STRENGTH AND DEFORMABILITY OF REINFORCED POLYMERS IN TENSION NORMAL TO THE FIBERS

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The strength and deformability of reinforced polymers in tension across the fibers is investigated. It is assumed that the polymer deforms as an ideal elastoplastic body. Relations are obtained for the nature of the deformation of the polymer between the fibers and the strength and deformability of the composite with allowance for the structural distribution of the components. Theoretical stress-strain diagrams are presented for composites with different reinforcement densities and resin elasticities. The theoretical values of the strength and deformation of reinforced polymers with the load applied across the fibers are compared with the results of experiments on model specimens of epoxy-Thiokol polymers.

Although much attention has been given to the properties of composites in the direction of reinforcement, it should not be forgotten that in most cases unidirectional reinforced polymers are also stressed across the fibers and in interlaminar shear. In these circumstances it is the strength of the bond between fibers that determines the onset of failure.

On the basis of an analysis of a simple model of a reinforced material with a parallel arrangement of the fibers we shall attempt to ascertain the effect of the characteristics of the reinforcement and the polymer and the structural distribution of the components on the strength and deformation of a composite in a direction perpendicular to the axis of the fibers. As distinct from [1, 2], our approach will be an engineering one and, accordingly, we shall introduce a number of assumptions.

The structures of reinforced polymers in tension across the fibers can be reduced to the model shown in Fig. 1a. For this model the following conditions are assumed to be satisfied:

1) the reinforced polymer is free of initial fabrication stresses;

2) the fiber and the polymer are isotropic materials, the fiber obeying Hooke's law and the polymer possessing elastoplastic properties, with a slight error in the stress–strain diagrams of actual resins [3] can be approximated for each specific strain rate by an ideal elastoplastic body;

3) the strength of the resin at the fiber–polymer interface (adhesion strength) is greater than the strength of the resin.

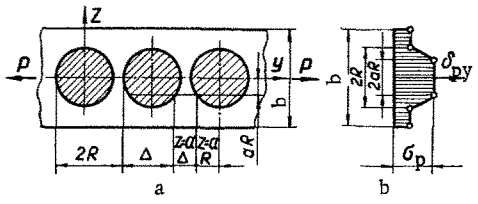

Fig. 1. Calculation model a); stress diagram (b).

We will determine the strength of a specimen in tension along the y axis, i.e., the stress corresponding to the moment at which the polymer cracks.

The strain of the specimen along the y axis is given in terms of the strains of its components by [4]

$$\varepsilon_y=\frac{\varepsilon_{py}\Delta+\varepsilon_{fy}2R}{\Delta+2R}=\frac{\varepsilon_{py}m+\varepsilon_{fy}}{m+1}. \quad (1)$$

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Expression (1) can be transformed as follows: for a polymer with ideal elastoplastic properties

$$\varepsilon_y=\varepsilon_l\frac{\frac{E_p}{E_f}+m\frac{\varepsilon_{py}}{\varepsilon_l}}{1+m}; \tag{2a}$$

for an elastic polymer

$$\varepsilon_y=\varepsilon_{py}\frac{\frac{E_p}{E_f}+m}{1+m}. \tag{2b}$$

In Eqs. (1) and (2a, b) ε_{py} and ε_{fy} are the strains of the polymer and the fiber along the y axis (z = 0); ε_l is the linear component of the deformation of the ideal elastoplastic polymer; E_p and E_f are the moduli of elasticity of the polymer and the fiber; $m = \Delta/2R$.

Expressions (1) and (2a, b) are approximate, since they do not take the Poisson effect into account. However, the assumption that the Poisson's ratios of the fiber and the polymer are equal, without introducing a serious error in most cases of practical importance, simplifies the further analysis.

