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Time representation: A taxonomy of temporal models

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Abstract. The objective of the paper is to provide a taxonomy of temporal systems according to three fundamental considerations: the assumed axiomatic theory, the expressiveness, and the mechanisms for inference which are provided. There is an discussion of the significance of the key features of the taxonomy for computer modelling of temporal events. A review considers the most significant representative systems with respect to these issues, including those due to Bruce, Allen and Hayes, Vilain, McDermott, Dechter *et al.*, Kahn and Gorry, Kowalski and Sergot, Bacchus *et al.*, and Knight and Ma. A tabular comparison of systems is given according to their main structural features. In conclusion, the characteristics of a general axiomatic system capable of representing all the features of these models is discussed.

Key Words: time representation, temporal system, axioms, semantic analysis

1. INTRODUCTION

In this article we consider the characteristics of modelling systems which have been proposed for capturing the temporal properties of events and processes in computer based systems. The objective is to give a taxonomy of systems, according to some fundamental features. We start with a brief discussion of what the fundamental features are, and why they are important for use in computer modelling.

Basic to all computer systems dealing with temporal events is an assumed theory of time. We require that this theory satisfies our intuitive notions of time, so that we can say that the real world is a model of the theory. By this, following the ideas of Suppes (1961), and of Funk (1983), we mean that the statements of the theory may be interpreted as true in the real world. The most common theoretical basis is the standard time-point system assumed by classical physics. In this theory, the time domain consists of a continuum of time points, isomorphic to the real line. Time intervals are taken as intervals on the real line, and duration of intervals is the real number difference of their start and end points. However, for many applications, particularly those in artificial intelligence and natural language understanding, the time-point system is not ideal for either the expression of temporal facts, or for the storage and organisation of incomplete temporal knowledge. For these applications, other theories have been proposed, for example, based on time intervals as primitive rather than time points.

The importance of the theory to a database system is as a basis for reasoning over the database. Inference may be performed over the stored data, by logical deduction from the axioms of the assumed theory. In some systems, no formal mechanism for inference is proposed, it being left to the user to draw inference from the database. In other systems a deduction system is proposed, in which rules are provided that allow deduction of true facts by forward chaining from the database. It is a characteristic of these systems that they are undirected, and do not allow the specific determination of a given query. Finally, some systems provide a consistency checker, and allow deduction by refutation. In these systems the user may enter a specific query, and the system checks whether it, or its negation, is inconsistent with the database. In this way the system may deduce whether a fact is: known true, known false, or unknown.

This view of temporal systems leads us to attempt a characterisation of temporal systems according to three basic elements, as follows:

- *The assumed axiomatic theory*: For all of the systems which we shall consider, there exists an underlying theoretical basis. For some systems this basis is formally described, and for others it remains assumed as intuitively agreed.
- *The expressiveness of the modelling language*: A computer based system may be viewed as a model of the fundamental theory, in the form of a finite data base of temporal facts. Given that the model is incomplete by reason of storage limitations, there is a drive for efficient storage and retrieval of incomplete temporal knowledge. Expressive modelling languages allow the storage of temporal information which is incomplete in various fashions.
- *The reasoning mechanisms which are provided*: Deductive inference may be performed on the stored data, with reference to the underlying theory, so that any fact which can be proved from the axioms of the theory and the stored temporal database may be assumed true by inference. In this way, the axioms plus database may be viewed as a deductive system from which facts may be retrieved by inference.

In summary, we can capture some fundamental characteristics of existing temporal systems with respect to the following set of questions:

- What are the assumed primitives?
- Is there a formal theory?
- What are its good/bad features of expression?
- What is its application domain?
- What reasoning mechanisms are there?
- Is there a consistency checker?

In Section 2, we address the major issues of theory which characterise systems at a fundamental level. The questions of expressiveness and inference are particular to proposed systems, and these are discussed in a review of some representative temporal models in Section 3. Section 3.10 provides a summary table characterising these mdoels. In Section 4, the characteristics of a general axiomatic system capable of representing the features of these models is discussed.

2. MAJOR THEORETICAL ISSUES

The theoretical nature of time is a question with a long philosophical tradition and the literature seems full of disputes and contradictory theories. This contrasts sharply with the commonly held view of time, which allows people to cope easily with time in their everyday life. However, there are several major issues which should be addressed in terms of the theoretical basis of proposed systems. These issues are as follows:

2.1. *The primitive nature of time*

This is the issue of what should be taken as the primitive elements of time. There are three known choices: points, intervals, or both of them. Additionally, there are two fundamentally different treatments of interval based systems. In the first, intervals are assumed to consist of points, and hence, the corresponding systems may be considered as models of point-based time theories. An example of this kind of interval is the *time-segment* of Bruce's model for temporal references (Bruce 1972). However, as Allen has commented (Allen 1981, 1983), modelling intervals by taking their ending-points can lead to problems: the annoying question of whether ending-points are in the interval or not must be addressed, seemingly without any satisfactory solution. The second treatment takes intervals as primitive objects without any definitions of the "ending-point" and "internal-point" structures. Allen's interval logic (Allen 1981, 1983; Allen and Hayes 1989), Vilain's temporal system (Vilain 1982; Vilain and Kautz 1986), Knight and Ma's extended temporal model (Knight and Ma 1992, 1993), are examples that treat intervals as primitive.

