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# Fractal Analysis of Climatic Data: Annual Precipitation Records in Spain

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With 2 Figures

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**Summary**

The rescaled range analysis was applied to the annual precipitation series from 10 weather stations in Spain for the period 1901–1989. The analysis reveals that the series of precipitations fits a fractal distribution, with a mean fractal dimension of $1.32 \pm 0.01$. This lies in the same order of magnitude as the fractal dimensions obtained from other macrometeorological and paleoclimatic registers. The favourable comparison between fractal dimensions of the variables on both small time scales and long-term time spans suggests that such values are characteristic of climatic change over the spectral range of 10 to $10^6$ years. The results contribute to the establishment of this assumption as a valid hypothesis for the interpolation of climatic change from one scale to the next, and also in applications such as hydrological design.

## 1. Introduction

Fractal geometry exploits a characteristic property of the real world (self-similarity) to find simple rules for the assembly of complex natural objects or processes. Self-similarity is a property of systems with structures that remain constant when the scale of observation is changed. Many natural phenomena manifest scale invariance and may be studied in the framework of fractal geometry (Sughihara and May, 1990). Turcotte (1992) presents numerous examples of geophysical and geological variables that are responsive to this approach.

Fractal distribution is defined (Mandelbrot, 1982) as one in which the number of objects $N$ with a characteristic size greater than $r$, scales with the relation

$$N = Cr^{-D} \tag{1}$$

where $C$ is a constant of proportionality and $D$ is a similarity or fractal exponent. The fractal exponent is an allometric scaling constant that performs somewhat like a dimension. A more rigorous interpretation of the exponent like the Hausdorf (true fractal) dimension requires an analysis of the scaling as the linear measurement, $r$, approaches zero. In practice, the Hausdorf dimension can never be produced due to the finite resolution of measuring instruments or photographic grain (Mandelbrot, 1975). Largely for this reason (and as usual in earth sciences), the present paper relates $D$ to the fractal dimension.

The applicability of Eq. (1) as a mathematical representation may be valid over an infinite range of scales, but in practice the scaling rules tend to hold over a finite range of scales between upper and lower limits, where the value of the "outer cutoff" is an important characteristic of the problem (Lovejoy and Mandelbrot, 1985). Fractal distribution thus provides a useful description of statistical distribution which is applicable whenever the scale invariance extends over a sufficient range of scales.

Very long time scales, usually associated with climate, were studied by Hurst (1951, 1956) who found that fluctuations in the Nile River outflows reveal long-term correlations. Hurst's empirical law concerns the dependence upon the lag time $s$ of the standard deviations of the increments in the signal (i.e. outflows chronology). The "rescaled range" $R(t, s)/S(t, s)$, has been found to be proportional to $s^H$, where $H$ is a typically different constant from 0.5. The $R/S$ rescaled range analysis has been applied to a number of macro-meteorological, climatic and paleoclimatic variables such as hydrological records of stream flow, sun-spot numbers and dendroclimatologic data (Mandelbrot and Wallis, 1968, 1969), long-range temperature records (Lovejory and Schertzer, 1983), rain fields (Lovejoy and Mandelbrot, 1985), long-range paleoclimatic data from the oxygen isotope curve of the Pacific core V28–239 (Flugeman and Snow, 1988) and instrumental temperature records (Bodri, 1994). The results of these analyses indicate fractal geometry of the variables and yield Hurst's exponent values in the interval $0.5 < H < 1$.

This paper analyzes a number of annual precipitation series registered over the last century in terms of fractal geometry, in order to determine the fractal dimension of their curves. If the fractal dimensions calculated on this centenary time scale are found to be similar in order of magnitude to those obtained for the longer-term climatic data, we have new proof that climatic changes are characterized by a general fractal dimension over the spectral range 10 to $10^6$ years, with a heavy statistical interdependence of data.

## 2. Data

Ten series of annual precipitations between 1901 and 1989 corresponding to 10 meteorological stations in Spain were selected (Table 1). Annual values were computed from daily figures kindly provided by Spanish Institute of Meteorology (Oñate, 1993). Homogeneity of the original records was tested by means of the application of run test (Thom, 1966), and was considered acceptable in all cases ($p < 0.05$; Oñate, 1993). Since there was a considerable variability of the original records (Table 2), 5-year moving average filters were computed to smooth the series.