With respect to the nature of the deformation of the resin, the specimen can be divided into three zones: plastic flow between the fibers, and elastic deformation between and outside the fibers (see Fig. 1). Here, it is assumed that $\varepsilon_y \leq \varepsilon_l$. For the zonal loads we have

$$P_1=2aR\sigma_p; \tag{3}$$

$$P_2=2\int_{aR}^{R}\sigma_{py^z}dz=2\int_{aR}^{R}\varepsilon_{py^z}E_p dz; \tag{4}$$

$$P_3=(b-2R)\varepsilon_y E_p, \tag{5}$$

where σ_p is the strength of the polymer; $\sigma_{py}{}^z$, $\varepsilon_{py}{}^z$ are the stress and strain in the resin along the y axis at $z \neq 0$; a is a coefficient characterizing the dimensions of the region of plastic flow

$$a=\sqrt{1-\left(\frac{1+m}{1+m'}\right)^2}. \tag{6}$$

Here, $m' = \Delta^{z=a}/2R^{z=a}$ is the parameter of the section in which, when the specimen is stretched to $\varepsilon_{y\,max}$, the relative elongation of the polymer between the fibers $\varepsilon_{py}{}^z = \varepsilon_l$. Using Eq. (1), we obtain

$$m'=\frac{\frac{\varepsilon_{py}}{\varepsilon_l}-\frac{E_p}{E_f}}{\frac{1}{m}\left(1-\frac{E_p}{E_p}\right)-\left(\frac{\varepsilon_p}{\varepsilon_l}-1\right)} \tag{7}$$

It follows from (6) and (7) that the polymer flow zone develops at $\eta = \varepsilon_p/\varepsilon_l > 1$; $m > 0$. In this case $R \geq a \geq 0$.

We will determine P_2. The strain $\varepsilon_{py}{}^z$ is found from the equations

$$\varepsilon_y=\frac{\varepsilon_{py^z}\Delta^z+\varepsilon_{fy^z}2R^z}{2R+\Delta}; \quad \varepsilon_{fy^z}=\varepsilon_{py^z}\frac{E_p}{E_f}, \tag{8}$$

where $R^z = \sqrt{R^2-z^2}$; $\Delta^z = 2R + \Delta - 2\sqrt{R^2-z^2}$:

$$\varepsilon_{py^z}=\varepsilon_y\frac{1}{1-\frac{2\sqrt{R^2-z^2}}{2R+\Delta}\left(1-\frac{E_l}{E_f}\right)}. \tag{9}$$

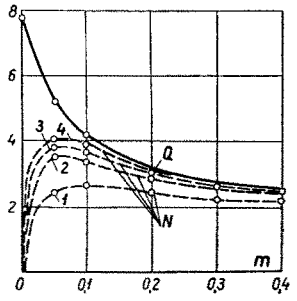

Fig. 2. $\eta = 2$ (1), 3 (2), 4 (3), 5 (4).

After substituting (9) in (4), we have

$$P_2 = 2\varepsilon_y E_p \left(\int_0^R \frac{dz}{1 - \frac{2\sqrt{R^2 - z^2}}{2R + \Delta}\left(1 - \frac{E_p}{E_f}\right)} - \int_0^{aR} \frac{dz}{1 - \frac{2\sqrt{R^2 - z^2}}{2R + \Delta}\left(1 - \frac{E_p}{E_f}\right)} \right). \tag{10}$$

The integrals of expression (10) are determined by means of the substitutions $z = R \sin t$, $\tan 1/2 = l$:

$$\int_0^R \frac{dz}{1 - \frac{2\sqrt{R^2 - z^2}}{2R + \Delta}\left(1 - \frac{E_p}{E_f}\right)} = QR, \tag{11}$$

where

$$Q = \frac{2(1+m)}{1 - \frac{E_p}{E_f}} \left(\sqrt{\frac{1}{1 - \left[\frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)\right]^2}} \times \operatorname{arctg} \sqrt{\frac{1 + \frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)}{1 - \frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)}} - \frac{\pi}{4} \right),$$

and

$$\int_0^{aR} \frac{dz}{1 - \frac{2\sqrt{R^2 - z^2}}{2R + \Delta}\left(1 - \frac{E_p}{E_f}\right)} = NR, \tag{12}$$

where

$$N = \frac{2(1+m)}{1 - \frac{E}{E}} \left\{ \sqrt{\frac{1}{1 - \left[\frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)\right]^2}} \times \operatorname{arctg} \left(\operatorname{tg} \frac{\arcsin a}{2} \sqrt{\frac{1 + \frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)}{1 - \frac{1}{1+m}\left(1 - \frac{E_p}{E_f}\right)}} \right) - \frac{\arcsin a}{2} \right\}.$$