2.2. *Ordering relations*

Whatever primitive time elements are taken, all time systems must adopt axioms defining some sort of ordering relations. Two fundamental issues are associated with time ordering: the density of time elements, and the linearity of the time axis. We discuss these issues in the following sections.

2.2.1. *Density of time*

The density question is associated with the choice of whether the set of time-elements should be modeled as a continuum (such as rationals or reals) or as a discrete set (such as integers). For time-points, can we always assume that between any two distinct time-points there is at least another time-point? For time-intervals, can we always assume that any interval can be decomposed into two distinct contiguous intervals? If so, then the primitive elements form a dense system. The alternative assumption is that of discrete time, "whereby each time (except the first and last if there is a beginning or end to time) is sandwiched between unique **previous** and **next** times" (Galton 1990).

The fact that the database must consist of a finite set of time-elements has no bearing on the density question at all, which is a question of the assumed theory only. This theoretical issue impinges upon the inferencing mechanisms which may be used to derive facts from the database, insofar that the denseness assumption is needed to prove the consistency algorithms.

2.2.2. *Linearity of time*

This issue refers to whether the time axis can be always considered as *linear* or *non-linear.* Linear time corresponds to the classical physical model of time, where the structure is that of the real time, extending indefinitely in both directions. The majority of time modelling approaches consider the time axis as being linear, that is, there is a total order over the whole set of time elements. However, non-linear time structures have been proposed, where the fundamental order relation allows topologies such as *branching time, parallel time* and *circular time,* etc.

It is questionable whether computer based systems really require non-linearity to be built into the temporal axioms, since its raison d'être appears to be involved with a lack of knowledge of temporal events, rather than with our intuition about time itself. For example, parallel time lines have been proposed as a way of modelling separate parallel processes. However, it is a limitation in our knowledge which gives rise to the parallelism. We believe that the two processes are actually operating in the same linear time — it is just that we have no knowledge of synchronisation. We do not need a theory of parallel time lines for this application; what we need is a model which allows us incomplete knowledge of synchronisation over a single linear time line. Similarly, branching time is proposed as a useful model to handle possible worlds, uncertainty about the past or the future and the effects of alternative actions when planning. However, it is arguable whether we need a theory which assumes that time itself branches in order to model possible worlds, rather than a model which expresses our limited knowledge of causality in possible worlds over a single linear time. For most applications, linearity is sufficient at the theoretical level. This corresponds with the usual assumption of classical physics where all events may be universally synchronised with a single time measure. Only if we wish to model relativity would we be unable to assume synchronisation of distant events at a theoretical level.

2.3. *Duration assignment*

In most applications, it is expected that a temporal system can support duration reasoning. For example, if it is known that interval I_a and interval I_b start together and that the duration of I_a is greater than duration of I_b, we may infer that I_b finishes before I_a. This inference can be made by use of duration knowledge.

The duration assignment to time elements may be characterised by a function from the set of time elements to $\mathbf{R}_0^+$, the set of non-negative real numbers. Intuitively, of course, the duration of the points should be sero, while the durations of intervals are positive. For point-based intervals, their durations may be derived from the distance between their greatest lower bound and least upper bound. However, for systems which treat intervals as primitive, their durations may be directly defined by an abstract function from intervals to positive reals. Given a duration assignment over time elements, some corresponding operators, such as *addition*, may be required to be defined, providing consistency of the whole system.

3. SOME REPRESENTATIVE MODELS

In this section, we review some representative temporal models, with respect to the fundamental issues addressed in the introduction.

3.1. *Bruce's temporal model*

An early attempt at mechanizing part of the understanding of time within an artificial intelligence is Bruce's model for temporal reference (Bruce 1972). In this system a formal framework, based upon first-order logic, is established for the analysis of tenses, time relations, and other references to time in natural language. The axioms of the framework are based on the following definitions: A *time-system* is a pair, (*time*, $\leqslant$), where *time* is a set whose elements are called *time-points*, and $\leqslant$ is a partial order over *time*. Because there is nothing that has been defined about *time* other than that it is partially ordered by $\leqslant$, the theory allows linear time or branching time, discrete time or dense time. The theory is thus more general than that for the standard point-based system, and inferencing mechanisms must be built on weaker axioms.