Trends in the series for the last century were examined using the Mann-Kendall trend analysis (Mitchell et al., 1966). Statistically significant and negative tendencies were found in three series, while another three had significant and positive tendencies. No statistically significant trends were found in the remaining four series (Table 3).

Table 1. *Meterological Station Locations*

| Station | Lat. (°N) | Long. (°W) | Alt. (m) |
|---|---|---|---|
| Albacete | 38.56 | 1.51 | 699 |
| Alicante | 38.22 | 0.30 | 82 |
| Badajoz | 38.52 | 6.58 | 186 |
| Huesca | 42.07 | 0.26 | 503 |
| Jaén | 37.48 | 3.48 | 503 |
| La Coruña | 43.22 | 8.25 | 57 |
| Salamanca | 40.58 | 5.41 | 797 |
| San Fernando | 36.27 | 6.12 | 30 |
| San Sebastián | 43.19 | 2.03 | 258 |
| Soria | 41.46 | 2.28 | 1063 |

Table 2. *Average Annual Precipitation (mm) and Variation Coefficient of Records*

| Station | Mean | CV |
|---|---|---|
| Albacete | 343.6 | 347.5 |
| Alicante | 333.3 | 497.5 |
| Badajoz | 499.5 | 331.7 |
| Huesca | 529.2 | 336.3 |
| Jaén | 586.1 | 341.6 |
| La Coruña | 892.7 | 214.9 |
| Salamanca | 419.1 | 312.1 |
| San Fernando | 604.9 | 350.0 |
| San Sebastián | 1423.9 | 209.3 |
| Soria | 546.2 | 271.5 |

Table 3. *Trend Analysis. Mann-Kendall Statistic $U(t)$ and Probability Level: *$p < 0.05$; **$p < 0.01$*

| Station | $U(t)$ | $p$ |
|---|---|---|
| Albacete | 1.06 | – |
| Alicante | −0.22 | – |
| Badajoz | 0.12 | – |
| Huesca | 1.24 | – |
| Jaén | −3.36 | ** |
| La Coruña | 4.10 | ** |
| Salamanca | 1.99 | * |
| San Fernando | −2.31 | * |
| San Sebastián | 4.93 | ** |
| Soria | −1.94 | * |

## 3. Methods

Detailed descriptions of the $R/S$ analysis can be found in the bibliography (e.g. Hurst, 1965; Mandelbrot and Wallis, 1969; Feder, 1988). Since the method is not widely known, however, a brief outline is provided here. The present study obtained 10 discrete records in time of annual precipitation. By choosing a time period $s$ (referred to as a lag) shorter than the total record $T$, we first calculate the increments $I$, the change in precipitation between two adjacent years, for this lag and then the average $A$ and the standard deviation $S$ of the increments. By constructing a set of new increments $(I - A)$ and sequentially adding them, we reconstruct an image of the original curve in this lag along a horizontal axis. Range $R$ is defined as the maximum value minus the minimum value of this curve, and by dividing $R$ by $S$ we obtain a dimensionless number $R/S$ (rescaled range value) for the lag in question. For each lag $s$ we obtained $(T - s)$ number of $R/S$ values. The average of all of them is the final representative value of each specific time span.

The record is said to satisfy Hurst's law if the plot of $R/S$ values versus time spans in log-log space is closely aligned along a straight trendline, the slope of which shall be designated as $H$. For many natural phenomena, Hurst and later Mandelbrot found that $R/S \sim (\log)^H$, where $H$ is now referred to as the Hurst exponent. Mandelbrot and Wallis (1969) discussed the effect of $R/S$ ratios for high and low values of $s$ when drawing conclusions from the plot. They concluded that when fitting a straight trendline, high and low values of $s$ should be given less weight than values far from both $s = 1$ and $s = T$. The present analysis therefore adopts time spans of 5, 7, 10, 20, 40, and 70 years.

