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PROBLEMS OF THE STABILITY OF COMPOSITE MATERIALS IN COMPRESSION ALONG INTERLAMINAR CRACKS: PERIODIC SYSTEM OF PARALLEL MACROCRACKS*

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One of the most common types of defects in actual structural composites is a disruption in interlaminar adhesion — in particular, the presence of cracks at the interfaces between the layers. The most accurate results from study of the stability of composites in compression along such defects can be obtained within the framework of the model of a piecewise-uniform medium, with the use of the three-dimensional linearized theory of stability of deformable bodies (TLTSDB) [2]. However, until recently, different authors have studied such problems using only applied structural models — rod, shell, etc. — in addition to approximate theories (such as continuum theory [3], in which it is impossible to distinguish between interlaminar and intralaminar cracks).

An exact approach to studying the stability of composite materials in compression along interlaminar cracks was used only in [4-11]. In [4], a general formulation of the problem was presented for a material consisting of alternating layers of filler and binder. A classification was proposed for interlaminar cracks in relation to their length and the thickness of the layers: macrocracks, structural cracks, and microcracks. In [8], these subjects were generalized to the case of a composite with an arbitrary laminated structure, including the possibility of the presence of a free surface (a bounded material). Numerical results were also presented in [4, 8, 11] for a microcrack between orthotropic layers. It was shown in [6] that the critical loading parameters are independent of the number of microcracks on one interface or the distance between the them. (A similar conclusion was reached in [3] for uniform materials). The author of [7, 10] studied the stability of a composite material in compression along two parallel structural cracks located at interfaces between layers (isotropic or orthotropic). In [5-7, 9, 10], numerous comparisons were made with experimental data, and it was shown in [7, 10] that applied schemes do not even qualitatively describe the phenomenon in question. As can be seen, all of the above-noted studies examined cases of the compression of a finite number of interlaminar cracks. In the present investigation, we use an exact formulation (the model of a piecewise-uniform medium with relations from the TLTSDB [2]) to study stability in compression along an infinite number of interlaminar cracks — a periodic system of parallel cracks.

It must also be mentioned that classical fracture criteria of the Griffith–Irwin type or its generalizations are invalid for compression along interlaminar cracks, i.e., all of the stress-intensity factors and the crack-openings are equal to zero in this situation. According to the criterion proposed in [2, 3], the beginning of the fracture process in the case of compression along cracks is associated with loss of stability in the neighborhood of the cracks. Thus, the problems examined in [4-11] and in this article are also problems of fracture mechanics.

1. Formulation of the Problem. We will examine a composite consisting of alternating compressible layers of thicknesses h^+ and h^-. All quantities pertaining to these layers will be designated by the indices "+" and "−". The material is in plane strain and is compressed along the layers by "dead" loads. The loads are applied in such a way that a uniform subcritical state is realized in the material (the contractions along the layers are identical):

$$\varepsilon_{11}^{0+} = \varepsilon_{11}^{0-}, \quad \sigma_{22}^{0+} = \sigma_{22}^{0-} = 0. \tag{1.1}$$

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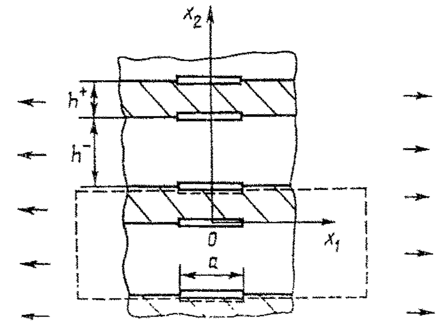

Fig. 1

With (1.1), we use the static method of studying static problems of the TLTSDB [1]. Let us also suppose that the material contains parallel cracks of length a. There is one crack on each interface (Fig. 1). The magnitude of a is of the same order as the thickness of the layers. Thus, these are macrocracks, to use the classification adopted in [4, 8]. Regardless of the causes of the cracks, we will model them mathematically as slits with edges that are free of forces. We introduce a Lagrangian coordinate system which, in the undeformed state, coincides with the cartesian coordinate system. This is illustrated by Fig. 1. In the Lagrangian system, a material with cracks is a periodic structure with a period along the ox_2 axis equal to $T = h^+ + h^-$.

Within the framework of the TLTSDB [1], the stability equations will then have the form

$$\omega^{+}_{ij\alpha\beta}\frac{\partial^2}{\partial x_i \partial x_\beta}u^{+}_{\alpha}=0, \quad nT<x_2<h^{+}+nT,$$
$$\omega^{-}_{ij\alpha\beta}\frac{\partial^2}{\partial x_i \partial x_\beta}u^{-}_{\alpha}=0, \quad -h^{-}+nT<x_2<nT; \quad n=0;\pm 1;\pm 2;\ldots \tag{1.2}$$

Besides (1.2), the components of the vector characterizing the perturbations of the displacements u and the asymmetric Kirchhoff tensor describing the perturbations of the stress state t must satisfy the continuity conditions in the region of ideal contact (rigid bonding) of the layers, the conditions expressing the fact that the crack edges are free of stresses, the conditions of decay of the perturbations with increasing distance from the cracks, and the conditions of periodicity along the ox_2 axis. Since the structure of the material and the applied load are periodic along ox_2, we need to examine buckling modes for which the following are satisfied

$$u_i(x_1,x_2)=u_i(x_1,x_2+kT), \quad t_{ij}(x_1,x_2)=t_{ij}(x_1,+kT),$$
$$k=1,2,\ldots;\ i,j=1,2, \tag{1.3}$$

i.e., modes which have a period along ox_2 that is a multiple of the period of the structure of the material.