Bruce then defines point-based intervals, termed *time-segments*, as chains which are convex in the sense that there are no points missing within the chains, where a chain is a totally ordered subset of *time-points*. The related issues about time-segments, such as: density and linearity, may hence be derived from the corresponding issues of the time-points which make up the time-segments. The ordering relations between segments are also inherited from the partial order over the time points. Bruce gives seven binary relations between *time-segments*, which can be derived from the ordering relations over their greatest lower bounds and the least upper bounds: *Before, During, Same-time, Overlaps, After, Contains* and *Overlappped*. In terms of these binary relations, a *tense* is defined as a special n-ary relation on time-segments with the following form:

$$R_1\_R_2\_\ldots\_R_{n-1}(S_1, S_2, \ldots, S_n) \equiv$$
$$R_1(S_1, S_2) \wedge R_2(S_2, S_3) \wedge \ldots \wedge R_{n-1}(S_{n-1}, S_n)$$

where each S_i is a time-segment and R_i is a binary relation between S_i and S_{i+1}.

S_1 is called the *time of speech*, $S_2, \ldots, S_{n-1}$ are called the *times of reference*, and S_n is called the *time of event*. For example, the following sentence

- He will have been going to be going to go

has the tense

$$Before\_After\_Before\_Before(S_1, S_2, S_3, S_4, S_5) \equiv$$
$$Before(S_1, S_2) \wedge After(S_2, S_3) \wedge Before(S3, S_4) \wedge Before(S_4, S_5)$$

where S_1 is the time of speech, S_2, S_3, S_4 are reference times, and S_5 is the time of event.

Bruce provides a natural language system, termed *CHRONOS*, which consists of a simple English sentence parser, a theorem prover, and a database of facts and events. The system accepts facts about events from the user and the information which is given by tense and time relations can be combined with other facts to allow inferences about the temporal ordering of events. However, a consistency checker for the database has not been provided explicitly. No heuristics are used in searching the network of temporal ordering links. Additionally, as argued by Allen (see next section), there are some problems in dealing with the treatment of open or closed intervals. Mechanisms for duration reasoning are not specified, although these may be defined by introducing a mapping from the time-points to the reals.

3.2. *The interval logic of Allen*

Allen introduces his temporal logic in order to provide a framework for the naive treatment of two major subareas of artificial intelligence: natural language processing and problem solving. Instead of adopting time points (or states which are associated with time points), he takes intervals as the primitive temporal quantity, as being the natural means of human reference to time. As an example, in (Allen 1983), Allen gives the following story:

> Ernie entered the room and picked up a cup in each hand from the table. He drank from the one in the right hand, put the cups back on the table, and left the room.

In this account we can identify several time intervals, e.g.: the time Ernie was in the room, the time between entering the room and picking up each cup, the time between putting down the cups and leaving the room, and many others. However, the claim is that intervals are sufficient for modelling all the temporal references in human accounts such as this. Even references to apparent point events, such as the time Ernie entered the room, or the time that he put down a cup, are best modelled as small time intervals. The argument is put forward that all apparently instantaneous events can be decomposed further if we examine them more closely. For example, "entering the room" may be decomposed into: opening the door, moving through the doorway, and closing the door. And again, "opening the door" can be decomposed into turning the handle and pushing the door open. As Allen puts it (Allen 1983)

There seems to be a strong intuition that, given an event, we can always "turn up the magnification" and look at its structure.

In order to express temporal relationships over time intervals, Allen took originally as primitive a set of nine (mutually exclusive) basic binary relations between any two intervals (Allen 1981), extended later to 13 (Allen 1983): *Equal, Before, Meets, Overlaps, Starts, Started-by, During, Contains, Finishes, Finished-by, Overlapped-by, Met-by, After.* These are based on Bruce's seven relationships, but whereas Bruce's relations are derived from the partial order within a point-based theory, Allen's are taken as primitive.

These relationships are later formally defined in terms of the single primitive relation "*Meets*" by Allen and Hayes (1989). This is done by positing the existence of related intervals for some relations. For example:

$$Before(i_1, i_2) \Leftrightarrow \exists i(Meets(i_1, i) \wedge Meets(i, i_2))$$

In Allen's system, consistency checking is performed by formation of the transitive closure, according to a transitivity table with 144 entries which describes the composition of the thirteen (mutually exclusive) relations. If no conflict is found according to the exclusivity, then the system is consistent. For example, for the system:

$$Before(a, b), Before(b, c)$$

we may use the transitivity entry:

$$Before(i_1, i_2) \wedge Before(i_2, i_3) \Rightarrow Before(i_1, i_3)$$

to deduce that $Before(a, c)$. Hence facts may be derived by forward chaining from the database, using the transitivity rules (termed truth propagation by Allen). Possible inconsistencies in a database can also be established by truth propagation. For example, from:

$$Before(a, b), Before(b, c), Before(c, a)$$

we can deduce $Before(a, c)$ from the first two predicates, and $After(a, c)$ from the third. Hence we have two distinct relations between a and c, which are not allowed due to the exclusivity of temporal relations.

Allen and Hayes show that the transitivity table in (Allen 1983) is a result of the their axioms in (Allen and Hayes 1989), following the intuitive reasoning by possibel cases which has been used to construct the table originally. Additionally, in (Allen 1983), Allen has suggested that duration reasoning may also be incorporated into the interval-based system by giving examples of rules for duration reasoning. For example:

$$During(a, b) \vee Starts(a, b) \vee Finishes(a, b) \Rightarrow$$
$$duration(a) < duration(b).$$

However no comprehensive mechanism has been proposed, and hence the duration reasoning is rather weak.

