlet oscillations
$\theta(X, Z, \tau)$ = dimensionless temperature
ρ = fluid mass density
ρ_w = wall density
τ = dimensionless time
$\psi(Z)$ = dimensionless periodic part of
Ω = dimensionless frequency of oscillations
ω = frequency of oscillations

Subscript

b = bulk temperature
h = heat flux
w = wall temperature

Superscript

$\sim$ = Laplace transform of

Introduction

Knowledge of the thermal interaction between the solid body and the fluid flow, the conjugated heat transfer problems, has received great attention recently, because of its importance in the design of engineering systems involving heat exchanging devices. Problems of this type, where the temperature fields of the fluid and of the solid must be found simultaneously, because they are coupled, lead to severe mathematical difficulties. As a consequence of such difficulties, approximate solution using quasi-steady approach is assumed.

Sucec [1] presented an improved quasi-steady approach for transient conjugated forced convection inside a parallel plate duct by assuming slug flow model. Walls and fluid were both at a constant temperature initially when a transient was initiated by either a step change in fluid inlet temperature with time, or a sinusoidal variation in time.

Because of the relative analytical simplicity, the slug flow model has been employed in the solution of the transient conjugated problems. In [2], Sucec solved, by the method of complex temperature, the problem of the transient conjugated forced convection between a plate, with base isolated, and a fluid whose temperature varies periodically with time. Similar problem was again solved by Sucec [3], whose plate is convectively cooled from below.

Sparrow and Farais [4] studied the transient conjugated problem inside a parallel plate duct, subject to periodic time variation of the inlet condition. Their solution was obtained in series form, assuming a slug flow velocity profile. Cotta et al. [5] advanced the analysis presented by [4] extending it to circular duct, and adopted the Sign Count Method to obtain the complex eigenvalues which appear in the solution in series form. Travelho and Santos [6], further advanced the analysis of [4] and [5] to the thermal entrance region in a parallel plate duct by eliminating the complex eigenvalues using Laplace transformation.

Kakaç and Yener [7] obtained an exact solution for the transient energy equation for laminar slug flow of an incompressible fluid in a parallel plate channel with external convection and a sinusoidal variation of fluid inlet temperature, but the duct wall thermal capacity was neglected. In their analysis the finite integral transform technique was used. In [8], Kakaç et al. present a theoretical and experimental study of the latter problem taking into account the duct wall thermal capacitance effects and a parabolic velocity profile. The analytical solution was obtained through extending the generalized integral transform technique. Sucec [9] presented an analytical solution, valid in the first time domain and in the transient thermal entrance region in a parallel plate channel, with finite thermal capacity walls which interact with an ambient medium outside the channel. The same physical situation was again analysed by Sucec [10], using a finite difference method.

In the present work, an analytical solution is found for the transient laminar forced convection in the thermal entrance region, inside a parallel plate duct, with finite duct wall thermal capacity, external convection and inlet temperature varying periodically with time. The periodic solution of the problem is obtained in terms of elementary (tabulated) functions by using Laplace transform.

Analysis

Consider unsteady forced convection, hydrodynamically developed laminar flow inside parallel plate duct separated by a distance $2L$ and whose walls are each of thickness l. The fluid entering the duct has a temperature which varies periodically in time. Convective heat exchange with the environment and duct wall thermal capacitance effects are considered, while axial conduction and viscous dissipation effects are neglected. The wall and fluid transport properties are assumed to be constant. The parallel plate duct under consideration is shown in Fig. 1.

Under these conditions, the energy equation has the form

$$\frac{\partial T}{\partial t}(x, z, t) + u\frac{\partial T}{\partial z}(x, z, t) = \alpha\frac{\partial^2 T}{\partial x^2}(x, z, t)$$

in $0 \leqslant x \leqslant L$, $z \geqslant 0$, $t \geqslant 0$; (1a)

the inlet and boundary conditions are given respectively by

$$T(x, 0, t) = T_0 + \Delta T_0 e^{i\omega t}, \quad 0 \leqslant x \leqslant L, t \geqslant 0, \tag{1b}$$

$$\left.\frac{\partial T}{\partial x}(x, z, t)\right|_{x=0} = 0, \quad z \geqslant 0, t \geqslant 0. \tag{1c}$$

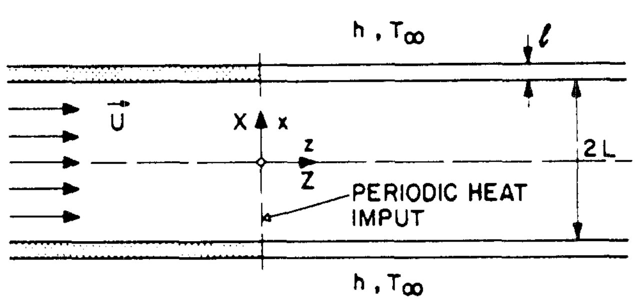

Fig. 1. Geometry of parallel plate duct.