The Hurst exponent is easily shown (Feder, 1988) to be 0.5 for a random record in time where observations are statistically independent. Mandelbrot and Wallis (1969) found that for many geophysical records $H \neq 0.5$, being a common characteristic for persistent data values of $H > 0.5$ and for antipersistent data values of $H < 0.5$.

According to Mandelbrot (1982), the Hurst exponent in the interval $0.5 < H < 1$ is related to fractal dimension $D$ by

$$D = 2 - H \tag{2}$$

which leads to the values of $D$ in the interval $1 < D < 1.5$. Mandelbrot and Wallis (1969) qualified this feature as a usual characteristic of many geophysical records, meaning some degree of long-term persistence in the data. In other words, even when the observations are sufficiently distant from each other in time they are not statistically independent.

Exponent $H$ is thus discussed for statistically independent random data dominated by "white noise" processes. But as we are dealing with low-pass filtered time series (5-year running means), we also discuss the $H$ exponent for 10 linear "red noise" time series. Each of these series can in the first approximation be equated to what is known as a first-order linear Markov process of the type:

$$x(t) = \beta x(t-1) + \varepsilon(t) \tag{3}$$

where $\beta$ is the "red noise" parameter representing the serial lag coefficient between succesive observations in the $x(t)$ series and $\varepsilon(t)$ is an independently distributed variable with a mean value of zero and variance $(1 - \beta^2)$ times that of $x(t)$. In general $\beta$ varies between 0 and 1 becoming the noise spectrum more and more "red" with increasing values of $\beta$ (Burroughs, 1992). Thus we have adopted $\beta = 0.9$. The $H$ exponents derived from the data were compared with those derived from the statistical linear model by means of the Mann-Whitney test (Zar, 1984).

## 4. Results and Discussion

Figure 1 shows the characteristic plot of $R/S$ values versus time spans for two of the meteorological stations. Given that the rest are virtually repetitive, they are ignored here. Plotted lines represent linear regressions for time spans of 5, 10, 20, 40 and 70 years (Mandelbrot and Wallis, 1969). In the detailed version of the plots (Fig. 2), the presence of clearly visible breaks are a characteristic feature of each diagram. There is a break in the trendline near a lag of $s \sim 33$ which reduces the scatter of the log $(R/S)$ values. The trendline slope is interrupted by an almost horizontal segment and quickly reestablishes after the break. Such characteristics also appear near a lag of $s \sim 44$ and less clearly near a lag of $s \sim 22$. Mandelbrot and Wallis (1969) link these characteristics to periodic phenomena with the wavelength equal to $s$. The second and third breaks might correspond re-

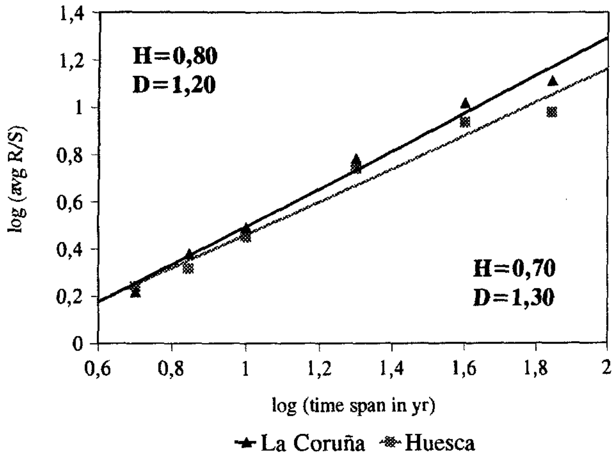


Fig. 1. Plot of averaged rescaled ($R/S$) values versus time spans for La Coruña and Huesca meteorological stations

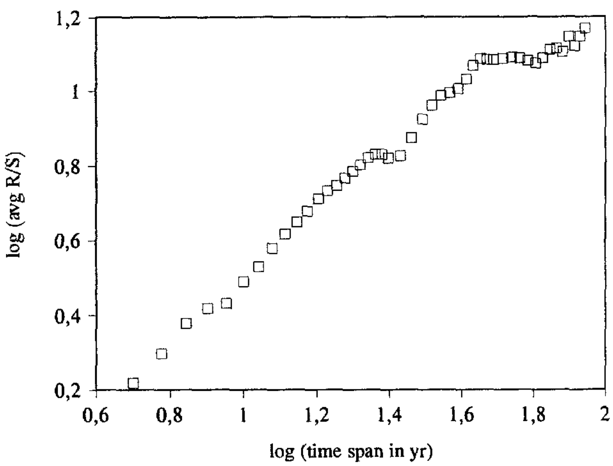


Fig. 2. Detailed version of Fig. 1 for La Coruña meteorological station. Values of $s$ range from 5 to 25 uniformly in increments of 1, then from 25 to 50 uniformly in increments of two, and from 50 onwards in increments of 3