The most disputed aspect of Allen's system is its exclusion of time points as primitive, although in the later paper (Allen and Hayes 1989), Allen and Hayes define a point as the "*meeting place*" of intervals or as a maximal set, termed "*nest*", of intervals that share a common intersection, at a subsidiary status within the theory; and use the concept of a "moment", i.e., a very short interval which is non-decomposable, to model some instantaneous events. Their contention is that nothing can be true at a point, for a point is not an entity at which things happen or are true (Allen 1983). Except for the assumption that moments have positive length, while points have zero length, another obvious structural difference between points and moments is that moments are treated as primitive objects, and hence can meet other intervals (although they are not allowed to meet other moments), while points are not treated as primitive objects and cannot meet anything (Allen and Hayes 1989).

However, as Galton shows in his critical examination of Allen's interval logic, Allen's theory of time is not adequate, as it stands, for reasoning correctly about continuous change (Galton 1990). This problem stems from Allen's determination to base his theory on time intervals rather than on time points, either banishing points entirely, or, latterly, relegating them to a subsidiary status within the theory. The following example of a ball thrown vertically into the air intuitively shows the problem involved with references to time points: The motion may be described qualitatively by the use of two intervals, interval i_1 where the ball is going up, and interval i_2 where the ball is coming down. According to classical physics, there is a point where the ball is stationary. In the interval calculus, we have two alternatives: we may assume that there is a small interval where the ball is stationary, or we may assume that interval i_1 "*Meets*" interval i_2. The first alternative does not seem tenable, not being consistent with the laws of physics. On the other hand, the second alternative also gives problems, since the interval calculus allows us to combine two intervals which meet; that is, $i_1 \oplus i_2 = i_3$. However, although both of the intervals i_1 and i_2 have the property "ball-in-motion", the combined interval i_3 doesn't have this property.

3.3. *Vilain's temporal system*

Noting that intervals are not the only mechanism by which human beings understand time, another common construct being that of time points, Villain (Vilain 1982; Vilain and Kautz 1986) proposes a system which handles time points in much the same way that it handles intervals. The logic of points is arrived at by expanding Allen's logic of intervals: adding new primitive relations and composition rules over them to Allen's interval logic. The new primitive relations may be classified into three groups:

(Point-Point) *Equal, Before, After,* which relate points to other points;

(Interval-Point) *Before, Started-by, Contains, Finished-by, After,* which relate intervals to points;

(Point-Interval) *Before, Starts, During, Finishes, After,* which relate points to intervals.

The mechanism by which Vilain's system makes deductions about points is an extension of that which it uses to make deductions about intervals. In an approach similar to that of Allen, the system maintains a "complete picture" of all relations over intervals and points by means of a transitive closure operation. The operation is performed over the expanded set of composition rules in the newer logic.

However there is a critical omission from the primitive relations between points and intervals in Vilain's system; for the "*Meets*" relation is defined only between intervals and is not allowed between points and intervals. Hence, the problems in modelling continuous change by Allen's system mentioned by Galton in (Galton 1990) still exist in Vilain's system. For example, the system is still not capable of modelling the processes of a ball thrown vertically into the air: Let interval i_1 refer to *ball-going-up*, point p refer to *ball-stationary*, and interval i_2 refer to *ball-coming-down*. On the one hand, it is easy to see that p is neither in i_1 nor i_2. On the other hand, according to Vilain's classifications of relations over points and interval, point p is not allowed to meet or be met-by any interval. Hence, we deduce that p is after i_1 and before i_2, that is, there is another time element between i_1 and p, and another time element between p and i_2. This is obviously contrary to our intuition of the processes.

N.B. In (Beek 1992), Beek has proposed an interval-based framework, *IA*, and point-based framework, *PA*, for representation of and reasoning about incomplete and indefinite qualitative temporal information. However, it is interesting to note that the frameworks, *IA* and *PA*, deal with temporal relations between intervals, and relations between points *separately*, that is, the interval-based framework *IA* deals with the thirteen temporal relations (defined by Allen (1981)) between intervals only, while the point-based framework *PA* deals with temporal relations between points only, which are addressed in Vilain and Kautz's point algebra (Vilain and Kautz 1986). Relations between intervals *and* points, such as that proposed in (Vilain 1982), are not addressed at all. Additionally, like Dechter *et al.*'s framework (see next section), time intervals are not defined as primitive. Indeed, time intervals, and temporal relations between intervals are defined in terms of points (rationals) and the corresponding order relations between points, respectively.