The energy balance on the duct wall yields,

$$-k\frac{\partial T}{\partial x}(x, z, t)\bigg|_{x=L} = \rho_\omega c_\omega l\frac{\partial T}{\partial t}(L, z, t) + h[T(L, z, t) - T_\infty], \quad z \geqslant 0, t \geqslant 0. \quad \text{(1d)}$$

Here the initial condition is not necessary, since this work's interest is the periodic solution to the problem (1). It is assumed the slug flow model ($u = U =$ constant) and the following dimensionless groups are introduced into equation 1(a–d)

$$X = \frac{x}{L}, \qquad Z = \frac{\alpha z}{UL^2}, \qquad \tau = \frac{\alpha t}{L^2}, \qquad \Omega = \frac{\omega L^2}{\alpha},$$

$$\mathrm{Nu}_0 = \frac{hL}{k}, \qquad \theta(X, Z, \tau) = \frac{T(x, z, t) - T_0}{\Delta T_0}, \qquad \theta_\infty = \frac{T_\infty - T_0}{\Delta T_0},$$

with these new variables the equations 1(a–d) become

$$\frac{\partial \theta}{\partial \tau}(X, Z, \tau) + \frac{\partial \theta}{\partial Z}(X, Z, \tau) = \frac{\partial^2 \theta}{\partial X^2}(X, Z, \tau), \quad 0 \leqslant X \leqslant 1, \tau \geqslant 0, \quad \text{(2a)}$$

$$\theta(X, 0, \tau) = \mathrm{e}^{i\Omega\tau}, \quad 0 \leqslant X \leqslant 1, \tau \geqslant 0, \quad \text{(2b)}$$

$$\frac{\partial \theta}{\partial X}(X, Z, \tau)\bigg|_{X=0} = 0, \quad Z \geqslant 0, \tau \geqslant 0, \quad \text{(2c)}$$

$$-\frac{\partial \theta}{\partial X}(X, Z, \tau)\bigg|_{X=1} = \frac{1}{a^*}\frac{\partial \theta}{\partial \tau}(1, Z, \tau) + \mathrm{Nu}_0[\theta(1, Z, \tau) - \theta_\infty]. \quad \text{(2d)}$$

The dimensionless parameter a^*, equation (2d), characterizes the effects of wall capacitance to heat transfer, and is defined by

$$a^* = \frac{\rho c_p L}{\rho_\omega c_\omega l}. \quad \text{(3)}$$

The problem (2) can be split into two parts as follows

$$\theta(X, Z, \tau) = \xi(X, Z) + \gamma(X, Z, \tau), \quad \text{(4)}$$

where $\xi(X, Z)$ is the solution of the following problem

$$\frac{\partial \xi}{\partial Z}(X, Z) = \frac{\partial^2 \xi}{\partial X^2}(X, Z), \quad \text{(5a)}$$

$$\xi(X, 0) = 0, \tag{5b}$$

$$\left.\frac{\partial \xi}{\partial X}(X, Z)\right|_{X=0} = 0, \tag{5c}$$

$$\left.\frac{\partial \xi}{\partial X}(X, Z)\right|_{X=1} + \mathrm{Nu}_0[\xi(1, Z) - \theta_\infty] = 0, \tag{5d}$$

and $\gamma(X, Z, \tau)$ satisfies the problem

$$\frac{\partial \gamma}{\partial \tau}(X, Z, \tau) + \frac{\partial \gamma}{\partial Z}(X, Z, \tau) = \frac{\partial^2 \gamma}{\partial X^2}(X, Z, \tau), \tag{6a}$$

$$\gamma(X, 0, \tau) = \mathrm{e}^{i\Omega\tau}, \tag{6b}$$

$$\left.\frac{\partial \gamma}{\partial X}(X, Z, \tau)\right|_{X=0} = 0, \tag{6c}$$

$$\left.\frac{\partial \gamma}{\partial X}(X, Z, \tau)\right|_{X=1} + \frac{1}{a^*}\frac{\partial \gamma}{\partial \tau}(1, Z, \tau) + \mathrm{Nu}_0 \gamma(1, Z, \tau) = 0. \tag{6d}$$

