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EFFECT OF INITIAL STRAINS ON THE PROPAGATION OF WAVES IN AN INCOMPRESSIBLE CYLINDER LOCATED IN AN IDEAL FLUID

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As is known, real materials invariably contain initial stresses that are capable of significantly altering the wave properties of hydroelastic systems. Thus, in the study of the laws governing the propagation of waves in solid cylinders located in a fluid, it is necessary to use models that more fully account for the behavior of the actual elastic medium. The model of a prestressed body is one such model.

Three-dimensional linearized problems of aerohydroelasticity for arbitrary bodies with initial stresses were formulated in general form in [1] and a method of solution was presented. This study also presented general solutions to coupled problems for an elastic body with initial stresses and a fluid, and it examined the propagation of torsion waves in a preloaded solid circular cylinder placed in a viscous compressible fluid.

In our investigation, we use the model referred to above to examine the propagation of axisymmetric longitudinal waves in a prestressed incompressible solid cylinder located in an ideal compressible fluid.

§1. We will examine an infinitely long incompressible solid circular cylinder of radius R located in an infinite ideal compressible fluid and subjected to uniform initial strains. The cylinder will be examined in a cylindrical coordinate system (r, θ, z_3) introduced in the initial strain state. We will consider the case when the cylinder is tensioned along the Oz_3 axis.

With allowance for the incompressibility of the cylinder and the chosen load, we will have

$$S_{11}^0 = S_{22}^0 = 0;\quad S_{33}^0 \neq 0,\qquad \lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1;\quad \lambda_1 = \lambda_2 \neq \lambda_3;\quad q_1 = q_2 \neq q_3, \tag{1.1}$$

where λ_i represents elongation factors along the Oz_i axes; S_{ii}^0 represents the magnitudes of the initial stresses along Oz_i ($i = \overline{1, 3}$).

We will use representations of general solutions for prestressed incompressible bodies [1, 3, 4] and a fluid [5] in the form

$$u_r = -\frac{\partial^2}{\partial r \partial z_3}\chi;\quad u_\theta \equiv 0;\quad u_3 = \lambda_1 q_1 \lambda_3^{-1} q_3^{-1} \Delta_1 \chi,\qquad V_r = \frac{\partial \varphi}{\partial r};\quad \Delta_1 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}, \tag{1.2}$$

where the potentials χ and φ are determined from the following equations [1, 5]:

$$[\Delta_1^2 + \lambda_1^{-4}\lambda_3^2 q_1^{-2} q_3^2 \frac{\lambda_1^2 \mu_{13} + S_{33}^0}{\mu_{13}} \frac{\partial^4}{\partial z_3^4} - \frac{\rho \lambda_2^{-2}}{\lambda_3^2 \mu_{13}} \frac{\partial^2}{\partial t^2}\Delta_1 + \frac{q_1 q_3^{-1}(\lambda_3^2 a_{33} + S_{33}^0) + q_1^{-1} q_3 \lambda_1^2 a_{11} - 2\lambda_1 \lambda_3 (a_{13} + \mu_{13})}{\lambda_1^2 q_1 q_3^{-1} \mu_{13} \lambda_3^2} \frac{\partial^2}{\partial z_3^2}\Delta_1 - \tag{1.3}$$

$$-\lambda_1^{-2} q_1^{-2} q_3^2 \frac{\rho \lambda_1^{-2}}{\mu_{13}} \frac{\partial^4}{\partial z_3^2 \partial t^2}]\chi = 0,\qquad \left[\Delta_1 + \frac{\partial^2}{\partial z_3^2} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right]\varphi = 0. \tag{1.4}$$

$$\frac{\partial \varphi}{\partial r}\Big|_{r=R} = -\frac{\partial^3}{\partial r \partial z_3 \partial t}\chi\Big|_{r=R}; \tag{1.5}$$

$$\tilde{Q}_r\Big|_{r=R} = P_{rr}\Big|_{r=R};\quad \tilde{Q}_{z_3}\Big|_{r=R} = 0;\quad P_{rr} = \rho_0 \frac{\partial \varphi}{\partial t}. \tag{1.6}$$