Values of the functions $Q(m)$, $N(m, \eta)$ for $E_p/E_f = 0.05$ are presented in Fig. 2. Finally, for P_2 we can write

$$P_2 = 2\varepsilon_y E_p (Q - N) R = 2\sigma_p \frac{\frac{E_p}{E_f} + m\eta}{1+m} (Q - N) R. \tag{13}$$

The strength of the specimen in tension along the y axis

$$\sigma_y = \frac{P_1 + P_2 + P_3}{b}. \tag{14}$$

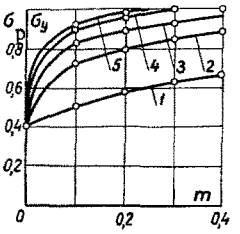

Fig. 3

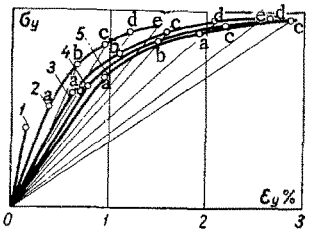

Fig. 4

Fig. 3. η = 1 (1), 2 (2), 3 (3), 4 (4), 5 (5).

Fig. 4. m = 0 (1), 0.1 (2), 0.2 (3), 0.3 (4), 0.4 (5); η = 1 (a), 2 (b), 3 (c), 4 (d), 5 (e).

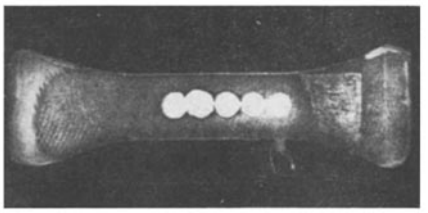

Fig. 5

After substitution in (14) we obtain

$$\sigma_y=\frac{2R}{b}\left\{a+\frac{\frac{E_p}{E_f}+m\eta}{1+m}\left[(Q-N)+\frac{b}{2R}-1\right]\right\}\sigma_p . \tag{15a}$$

For an elastic resin expression (15a) simplies to

$$\sigma_y=\frac{2R}{b}\frac{\frac{E_p}{E_f}+m}{1+m}\left(Q+\frac{b}{2R}-1\right)\sigma_p . \tag{15b}$$

For regular fiber reinforcement the ratio 2R/b takes the following values: for a rectangular distribution closed up along the z axis 2R/b = 1; for a square distribution 2R/b = 1/(1 + m); for a triangular distribution 2R/b = $(2/\sqrt{3})[1/(1+m)]$. If the polymer is locally reinforced with fibers, then the ratio 2R/b is given by

$$\frac{2R}{b}=\frac{1}{\frac{B}{2Rn}-1}; \qquad n=\frac{D+\delta}{2R+\delta}, \tag{16}$$

where B is the width of the specimen; n is the number of fibers in a row along the z axis; D is the width of the reinforced zone; δ = 2Rm for a square distribution; $\delta \cong (0.86m-0.14)2R$ for a triangular distribution. The secant modulus of elasticity of the specimen, defined as $E_y = \sigma_y/\varepsilon_y$, is equal to

$$E_y=\frac{2R}{b}\left\{a\frac{1+m}{\frac{E_p}{E_f}+m\eta}+\left[(Q-N)+\frac{b}{2R}-1\right]\right\}E_p . \tag{17}$$

Values of the strength of structures with a square distribution of the fibers, calculated for various $\varepsilon_p/\varepsilon_l$ at E_p/E_f = 0.05, are presented in Fig. 3. The stress-strain diagrams and the secant moduli, constructed from the data of Fig. 3, for ε_l = 3%, are presented in Fig. 4.