In order to obtain a periodic solution the temperature function $\gamma(X, Z, \tau)$ is assumed to be of the following type

$$\gamma(X, Z, \tau) = \psi(X, Z)\,\mathrm{e}^{i\Omega(\tau - Z)}. \tag{7}$$

By introducing this definition into problem (6), one has,

$$\frac{\partial \psi}{\partial Z}(X, Z) = \frac{\partial^2 \psi}{\partial X^2}(X, Z), \tag{8a}$$

$$\psi(X, 0) = 1, \tag{8b}$$

$$\left.\frac{\partial \psi}{\partial X}(X, Z)\right|_{X=0} = 0, \tag{8c}$$

$$\left.\frac{\partial \psi}{\partial X}(X, Z)\right|_{X=1} + c^* \psi(1, Z) = 0, \tag{8d}$$

where the starred parameters are defined as follows

$$c^* = \mathrm{Nu}_0 + ib^*;\; b^* = \frac{\Omega}{a^*} = \frac{\omega L \rho_w c_w l}{k}. \tag{9a, b}$$

Method of solution

The interest of this work is to obtain the periodic (time dependent) solution, that is, the solution of the problem described by equations 8(a–d). This is obtained through the use of the Laplace transform technique. By taking the Laplace transform with respect to the Z variable of the equations 5(a–d), one obtains

$$\frac{d^2\tilde{\psi}}{dX^2} = s\tilde{\psi} - 1, \tag{10a}$$

$$\left.\frac{d\tilde{\psi}}{dX}\right|_{X=0} = 0, \tag{10b}$$

$$\left.\frac{d\tilde{\psi}}{dX}\right|_{X=1} + c^*\tilde{\psi}(1, s) = 0, \tag{10c}$$

where $\psi(X, s)$ is defined as

$$\tilde{\psi}(X, s) = \underset{Z\to\infty}{\mathscr{L}}, \quad (X, Z) = \int_0^\infty (X, Z)\,e^{-sZ}\,dZ, \tag{11}$$

and s is the Laplace transform parameter.

The solution of the problem (10) results in

$$\tilde{\psi}(X, s) = \frac{1}{s} - \frac{c^*}{s}\left[\frac{e^{\sqrt{s}X} + e^{-\sqrt{s}X}}{\sqrt{s}(e^{\sqrt{s}} - e^{-\sqrt{s}}) + c^*(e^{\sqrt{s}} + e^{-\sqrt{s}})}\right], \tag{12}$$

rearranging the equation (12), one has

$$\tilde{\psi}(X, s) = \frac{1}{s} - \frac{c^*}{s}\frac{e^{-\sqrt{s}}(e^{\sqrt{s}X} + e^{-\sqrt{s}X})}{(\sqrt{s} + c^*)}\left[\frac{1}{1 - \left(\frac{\sqrt{s} - c^*}{\sqrt{s} + c^*}\right)e^{-2\sqrt{s}}}\right]. \tag{13}$$

Since the present interest is to obtain a solution to the problem (7) for small values of Z (thermal entrance region) the term in the brackets, equation (13), is expanded into a series for large s. This way the equation (13) can be written as

$$\tilde{\psi}(X, s) = \frac{1}{s} - \frac{c^*}{s}\sum_{n=0}^{\infty}\left[\frac{e^{-\sqrt{s}(2n+1\pm X)}}{(\sqrt{s} + c^*)}\left(\frac{\sqrt{s} - c^*}{\sqrt{s} + c^*}\right)^n\right], \tag{14}$$

where the $\pm$ sign means that c^* multiplies two series, one with the $+$ sign and the other with $-$ sign.

By using a table of transforms [11], one can find the inverse transformation of equation (14). By comparing with the results already known in the literature [7], one can see that a good agreement has already been reached with two terms, for the values of Z considered. More terms in the series would only slightly improve the present solution for larger Z, which is not the goal of this work. Thus,

$$\psi(X, Z) = 1 - \operatorname{erfc}\left(\frac{1 \pm X}{2\sqrt{Z}}\right) + \operatorname{erfc}\left(\frac{3 \pm X}{2\sqrt{Z}}\right) - 4c^*\sqrt{\frac{Z}{\pi}}\,e^{-(3 \pm X)^2/4Z}$$
$$+ e^{(1 \pm X)c^* + c^{*2}Z} \operatorname{erfc}\left(\frac{1 \pm X}{2\sqrt{Z}} + c^*\sqrt{Z}\right)$$
$$- [1 - 4c^{*2}Z - 2c^*(3 \pm X)]\, e^{(3 \pm X)c^* + c^{*2}Z} \operatorname{erfc}\left(\frac{3 \pm X}{2\sqrt{Z}} + c^*\sqrt{Z}\right). \tag{15}$$

The solution method adopted here, is more appropriate than the one obtained by direct eigenfunction solution of equation (8), because the present solution requires a smaller number of terms for an adequate convergence at small values of variable Z.