The kinematic and dynamic boundary conditions on the surface of the cylinder have the form

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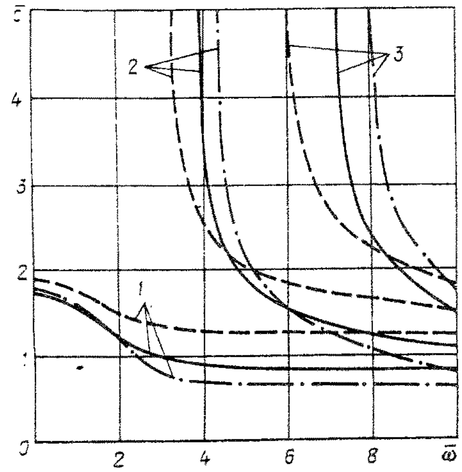

Fig. 1

In Eqs. (1.2)-(1.6), ρ is the density of the cylinder, ρ_0 is the density of the fluid, c_0 is sonic velocity in the fluid, and V_r and P_{rr} are components of the velocity vector and stress tensor in the fluid, respectively.

The components of the stresses in the cylinder at r = const will be represented in the form [1, 3, 4]

$$\tilde{Q}_r = \lambda_1^2 [a_{13}\lambda_1 q_1 \lambda_3 q_3^{-1} \Delta_1 \frac{\partial}{\partial z_3}\chi - \lambda_1^2 a_{11} \frac{\partial^3}{\partial r^2 \partial z_3}\chi - \frac{\lambda_1^2 a_{12}}{r}\frac{\partial^2}{\partial r \partial z_3}\chi + \lambda_1^{-1} q_1 p]; \quad \tilde{Q}_\theta \equiv 0;$$

$$\tilde{Q}_{z_3} = \lambda_1^2 [\lambda_1 q_1 \lambda_3 q_3^{-1} \mu_{13} \frac{\partial}{\partial r}\Delta_1 \chi - \lambda_3^2 \mu_{13} \frac{\partial^2}{\partial r \partial z_3^2}\chi]; \tag{1.7}$$

$$p = \lambda_1^{-1} q_1^{-1} \{[\lambda_1^2 a_{11} - \lambda_1 \lambda_3 q_1 q_3^{-1}(a_{13} + \mu_{13})]\Delta_1 + \lambda_3^2(\lambda_1^2 \mu_{13} + S_{33}^0)\frac{\partial}{\partial z_3} - \rho \frac{\partial^2}{\partial t^2}\}\frac{\partial}{\partial z_3}\chi. \tag{1.8}$$

Equations (1.3)-(1.4), together with boundary conditions (1.5)-(1.6), describe a hydroelastic problem for an incompressible infinite solid circular cylinder in an ideal compressible fluid.

§2. We will seek the solutions of Eqs. (1.3)-(1.4) in the class of travelling waves. We represent the potentials χ and φ in the form

$$\chi(r, z_3, t) = X(r)\exp[i(kz_3 - \omega t)]; \tag{2.1}$$

$$\varphi(r, z_3, t) = \Phi(r)\exp[i(kz_3 - \omega t)], \tag{2.2}$$

where k is the wave number and ω is the frequency.

Inserting Eqs. (2.1)-(2.2) into (1.3) and (1.4), we obtain ordinary differential equations relative to the functions X(r) and Φ(r)

$$(\Delta_1^2 + b\Delta_1 + c)X = 0; \quad \Delta_1 \equiv \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}, \tag{2.3}$$

$$(\Delta_1 - \alpha^2)\Phi = 0; \tag{2.4}$$

$$\alpha^2 = k^2 - \frac{\omega^2}{c_0^2}; \quad b = \frac{\rho\lambda_1^{-1}}{\lambda_3^2 \mu_{13}}\omega^2 - \frac{q_1 q_3^{-1}(\lambda_3^2 a_{33} + S_{33}^0) + q_1^{-1} q_3 \lambda_1^2 a_{11} - 2\lambda_1\lambda_3(a_{13} + \mu_{13})}{\lambda_1^2 \mu_{13} q_1 q_3^{-1}}k^2;$$

$$c = \lambda_1^{-4}\lambda_3^2 q_1^{-2} q_3^2 \frac{\lambda_1^2 \mu_{13} + S_{33}^0}{\mu_{13}}k^4 - \lambda_1^{-4} q_1^{-2} q_3^2 \frac{\rho}{\mu_{13}}k^2\omega^2. \tag{2.5}$$