3.4. *Dechter, Meiri and Pearl's TCSP*

Dechter *et al.* (1991) have presented a unified approach to temporal reasoning based on constraint-network formalism. In this framework of temporal constraint satisfaction problems (*TCSP*), variables represent time points, and temporal information is represented by a set of unary and binary constraints, each specifying a set of permitted intervals. The unique feature of this framework lies in the inclusion of duration information, namely, time differences between events. Algorithms are presented for performing some reasoning tasks, such as finding all feasible times at which a given event can occur, finding all possible relationships between two given events, and generating one or more scenarios consistent with the information provided. A *TCSP* involves a set of variables, $X_1, \ldots, X_n$, having continuous domains; each variable represents a time point. Each constraint is represented by a set of intervals: $\{I_1, \ldots, I_n\}$, where these intervals are similar to Bruce's time-segments, that is, they are point-based, may be closed, open, or semi-open. A simple temporal problem (*STP*)is a *TCSP* in which all constraints specify a single interval. The duration of an interval may be defined by the distance between its greatest lower bound and least upper bound. Relations between intervals, such as the thirteen relations defined by Allen, may be derived from the known total order relation among their greatest lower bound and least upper bound. Consistency checking for a *TCSP* is transformed to a corresponding examination of its graphic representation.

The theory is formally stated, with points and real numbers as primitives, and intervals being constructed out of points. It assumes a dense set of time-elements, but time may be branching. Duration reasoning is encompassed by the system by means of a consistency checking algorithm. The limitation of the *TCSP* model is it's assumption that point based intervals have the same open\closed nature, that is, either intervals are all assumed to be closed, or they are all assumed to be open (semi-open). This assumption can lead to problems: if intervals are all closed then adjacent intervals have ending-points in common, which when adjacent intervals correspond to states of truth and falsehood of some property, can lead to situations in which a property is both true and false at an instant. Similarly, if intervals are all open, there will be points at which the truth or falsity of a property will be undefined (The solution in which intervals are all taken as semi-open, so that they sit conveniently next to one another, seems arbitrary and unsatisfactory).

3.5. *The time specialist of Kahn and Gorry*

In order to store, retrieve, and reason about temporal information, Kahn and Gorry (1977) have designed and implemented a module, called the *time specialist*, to maintain separate mechanisms for dealing with dated and undated information. The time specialist is endowed with the capacity to order temporal facts in three major ways:

(1) relating events to dates,
(2) relating events to special "reference events",
(3) relating events together into before-after chains.

The time specialist can answer different types of questions such as:

- Did event X happen at time expression T?
- When did event X happen?
- What happened at time expression T.

The time specialist is able to make deductions and check if they are consistent with the facts known in the database. However, it is weak if the time indications are not definite. Also, each of the three methods to organize temporal statements has its own special data structures and routines to work with those structures. For a given set of temporal facts, it is up to the user, not the time specialist, to choose the most appropriate methods.

The time specialist can check the consistency of the latest fact with facts previously accepted, and try to resolve inconsistencies through interaction with the user. In such an interaction, the user may withdraw either the new fact, or some old facts whose removal would lead to consistency. However, removing old facts may involve undoing some prior deductions. In order to be able to do this, a deduced fact is marked by those facts used to deduce it.

No formal theory is stated as a basis for the time specialist. The basis for temporal reasoning is contained in the algorithms which make up the system.

3.6. *The temporal logic of McDermott*

McDermott (1982) has developed a first-order temporal logic to provide a versatile "common-sense" model for temporal reasoning. In accordance with the "naive physics" advocated by Hayes (1978), McDermott adopts an infinite collection of states (points) as the primitive temporal elements and adds several crucial axioms. Every state has a time of occurrence, $d(s)$, a real number called its *date.* Time is assumed to be a continuum, with an infinite numbers of states between any two distinct states, where states are partially ordered by the "*no later than*" order relation "$\leqslant$". The future (not the past) is branching, that is, there are many possible futures branching forward in time from the present. Each single branch, called a "*Chronicle*", consists of a connected series of states and is isomorphic to the real line. Developing his theory, McDermott examines three major problems that a temporal reasoning system must face: reasoning about causality, reasoning about continuous change, and planning actions.

McDermott's system has formal axioms with time-points (states) and reals as primitives. The theory assumes a partial ordering relation, which gives rise to branching time. Reasoning is via the assumed theory of the real numbers, and no special mechanisms are needed. We can represent a time state, s, as the pair (C_s, t), where $t = d(s)$ and C_s is the set of chronicles that s belongs to. Possible events may be associated with time states.

For illustration, we shall consider the example of a man, called John, planning a trip to the theatre. We assume that a decision will be made to go by train or bus. If the decision is made to go by train at time s_{train1}, where $d(s_{\text{train1}}) = t_1$, then John will arrive at the theatre at time s_{train2}, and the play will start at time s_{train3}, where $d(s_{\text{train3}}) = t_3$. All of these time states lie on a chronicle c_{train}. Alternatively, if the decision is made to go by bus at time s_{bus1}, where $d(s_{\text{bus1}}) = t_1$, then he will arrive at the theatre at time s_{bus2}, and the play will start at time s_{bus3}, where $d(s_{\text{bus3}}) = t_3$. All of these time states lie on chronicle c_{bus}. These events and states may be represented by the following data:

(decides-to-take-train,	c_{train},	t_1)
(arrives-at-theatre-by-train,	c_{train},	t_2)
(play-starts,	c_{train},	t_3)
(decides-to-take-bus,	c_{bus},	t_1)
(arrives-at-theatre-by-bus,	c_{bus},	t'_2)
(play-starts,	c_{bus},	t_3).