The tabulated function $W(r)$ [12], with r being complex number, is defined in terms of the error function of complex argument as,

$$W(r) - e^{-r^2} \operatorname{erfc}(-ir) \tag{16}$$

the introduction of this function in equation (15) yields

$$\psi(X, Z) = 1 - \operatorname{erfc}\left(\frac{1 \pm X}{2\sqrt{Z}}\right) + \operatorname{erfc}\left(\frac{3 \pm X}{2\sqrt{Z}}\right) - 4c^*\sqrt{\frac{Z}{\pi}}\,e^{-(3 \pm X)^2/4Z}$$
$$+ e^{-(1 \pm X)^2/4Z} W\left[i\left(\frac{1 \pm X}{2\sqrt{Z}} + c^*\sqrt{Z}\right)\right]$$
$$- [1 - 4c^{*2}Z - 2c^*(3 \pm X)]\, e^{-(3 \pm X)2/4Z} W\left[i\left(\frac{3 \pm X}{2\sqrt{Z}} + c^*\sqrt{Z}\right)\right]. \tag{17}$$

Equation (7), combined with equation (17), constitute the solution for the periodic temperature field in the fluid.

Once the periodic dimensionless temperature field $\gamma(X, Z, \tau)$ is determined, the quantities of interest such as dimensionless wall temperature, wall heat flux and fluid bulk temperature can be evaluated. Therefore, the periodic dimensionless temperature $\gamma_w(Z, \tau)$ is given by

$$\gamma_w(Z, \tau) \equiv \gamma(1, Z, \tau) = \psi(1, Z)\, e^{i\Omega(\tau - Z)}, \tag{18}$$

by setting $X = 1$ in equation (17), results in

$$\begin{aligned}\psi_w(Z) \equiv \psi(1, Z) = & -4c^* \sqrt{\frac{Z}{\pi}} \left(e^{-1/Z} + e^{-4/Z}\right) + \operatorname{erfc}\left(\frac{2}{\sqrt{Z}}\right) \\ & + W(ic^* \sqrt{Z}) + 4c^*(1 + c^* Z)\, e^{-1/Z} W\left[i\left(\frac{1}{\sqrt{Z}} + c^* \sqrt{Z}\right)\right] \\ & + (4c^{*2} Z + 8c^* - 1) e^{-4/Z} W\left[i\left(\frac{2}{\sqrt{Z}} + c^* \sqrt{Z}\right)\right].\end{aligned} \tag{19}$$

The periodic dimensionless wall heat flux $\gamma_h(Z, \tau)$ is expressed by

$$\gamma_h(Z, \tau) = -\frac{\partial \gamma}{\partial X}(X, Z, \tau)\bigg|_{X=1} = -\frac{\partial \psi}{\partial X}(X, Z)\bigg|_{X=1} e^{i\Omega(\tau - Z)}, \tag{20}$$

with the aid of equation (8d), one has

$$\psi_h(Z) \equiv -\frac{\partial \psi}{\partial X}(X, Z)\bigg|_{X=1} = c^* \psi(1, Z). \tag{21}$$

The periodic dimensionless fluid bulk temperature $\gamma_b(Z, \tau)$ is obtained from its definition, as being

$$\gamma_b(Z, \tau) = \int_0^1 \gamma(X, Z, \tau)\, dX = \int_0^1 \psi(X, Z)\, dX\, e^{i\Omega(\tau - Z)}, \tag{22}$$

by defining the periodic dimensionless fluid bulk temperature as being, $\psi_b(Z)$ and after applying the Laplace transform with respect to Z, one gets

$$\tilde{\psi}_b(s) = \int_0^1 \tilde{\psi}(X, s)\, dX. \tag{23}$$

By inserting $\tilde{\psi}(X, s)$ from equation (14) into equation (23), one obtains after a direct integration, the following equation

$$\tilde{\psi}_b(s) = \frac{1}{s} - \frac{c^*}{s^{3/2}} \frac{(1 - e^{-2\sqrt{s}})}{(\sqrt{s} + c^*)} \sum_{n=0}^{\infty} \left[e^{-2n\sqrt{s}} \left(\frac{\sqrt{s} - c^*}{\sqrt{s} + c^*}\right)^n \right]. \tag{24}$$