Since the function X(r) is bounded at r = 0, the general solution of differential equation (2.3) can be represented in the form

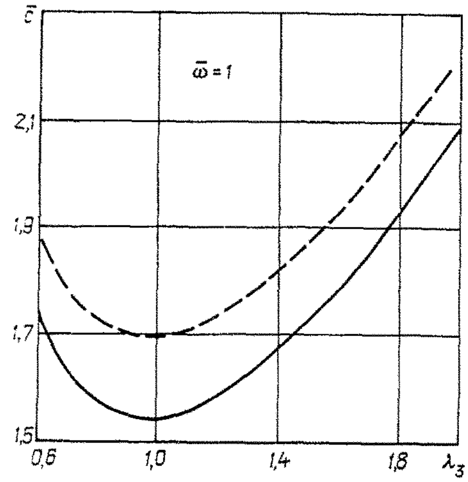

Fig. 2

$$X(r)=C_1Z_0(\gamma_1 r)+C_2Z_0(\gamma_2 r),$$
$$\gamma_{1,2}^2=-\frac{b}{2}\pm\sqrt{\left(\frac{b}{2}\right)^2-c};\quad \gamma_i=\sqrt{|\gamma_i^2|}; \tag{2.6}$$

$$Z_0(\gamma_i r)=\begin{cases} J_0(j_i r),\gamma_i^2<0,\\ I_0(j_i r),\gamma_i^2>0, \quad i=\overline{1,2},\ 0<r<R. \end{cases} \tag{2.7}$$

With the condition that $\Phi(r)\to 0$, $r\to\infty$, we represent the general solution of Eq. (2.4) in the form

$$\Phi(r)=AK_0(\alpha r). \tag{2.8}$$

In Eqs. (2.7-2.8), $J_0(\gamma_i r)$, $I_0(\gamma_i, r)$, and $K_0(\alpha r)$ are cylindrical Bessel functions.

Inserting solutions (2.6) and (2.8) into boundary conditions (1.5)-(1.6) and performing several transformations, we obtain a system of linear homogeneous algebraic equations in the unknowns C_1 and C_2. Proceeding on the basis of the condition for the existence of a nontrivial solution, we derive the dispersion equation for the given system

$$\det\|d_{ij}\|=0;\quad i,j=\overline{1,2}, \tag{2.9}$$

$$d_{1i}=\lambda_1^2\{a_{13}\lambda_1\lambda_3 q_1 q_3^{-1}\gamma_i^2 Z_0(\gamma_i R)-\lambda_1^2 a_{11}\gamma_i^2\left[Z_0(\gamma_i R)-\frac{Z_1(\gamma_i R)}{\gamma_i R}\right]-\lambda_1^2 a_{12}\gamma_i\frac{Z_1(\gamma_i R)}{R}+$$
$$\lambda_1^{-2}[\lambda_1^2 a_{11}-\lambda_1\lambda_3 q_1 q_3^{-1}(a_{13}+\mu_{13})]\gamma_i^2 Z_0(\gamma_i R)-\lambda_3^2(\lambda_1^2\mu_{13}+$$
$$+S_{33}^0)k^2 Z_0(\gamma_i R)+\rho\omega_2 Z_0(\gamma_i R)]\}+\rho_0\omega^2\gamma_i Z_1(\gamma_i R)\frac{K_0(\alpha R)}{\alpha K_1(\alpha R)}; \tag{2.10}$$
$$d_{2i}=\gamma_i Z_1(\gamma_i R)(\mu_{13}\lambda_1 q_1\lambda_3 q_3^{-1}\gamma_i^2-\mu_{13}k^2);\qquad Z_1(\gamma_i R)=\begin{cases}-J_1(\gamma_i R),\gamma_i^2<0,\\ I_1(\gamma_i R),\gamma_i^2>0,\quad i=\overline{1,2}.\end{cases}$$