spectively to two and three times the wavelength of the well-known periodic element in solar activity. Many other climatic records from instrumental as well as proxy data (see review by Burroughs, 1992), also indicate the sunspot cycle and the multiples of the 11-year period. The large time lag region is dominated by a plateau in which the slope of the curve approaches 0. The reason is that the standard deviation of the increments become essentially independent of the lag time for the larger lags near 100 $s$. Most surprisingly is the break that less clearly appears near a lag of $s \sim 18$, which may be linked to the 20-year cycle detected in other rainfall records (Currie and O'Brien, 1988; Hamseed, 1983; Tyson, 1986). No conclusions should be advanced at this point about whether it is due to lunar tidal effects or to some other cause. As Burroughs (1992) remarks, until we have looked at all the evidence, all we can say is that on the basis of instrumental records, the case for many aspects of precipitation being modulated by a 20-year cycle is apparently formidable.

Table 4. *H Values and Fractal Dimensions for the Series*

| Station | $H$ | $D$ |
|---|---|---|
| Albacete | $0.70 \pm 0.02$ | 1.30 |
| Alicante | $0.71 \pm 0.02$ | 1.29 |
| Badajoz | $0.79 \pm 0.03$ | 1.21 |
| Huesca | $0.70 \pm 0.06$ | 1.30 |
| Jaén | $0.56 \pm 0.02$ | 1.44 |
| La Coruña | $0.80 \pm 0.05$ | 1.20 |
| Salamanca | $0.68 \pm 0.05$ | 1.32 |
| San Fernando | $0.59 \pm 0.02$ | 1.41 |
| San Sebastián | $0.64 \pm 0.04$ | 1.36 |
| Soria | $0.68 \pm 0.02$ | 1.32 |
| Mean | $0.68 \pm 0.01$ | $1.32 \pm 0.01$ |

The presence of several periodic elements complicates the panorama, but it does not alter the validity of the $R/S$ diagrams. Table 4 shows estimates for slope $H$ and the calculated fractal dimensions $D$. There is no apparent link between the trends in the data sets and the fractal dimension values obtained. Values of $D$ lie in the range 1.20–1.44 with a mean of 1.32. This coincides with those obtained in previous studies. Furthermore, all estimated values of $H$ lie in the range 0.5–1, indicating long-term persistence of the data.

The $H$ exponents derived form the observed time series deviate significantly from those obtained from the 10 linear markovian models (Mann-Whitney test: $U = 100$; $p < 0.001$). All of these lie in the range 0.17–0.34 with a mean of 0.23. This indicate that indeed long term memory processes are relevant for climatic variations.

## 5. Conclusions

It is hard to understand why the Hurst exponents of natural phenomena are generally greater than 0.5, and more specifically why the precipitation fluctuations have such a dependence. Values of $H$ exceeding 0.5 infer that the span of statistical interdependence of climatic data is large. The good agreement of fractal dimensions in the range $1 < D < 1.5$ on both short and long-term time scales, indicate a persistence in the climatic records over the spectral range of 10 to $10^6$ years. The similarity in the irregularity characteristics of the records suggest that the system has a "memory" of past conditions over at least this range.

Further quantitative studies of the climatic elements in the whole hierarchy of time scales may have a comparative basis in the fractal dimension, which is independent of specific units of measurement. Although the distinction between macrometeorology, climatology and paleoclimatology is unquestionably useful for related items, it is not intrinsic to the subject matter: the characteristics of records on different time scales and different units can be compared in the light of fractal geometry. This potential opens up new opportunities in climatic research, particularly in the interpolation of climatic change from one scale to the next, and also in applications such as hydrological design.

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