The inverse transform of equation (24) is obtained by using a similar procedure used in equation (14). Therefore,

$$
\begin{aligned}
\psi_b(Z) = 1 &+ \frac{1}{c^*} - 2\sqrt{\frac{Z}{\pi}}(1 - 2\,e^{-1/Z} - 3\,e^{-4/Z}) - \frac{2}{c^*}\,\mathrm{erfc}\left(\frac{1}{\sqrt{Z}}\right) \\
&- \left(4 + \frac{3}{c^*}\right)\mathrm{erfc}\left(\frac{2}{\sqrt{Z}}\right) - \frac{1}{c^*}\,W(ic^*\sqrt{Z}) \\
&- \left(4Zc^* - \frac{2}{c^*} + 4\right)e^{-1/Z}W\left[i\left(\frac{1}{\sqrt{Z}} + c^*\sqrt{Z}\right)\right] \\
&- \left(4c^*Z - \frac{3}{c^*} + 8\right)e^{-4/Z}W\left[i\left(\frac{2}{\sqrt{Z}} + c^*\sqrt{Z}\right)\right].
\end{aligned}
\tag{25}
$$

In order to visualize the periodic part of solution presented here, the wall and fluid bulk temperature and the wall heat flux are expressed in terms of amplitudes $A(Z)$ and phase lags of oscillations $\phi(Z)$. Thus, $A_w(Z)$, $A_h(Z)$, $A_b(Z)$, $\phi_w(Z)$, $\phi_h(Z)$ and $\phi_b(Z)$ correspond respectively to the amplitudes and phase lags of wall temperature, wall heat flux and fluid bulk temperature and are calculated by

$$A(Z) - |\psi(Z)| = \{[\mathrm{Re}(\psi)]^2 + [\mathrm{Im}(\psi)]^2\}^{1/2}, \tag{26}$$

$$\phi(Z) = \mathrm{tg}^{-1}\left\{\frac{\mathrm{Im}[\psi(Z)]}{\mathrm{Re}[\psi(Z)]}\right\} - \Omega Z, \tag{27}$$

where $\mathrm{Re}(\psi)$ and $\mathrm{Im}(\psi)$ are the real and imaginary parts of equations (19), (21) and (25).

Results and discussion

The preceding analytical solution indicates that the calculated dimensionless quantities such as wall temperature, fluid bulk temperature and wall heat flux depend on the dimensionless parameters b^* and Nu_0. The former represents the ratio between the thermal capacity of the wall and the heat transfer by conduction across the fluid, and the latter, represents the effectiveness heat transfer to the ambient.

Amplitudes and phase lags of those quantities mentioned above, were evaluated as function of dimensionless axial coordinate Z, for representative values of both parameters b^* and Nu_0. Numerical values for the parameter b^* followed those in the literature [4–6], while two values of Nu_0 were investigated, namely $\mathrm{Nu}_0 = 1$ and 100.

Two cases of the present solution are evaluated separately, $\mathrm{Nu}_0 = 0$ and $b^* = 0$ ($a^* \to \infty$). If $\mathrm{Nu}_0 = 0$, the effect of external convection is eliminated and, in this case, the

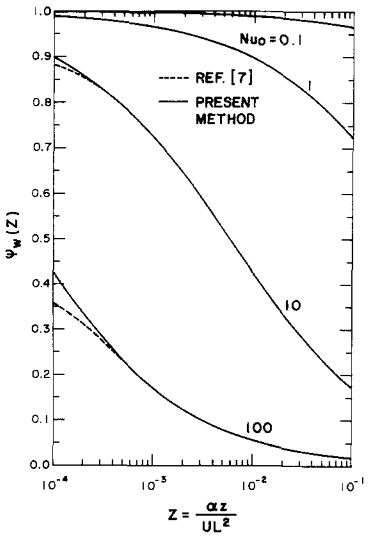

Fig. 2. Comparison between wall temperatures calculated by using the present method and by using series solution.

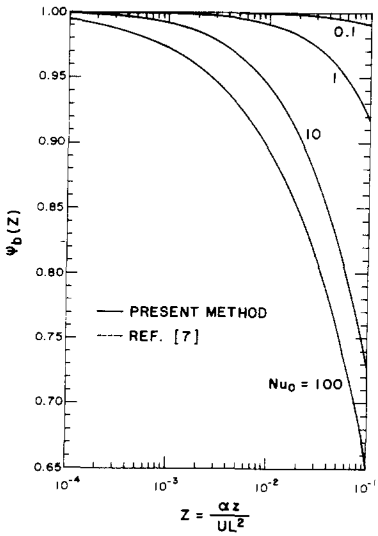

Fig. 3. Comparison between fluid bulk temperatures by using the two methods.

insulated walls problem [6] is recovered. For $b^* = 0$ ($a^* \to \infty$), the effect of the capacity at the wall can be neglected and therefore, one obtained the problem for time-varying inlet without participating walls.