The experiments were performed on monolithic model specimens of cast resin reinforced with aluminum alloy rods (Fig. 5). The aluminum rods were chosen as a material capable of simulating the modulus of elasticity of glass fibers. The rods, 9 mm in diameter with a clean degreased surface, were fixed in the mold at a distance Δ = 1 mm apart and covered with a compound based on ÉD-5 resin (100 pts by weight) with triethanolamine (10 pts by weight) as curing agent and grade NV liquid Thiokol as plasticizer. Two modifications of the polymer, containing 20 and 40 pts by weight Thiokol, were investigated.

At the same time, we also molded unreinforced specimens of the polymer. The polymerization conditions were as follows: 60°C for 4 h, 80°C for 4 h, 100°C for 4 h, 120°C for 4 h, slow cooling. The degree

TABLE 1

Thiokol content of polymer, pts by wt.	Polymer specimens						Reinforced specimens		
	σ_p, kgf/cm^2	σ_{pr}*, kgf/cm^2	E_p, kgf/cm^2	ε_p, %	ε_l, %	η	σ_y, kgf/cm^2	ε_y, %	Mode of failure
20	780±20	600±30	$3 \cdot 10^4$	6±1	2,6	2,3	110±20	0,2+0,02	Adhesion
40	120±10	—	$3 \cdot 10^3$	30+5	4	7,5	100±15	3+1	Cohesion

* σ_{pr} is the proportional limit of the polymer.

of polymerization of the specimens thus obtained was 90-96%. Four batches of five specimens each were prepared by mechanical means. The total length of the specimens was 155 mm, the cross section of the neck measured 25 × 8 mm^2. The specimens were loaded in a UG 20/2 No. 158 testing machine, in which the grips separated at a rate of 50 mm/min. The stress–strain diagram was constructed by recording the strain corresponding to a certain load without interrupting the loading of the specimen. The strains were measured by resistance gages with a base of 10 mm connected to ID-61M No. 363 strain meter. The gages were bonded either directly to the specimen or to a strain-measuring device in the form of a U-shaped piece of spring steel attached to the neck of the specimen. This device was employed for investigating the deformation of specimens of the pure polymer.

The results of the tests on the specimens and the approximation of their stress-strain diagrams by an ideal elastoplastic body are presented in Table 1.

In accordance with Eq. (2a) plastic deformation of the polymer in reinforced specimens should occur after $\varepsilon_y \geq 0.36\%$; consequently, the resin with 20 pts by weight Thiokol deformed elastically up to a relative elongation equal, according to (2b), to $\varepsilon_{py} \cong 1.47\%$. Then followed adhesion failure at $\sigma_{adh} = 0.0147 \cdot 3 \cdot 10^4$ kgf/cm^2 $\cong$ 440 kgf/cm^2.

Theoretically, the strength of a reinforced polymer of the given composition, determined from Eq. (15b), where $\sigma_p = \sigma_{adh}$, is equal to σ_y = 125 kgf/cm^2, i.e., close to the experimental value. The calculated values of the breaking strength and strain of reinforced polymers of the second composition, calculated from Eqs. (15a) and (2a), respectively, are equal to σ_y = 95 kgf/cm^2 and ε_y = 2.7%. In specimens of the given composition the adhesion strength exceeded the cohesion strength (~ 120 kgf/cm^2) and failure occurred as a result of the total deformation of the resin.

SUMMARY

The strength of reinforced polymers in tension across the fibers depends importantly on the density of the reinforcement. The greater the density of the reinforcement, i.e., the volume fiber content, the lower the strength of the composite. In the limit the strength of the composite reaches that of the unreinforced polymer. An increase in the elasticity of the polymer without loss of strength is accompanied by increases in the strength and deformability of the composite. The relations obtained are confirmed by experiments on model specimens of an epoxy-Thiokol polymer.

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