Here, s_{train1} has been represented by the pair (c_{train}, t_1), s_{train2} by $(c_{\text{train}}, t_{\text{train2}})$ etc.

In this example, illustrated in Figure 1, we see that time states divide into two separate chronicles c_{train} and c_{bus}, from the state s_0 as a result of the John's decision. Although it is obviously possible for us to compare times on different chronicles by means of the t component, McDermott uses the "*no later than*" relation over time states which is restricted to states on the same chronicle. This is to prevent us from making "no later than" comparisons for events which cannot both occur in reality. For example, we are not allowed to ask whether he arrives at the theatre by bus before he arrives by train, since he cannot do both. These two events are said to be in different possible worlds (i.e. chronicles).

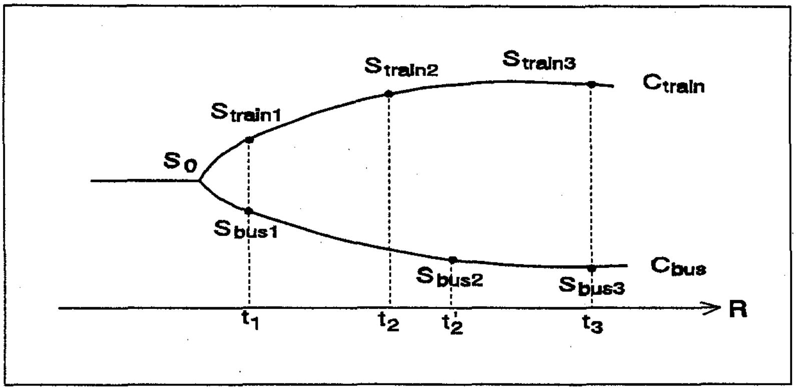

Fig. 1.

McDermott also provides axioms which ensure that chronicles branch only into the future, and this limits the expressiveness of the logic. For, in the example, we have the event "play starts" on two different chronicles which cannot be compared. Using McDermott's logic we must view these as two separate events: "play starts after John's arrival by train", and "play starts after john's arrival by bus". Since we may judge that the play is independent of John, we may wish to join the two chronicles at the state that play starts.

It is in fact arguable whether we need to consider time as branching in order to model possible worlds. In fact, it is possible to conceptualise the world number, or chronicle, as related to the event data, and not to the time. For example, we can regard the predicate:

(decides-to-take-train, c_{train}, t_1)

as relating:

(event, possible_world, time)

rather than:

(event, time_state)

In this case, time elements are standard linear dense time points, and the axioms for chronicles can be specified independently of those for time.

3.7. *Kowalski and Sergot's event calculus*

The *event calculus* of Kowalski and Sergot (1986) is an approach for representing and reasoning about time and events within a logic programming framework. It is based in part on the situation calculus (McCarthy 1963; McCarthy and Hayes 1969), but focuses on the concept of events as highlighted in semantic network representations of case semantics. Its main intended application is the representation of events in updating databases (Kowalski 1992) and discourse representation.

Primitives of the theory are events, which are considered to be structureless "points" in time, where "point" is used here only to convey the lack of internal structure. Events start and finish periods of time, during which states are maintained. Events are considered to be after the time periods that they finish and before the time periods that they start, not fully contained within either of these periods.

Sadri (1987) has illustrated a number of the general characteristics of the event calculus:

(1) Event descriptions can be assimilated in any order, independent of the order in which events actually take place.
(2) Events can be used for temporal references and need not be associated with absolute times.
(3) Events can be simultaneous.
(4) Events can be partially ordered.
(5) All updates are additive. The effect of deletion is obtained by adding information about the end of periods.
(6) The event calculus rules are in Horn clause logic augmented with negation by failure.
(7) The event calculus allows events to be input with incomplete descriptions.

In (Kowalski 1992), Kowalski specially investigates the case of the event calculus connected with database updates. The way in which relational databases, historical databases, modal logic, and the situation calculus deal with database updates is discussed in detail. It is claimed that the event calculus may overcome the computational aspects of the frame problem in the situation calculus, and it can be implemented with an efficiency approaching that of destructive assignment in relational databases. Bernard *et al.* (1991) have recently presented an adaptation of the event calculus to the problem of determining the temporal structure of operations that must be performed during the realization of some complex objectives. In (Borillo and Gaume 1990), an extension to Kowalski's event calculus model is proposed by Borillo and Gaume, by means of the additional spatial component, and the introduction of uncertainty and a general abstract relation among propositions.

The formal theory of Kowalski and Sergot's *event calculus* may be taken as the Horn clause system plus negation by failure. However, the use of negation by failure introduces a procedural element into the axioms. In this respect, the system is thus akin to the time specialist, in that the theory is presented in terms of algorithms.