Comparison between the present solution and the series solution obtained by Kakaç and Yener [7] to the case of $b^* = 0$ and $Nu_0 = 0.1$, 1, 10 and 100, are shown in Figs 2, 3 and 4 for the wall temperature, fluid bulk temperature and wall heat flux respectively. As shown in these figures, the agreement between the solutions is very good in the thermal entrance region, which depends on the parameter Nu_0. For small values of Nu_0, the thermal entrance region extends up to larger values of the variable Z. This is expected due to the external thermal resistance to be larger. Therefore, it is necessary a longer physical distance for the same amount of energy to be changed with the ambient, when compared with larger values of Nu_0. The discrepancies between the solutions for small values of Z, Figs 2 and 4, related to wall temperature and wall heat flux respectively, are due to relative small number of eigenvalues used in the series solution, in the case, it was used 25 eigenvalues. On the other hand, the solution presented here yields better results for small values of Z (near $Z = 0$). This happens because the expansion of the term in the brackets, equation (13), in the transformed plane, was made in order to obtain good precision in the asymptotic region. From Fig. 3, which represents the fluid bulk temperature, the agreement is seen to be excellent for the values of Nu_0 investigated, where the curves to both solutions are coincident. The comparison between the present solution and the series solution with small number of eigenvalues allows one to identify when the difference between the solutions appears and the magnitude of this difference.

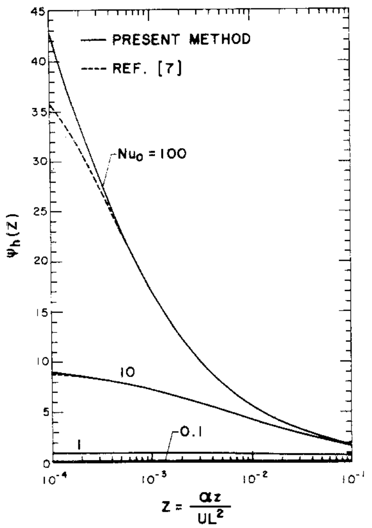

Fig. 4. Comparison between wall heat flux by using the two methods.

Amplitudes and phase lags for dimensionless wall temperature are plotted in Figs 5(a, b). As shown in these figures, the oscillations in the thermal entrance region are influenced by the parameters b^* and Nu_0. When Nu_0 is small, $\mathrm{Nu}_0 = 1$, i.e. when the external thermal resistance is larger, the oscillations in the thermal entrance region depend strongly on the parameter b^*. For larger values of b^*, the thermal wave has little penetration along the duct length, rapidly decaying with the dimensionless axial distance. Therefore, oscillations in fluid temperature are damped within a short nondimensional distance from the duct inlet. This is to be expected due to larger thermal capacitance of the walls. For small values of b^*, the thermal wave has a penetration more gradual along the duct because of smaller walls thermal capacitance, requiring a longer nondimensional length for the same energy to be stored in the walls. For $\mathrm{Nu}_0 = 100$, i.e. when the external thermal resistance is very low, the influence of the parameter b^* on the oscillations in the thermal entrance region is much smaller. It can be noticed from Fig. 5b, that up to the scale of the graph, the amplitude was not influenced by parameter b^*, in the range considered, coinciding with the limit case of $b^* = 0$ ($a^* \to \infty$). The phase lag, on the other hand, was only slightly advanced for increasing b^*.

Figures 6(a, b) show the amplitudes and phase lags of the dimensionless fluid bulk temperature for the same numerical values of parameters b^* and Nu_0 used in Figs 5(a, b). As shown in these figures, the bulk and wall temperatures amplitudes and phase lags present similar behavior. However, for the bulk temperature, the effects of the both parameters b^* and Nu_0 are less pronounced. The reader should notice that the fluid bulk temperature amplitudes are less attenuated than those for wall temperature.

The influences of the parameters b^* and Nu_0 on the dimensionless wall heat flux

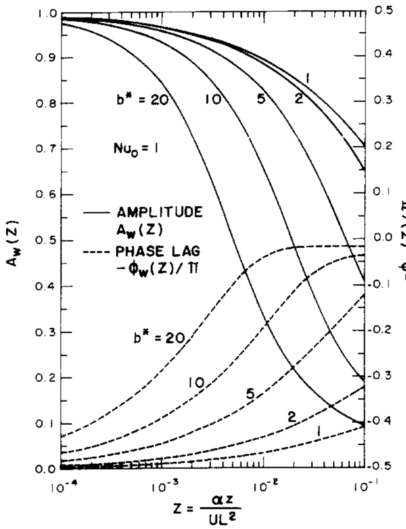

Fig. 5(a). Axial distributions of amplitudes and phase lags for dimensionless wall temperature for various values of b^* ($Nu_0 = 1$).