§3. Dispersion equation (2.9) was solved numerically on a computer. The elastic properties of the cylinder material were described by a Treloar potential [2]. Here, the elastic body and the fluid were characterized by the following parameters

$$\rho=1200\,\frac{\text{kg}}{\text{m}^3};\ \mu=1{,}2\cdot 10^6\ \text{Pa};\quad \nu=0{,}5;\ \rho_0=1260\,\frac{\text{kg}}{\text{m}^3};\quad c_0=1927\,\frac{\text{m}}{\text{sec}},$$

where μ is the shear modulus; ν is the Poisson's ratio.

The results of the computation are shown in Figs. 1-3. The dimensionless quantities $\bar{c}$ and $\bar{\omega}$ in the figures were introduced as follows

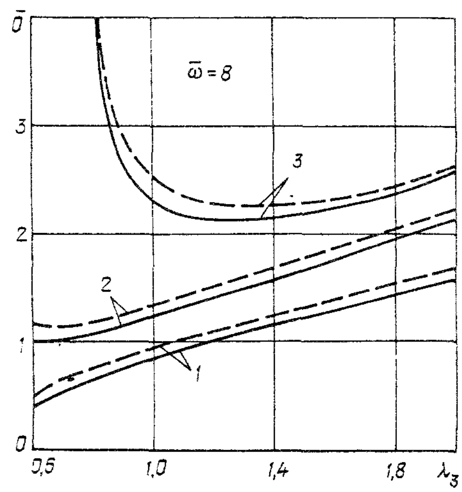

Fig. 3

$$\bar{c}=\frac{c}{c_s};\quad \bar{\omega}=\frac{\omega R}{c_s},$$

where $c = \omega/k$ is phase velocity; $c_s^2 = \mu/\rho$ is the velocity of the shear wave.

Figure 1 shows the dependence of dimensionless phase velocity $\bar{c}$ on frequency $\bar{\omega}$ for the given system (incompressible cylinder and fluid). The figure shows the dispersion curves corresponding to the first three modes. The results were obtained for $\lambda_3 = 0.8$ (dot–dash lines), $\lambda_3 = 1.5$ (dashed lines), and $\lambda_3 = 1$ (solid lines). The case $\lambda_3 = 0.8$ corresponds to initial compression, $\lambda_3 = 1.5$ corresponds to tension, and $\lambda_3 = 1$ corresponds to the absence of initial strains. An analysis of the results in Fig. 1 shows that initial strains have a significant effect on the frequencies at which the modes appear. Compression leads to an increase in the critical frequencies, while tension leads to their decrease.

Figures 2 and 3 show the dependence of phase velocity $\bar{c}$ on elongation λ_3 for a cylinder in a vacuum (dashed lines) and for a cylinder interacting with a fluid (solid lines). The curves in Fig. 2 were obtained for waves propagating with a frequency equal to 1. Figure 3 shows the function $\bar{c} = f(\lambda_3)$ for $\bar{\omega} = 8$. It follows from the graphs shown in these figures that phase velocity increases with either initial compression ($\lambda_3 < 1$) or initial tension ($\lambda_3 > 1$) of the incompressible cylinder for waves with a frequency close to the critical frequency (curves 3 in Figs. 2 and 3). The pattern of change of phase velocity as a function of preliminary strain is different for modes propagating with frequencies that differ greatly from the critical values. These modes are characterized by a decrease in velocity in compression and an increase in velocity in tension (curves 1 and 2 in Fig. 3). It is also apparent that the phase velocities of the modes are somewhat lower when the cylinder interacts with the fluid than when it is located in a vacuum.

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