3.8. *Bacchus, Tenenberg and Koomen's BTK*

Bacchus, Tenenberg and Koomen present a many-sorted temporal logic, termed *BTK* (Bacchus *et al.* 1991), for reasoning about propositions whose truth values might change as a function of time. In order to provide a clear semantics and a well-studied proof theory, Bacchus *et al.* partition both the universe of discourse and the symbols of their language into two sorts, temporal and non-temporal, by which time is given a special syntactic and semantic status without having to resort to reification. In *BTK*, propositions are associated with time objects by including temporal arguments to the functions and predicates, where terms and wffs are defined in the standard fashion, with the only restriction being that arguments of the correct sort must be given for each function and predicate.

Actually, *BTK* is sorted in much the same way as Shoham's *reified logic* (Shoham 1987a, b). Unlike Shoham's first-order logic in which propositions are expressed just with respect to a pair of time points (denoting a time interval), propositions in *BTK* can be expressed and interpreted with respect to any number of temporal arguments: there is neither a syntactic commitment to the number of temporal objects that any function or predicate may depend upon, nor is there any commitment to interpreting the temporal objects as either intervals or points.

It is interesting to noted that, in their paper (Bacchus *et al.* 1991), Bacchus *et al.* have shown that Shoham's logic can in fact be subsumed by *BTK* by defining two transformations, a syntactic transformation, π_{syn}, and a semantic transformation, $\pi_{sem} \cdot \pi_{syn}$ maps sentences of Shoham's logic to sentences of *BTK*, while π_{sem} maps models of Shoham's logic to mdoels of *BTK*. Additionally, Bacchus *et al.* argue that Shoham's categorization of propositions over point-based time intervals may also be translated to *BTK*, and the ontology of *BTK* is richer since it allows time intervals to be the primitive temporal objects rather than being defined as pairs of time points.

The major difficulty involved in reasoning in a *BTK* system lies in reasoning with the temporal terms, while the complexity of reasoning is highly dependent on the nature of the temporal domain. However, in *BTK*, there is no axiomatisation characterising the time structure. This question is left open, so that the temporal domain of *BTK* may be defined to be any temporal structure which can be characterised by a set of axioms, for example that of Bruce (1972), of Allen and Hayes (1989), or of McDermott (1982). A complete proof theory may then be generated by adding the axioms for the temporal domain to the fundamental axiomatisation of the logic.

3.9. *Knight and Ma's ETM*

As mentioned in Sections 3.2 and 3.3, there are some difficulties with Allen's and Vilain's approaches in the qualitative modelling of everyday occurrences. The authors have proposed an extended temporal model, *ETM* (Knight and Ma 1992), which treats both intervals and points as primitive time elements on the same footing, and supports duration reasoning and consistency checking.

The definition of a temporal system supporting duration reasoning consists first of a definition of an underlying well-ordered set E. The elements of the elementary set E may be both points and intervals with a duration assignment which is defined by a mapping from the primitives to the non-negative reals. The temporal system is then defined as the closure, T, of E under the binary operations $\oplus$, representing the combination of adjacent elements, and the conventional addition of the corresponding durations. This model provides axioms for a single successor relation, termed "*Meets*", over time intervals and points, and supports duration reasoning, which has been a problematic aspect in many temporal systems. Excepting the axiom that the duration of an interval is positive while the duration of a point is zero, the differentiating property between intervals and points which is proposed is that while intervals may meet points or intervals, points are not allowed to meet points, although they can meeõt (or be met-by) intervals. This characteristic is in line both with modelling requirements where points are defined as separators or end-points of intervals, and with the denseness of points on the real line. But this is the only extra requirement which is made of elements if they are to be points. According to their definitions, points, as primitive elements of *ETM*, are different from either Allen's points or moments. It seems that Allen's moments may be taken as the elementary intervals in E.

An intuitive graphical representation of an incomplete temporal system, (K, M_K, D_K), is introduced in terms of a directed, partially weighted graph, where K is a set of time elements, and M_K, D_K are the "*Meets*" knowledge and duration knowledge over K, respectively. And necessary and sufficient condition for the consistency of an incomplete system (K, M_K, D_K) (Knight and Ma 1992), and the corresponding limited system (K, M_K) (Knight and Ma 1993), is formally presented.

If we let intervals i_1 and i_2 refer to *ball-going-up, ball-coming-down* respectively, and point p refer to *ball-stationary*, we can now satisfactorily model the processes of a ball thrown into the air (see Sections 3.2 and 3.3) as: $Meets(i_1, p)$ and $Meets(p, i_2)$.

Additionally, although intervals are taken as primitive, as in Allen's system, the *ETM* allows formal expression of open and closed nature of intervals with the following meaning:

interval i is left-open at point p iff $Meets(p, i)$;
interval i is right-open at point p iff $Meets(i, p)$;
interval i is left-closed at point p iff $\exists i'(Meets(i', i) \wedge Meets(i', p))$;
interval i is right-closed at point p iff $\exists i'(Meets(i, i') \wedge Meets(p, i'))$.

which is in fact consistent with the conventional meaning of the "open" and "closed" nature for point-based intervals.