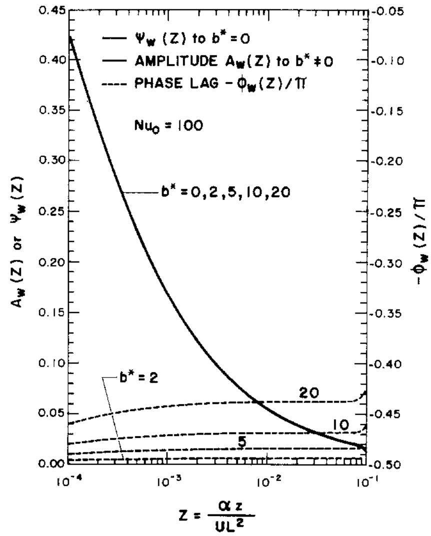

Fig. 5(b). Same with $Nu_0 = 100$.

amplitudes and phase lags are illustrated in Figs 7(a, b). For $Nu_0 = 1$, the amplitudes for wall heat flux are larger for larger values of b^*. This is expected since due to a larger walls heat capacitance, more heat is transferred to the walls, what can be seen in Fig. 5(b) by noticing the large temperature gradient. As Nu_0 increases, i.e. when the external thermal resistance decreases, the amplitudes for wall heat flux increase

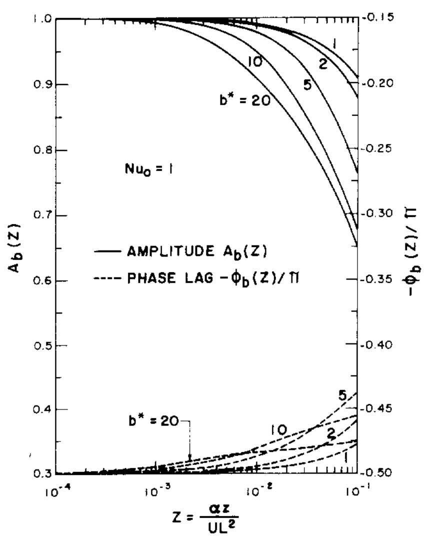

Fig. 6(a). Axial distributions of amplitudes and phase lags for dimensionless fluid bulk temperature for various values of b^* ($Nu_0 = 1$).

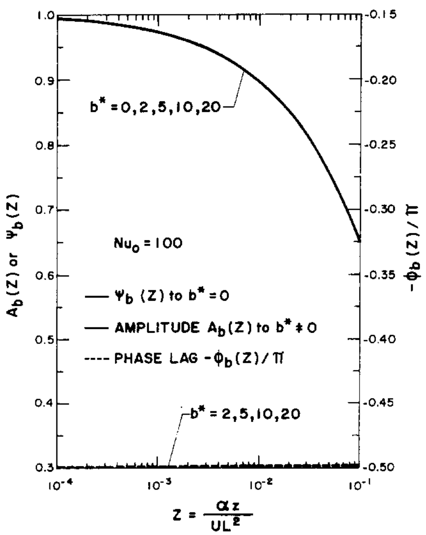

Fig. 6(b). Same with $Nu_0 = 100$.

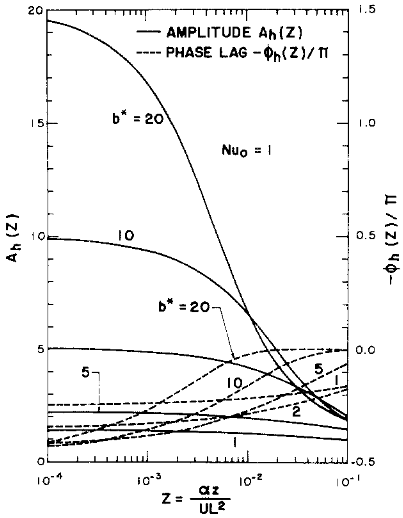

Fig. 7(a). Axial distributions of amplitudes and phase lags for dimensionless wall heat flux for various values of b^* ($Nu_0 = 1$).

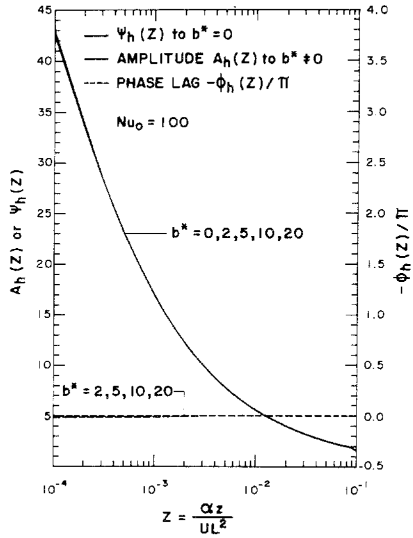

Fig. 7(b). Same with $Nu_0 = 100$.

independently of the walls thermal capacity. Figure 7(b), shows amplitudes and phase lags practically coincident over the range of the parameter b^* investigated.

Finally, it should be noticed that the values of the amplitudes reached for $Nu_0 = 100$, Figs 5(b), 6(b) and 7(b), are the values got by the particular case, $b^* = 0$, shown in Figs 2, 3 and 4, for wall temperature, fluid bulk temperature and wall heat flux.

Conclusion

An analytical solution, valid for the thermal entrance region, based on the Laplace transform technique, has been found for the heat transfer problem with inlet temperature varying periodically with time, external convection, finite walls thermal capacity and laminar flow inside a parallel plate duct. This solution was obtained through an infinite series truncation, which showed to be a very good approximation. Results for practical interesting quantities such as wall temperature, fluid bulk temperature and wall heat flux are presented in terms of the amplitudes and phase lags as function of dimensionless axial coordinate. The influences of the two governing parameters, i.e. the walls thermal capacity b^* and the external thermal resistance Nu_0, on the amplitudes and phase lags, are investigated in detail. It can be summarized here that the thermal capacity of the walls b^* plays an important role in damping the amplitudes of studied quantities, in the thermal entrance region, when the external thermal resistance is high. When the external thermal resistance is low, that is, larger values of Nu_0, the thermal capacity of the walls has no decisive influence in the speed of propagation of the thermal wave in the thermal entrance region and, therefore, the external convection takes place as the controlling factor in damping the amplitudes.

This work presents an analytical solution with no experimental data for its verification. However, a comparison is made with another solution presented in the literature, whose main ideas were checked experimentally [7]. The comparison was made only for the cases already studied in former works. It is the authors opinion that an experimental verification of the present results would be a valuable contribution.

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References

1. Sucec, J., An improved quasi-steady approach for transient conjugated forced convection problems. *Int. J. of Heat and Mass Transfer* 24 (1981) 1711–1722.
2. Sucec, J., Transient heat transfer between a plate and a fluid whose temperature varies periodically with time. *J. of Heat Transfer* 102 (1980) 126–131.
3. Sucec, J., Unsteady heat transfer between a fluid, with time varying temperature and a plate: an exact solution. *Int. J. of Heat and Mass Transfer* 18 (1975) 25–36.
4. Sparrow, E.M. and de Farias, F.N., Unsteady heat transfer in ducts with time-varying inlet temperature and participating wall. *Int. J. of Heat and Mass Transfer* 11 (1968) 837–853.
5. Cotta, R.M., Mikhailov, M.D. and Ozisik, M.N., Transient conjugated forced convection in ducts with periodically varying inlet temperature. *Int. J. of Heat and Mass Transfer* 30 (1987) 2073–2082.
6. Travelho, J.S. and dos Santos, W.F.N., Solution for transient conjugated forced convection in the thermal entrance region of a duct with periodically varying inlet temperature. *J. of Heat Transfer* (to appear).
7. Kakaç, S. and Yener, Y., Exact solution of the transient forced convection energy equation for timewise variation of inlet temperature. *Int. J. of Heat and Mass Transfer* 16 (1973) 2205–2214.
8. Kakaç, S., Li, W. and Cotta, R.M., Unsteady laminar forced convection in ducts with periodic variation of inlet temperature. *J. of Heat Transfer* 112 (1990) 913–920.
9. Sucec, J., Exact solution for unsteady conjugated heat transfer in the thermal entrance region of a duct. *J. of Heat Transfer* 109 (1987) 295–299.
10. Sucec, J., Unsteady conjugated forced convective heat transfer in a duct with convection from the ambient. *Int. J. of Heat and Mass Transfer* 30 (1987) 1963–1970.
11. Roberts, G.E. and Kaufmann, H., *Tables of Laplace Transforms*. Philadelphia, PA: W.B. Saunder Co. (1966).
12. Abramowitz, M. and Stegun, I.A., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. National Bureau of Standards Appl. Math. Ser. 55 (1964).