In terms of "*Meet*", 30 relations over intervals and points may be formally defined. This is indeed an extension of Vilain's primitive relations (see Section 3.3), by means of adding four critical relations: *Meets, Met-by* that relate intervals to points, and symmetrically, *Meet, Met-by* that relate points to intervals.

The consistency condition given in *ETM* implies an inferencing mechanism including duration reasoning. It is straightforward to prove that all Allen's duration reasoning rules are explicit results of the inferencing mechanism, by using the consistency condition.

A limitation of this system is the assumption of discreteness. The theory on which the system is based assumes a discrete set as time domain. However, since any computer based system must be in a finite form, this requirement does not in fact place any restriction on the application field. In Section 4, it is proposed that the system may also be based on a dense time domain.

3.10. *Overview of models*

Based on the above discussions, we present an overview of these representative temporal models in terms of Table 1.

4. CONCLUSION

In this paper, we have examined the bases of various temporal systems, concentrating on the differences of approaches taken. However, apart from differences of terminology, the models show a commonality of structure at a fundamental level. All the systems rely on theories based on a primitive set of time elements, which may be points, intervals, or both of them. The systems are axiomatised by primitive order relations over the time elements. This suggests the question as to whether a general axiomatic system is possible, which will express this common structure at a theoretical level. We first discuss the properties that we might wish for a general axiomatic system, and then how it might be possible to achieve it.

To start we ask the question as to what we might ideally require of a general axiomatic system. Firstly, we might require that it should take both intervals and points as primitive time elements, and thus allow point-based, interval-based, or point- and interval-based models. Secondly, primitive order relations should be defined over the primitive time elements, from which the order relations for the main temporal systems, outlined in Section 2, may be derived. For point based systems such as those of Bruce, of Dechter, Pearl and Meier, and of McDermott, the primitive order is "no later than". For interval based systems, it is "*Meets*", in terms of which thirteen possible temporal relations can be defined. Thirdly, a primitive duration function is needed, assigning a real number to each time element.

To ensure the generality of the axiomatisation, it should allow discreteness or denseness of time, which could be specified by additional axioms if required. It should also provide a special axiom for linearity of time, without which the time structure is branching. Finally, we would like a consistency checking algorithm for any finite database of temporal facts to be proved from the axioms, so that inference by refutation is possible.

In (Allen and Hayes 1989), Allen and Hayes have provided formal axioms

Table 1

Issue → Model ↓	Primitive	Ordering relation	Theory	Duration reasoning	Inference mechanism
Bruce's CHRONOS	point-based intervals	7 binary relations	formal	no	refutation (no consistency checker provided)
Allen and Hayes' *interval logic*	primitive intervals	13 binary relations formed by "Meets"	formal	weak	deductive rules (transitivity table)
Vilain's *temporal system*	primitive intervals and points	26 intuitive binary relations	no formal axioms	no	deductive rules (transitivity table)
Dechter, Meiri, and Pearl's *TCSP*	point-based intervals	total order	formal	yes	refutation (consistency checker provided)
Kahn and Gorry's *time specialist*	points (event dates)	"before-after" chains	no formal axioms	no	not formal
McDermott's *temporal logic*	points (states)	"no later than" binary order	formal	no	assumed theory of the real numbers
Kowalski and Sergot's *event calculus*	points (events)	partial order	no formal axioms	no	resolution (negation by failure)
Bacchus, Tenenberg and Koomen's *BTK*	not specified	not specified	not presented	no	none
Knight and Ma's *ETM*	primitive intervals and points	successor relation	formal	yes	refutation (consistency checker provided)

for their interval based system, including a special axiom, ⟨M2⟩, for the linearity of time. The primitive order assumed in Allen's theory is the "*Meets*" relation between time intervals, which may be used to define all the thirteen possible temporal relations between intervals. In *ETM*, it is also used to define the three relations between two points: *Before*, *Equal* and *After*. Hence, the fundamental order relation for time point systems, "*no later than*", may be defined in terms of "*Meets*".

The problem is that neither Allen's system, nor *ETM* are a sufficient basis for this purpose: Allen's system does not include points, while *ETM* deals with finite sets of time elements only.

However, Allen and Hayes' interval based axiomatisation of time (Allen and

Hayes 1989) is in fact very appropriate for extending to a general time theory. What is needed is an extension of Allen's axiomatisation to include points. In *ETM* there is a critical axiom that a point cannot meet another point, and it seems that this is likely also to be necessary for the general axiomatisation. In fact, in (Ma and Knight (in press)), the authors have proposed a general temporal theory which addresses both intervals and points as primitive time elements of equal footing. The fundamental axiomatisation is independent of the specification of density and linearity, while additional axioms specifying the *linearity* and *density* of time are separately presented. It is shown that Allen and Hayes' interval based theory may be subsumed, and the authors's *ETM* may be taken as a special finite model of the general theory.

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