Let us specify the formulation of the stability problem for the case $k = 1$, a buckling mode with a period equal to the period of the structure of the material. We need to solve a generalized, completely determined eigenvalue problem in the loading parameters that go into the components of the tensors ω^+ and ω^- for Eqs. (1.2) (with $n = 0$). The boundary conditions are

$$t^{+}_{2i}(x_1,0)=0; \quad t^{-}_{2i}(x_1,0)=0, \quad x_1\in[-a/2,\ a/2],$$
$$u^{+}_{i}(x_1,0)=u^{-}_{i}(x_1,0); \quad t^{+}_{ij}(x_1,0)=t^{-}_{ij}(x_1,0), \quad x_1\notin[-a/2,\ a/2],$$
$$t^{+}_{2i}(x_1,h^{+})=0; \quad t^{-}_{2i}(x_1,-h^{-})=0, \quad x_1\in[-a/2,\ a/2],$$
$$u^{+}_{i}(x_1,h^{+})=u^{-}_{i}(x_1,-h^{-}); \quad t^{+}_{2i}(x_1,h^{+})=t^{-}_{2i}(x_1,-h^{-}), \quad x_1\notin[-a/2,\ a/2] \tag{1.4}$$

while the conditions expressing the decay of the perturbations with increasing distance from the cracks are

$$u^{+}_{i}\to 0, \quad u^{-}_{i}\to 0 \ \text{for}\ x_1\to\pm\infty. \tag{1.5}$$

The last condition of (1.4) follows from (1.3) when $k = 1$.

TABLE 1

Type of fibers	h^+/a	h^-/a	c_f %	E_f GPa	ν_f	ε_{cr}, %	ε_{cr}^s, %	ε_{cr}^m, %
Boron	0.5	1	50	400	0,21	1,00	1,30	1,37
Tornel-300	0,5	1	50	239	0,21	1,76	2,10	2,26

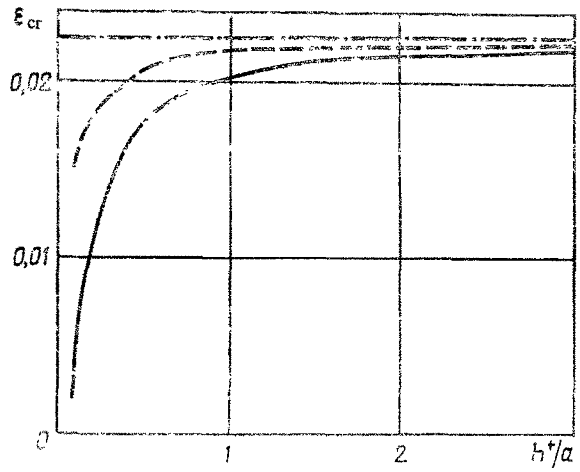

Fig. 2

We proceed similarly in formulating the problems for buckling modes with a period equal to two, three, etc. periods of the structure. However, it must be pointed out that a buckling mode with a period equal to one period of the structure was realized for all of the specific composites for which calculations were performed.

2. Method of Solution. In solving the given problem numerically by the finite-differences method, we will restrict ourselves to the case of linearly elastic orthotropic layers. Since actual composites undergoing compression usually fail with small strains, we can conduct our investigation within the framework of the second variant of the theory of small subcritical strains of the TLTSDB [2]. We proceed as in [2] in resolving any questions relating to proof of the convergence of the computational scheme employed or selection of the gradient iteration process for finding the lowest eigenvalue.

The scheme we are using represents a first-order approximation in the grid analog of the norm of the space C. The finite-difference equation for each layer has the form

$$Au_i + \mu Bu_i = 0, \quad i = 1,2 \tag{2.1}$$

with the corresponding conditions at the interface. The form of the difference operators A and B was shown in [11]. They were obtained by the variational-difference method. We used a gradient iteration process to find the lowest eigenvalue of the difference problem. Here, each successive approximation was chosen in the form [11]:

$$\begin{gathered} u^l = u^{l-1} + \alpha^{l-1} r^{l-1}, \text{ where} \\ r^{l-1} = Au^{l-1} - \mu^{l-1} Bu^{l-1}, \ l = 1,2,\ldots; \\ \mu^{l-1} = \frac{(Au^{l-1}, u^{l-1})}{(Bu^{l-1}, u^{l-1})}, \end{gathered} \tag{2.2}$$

where l is the number of the iteration; r is the error vector; μ is the approximation of the eigenvalue; α is the convergence parameter. It was shown in [11] that there exists an optimum convergence parameter for (2.2) for which μ^l monotonically converges upward to the minimum eigenvalue of difference problem (2.1) with an increase in the number of iterations. Here, α is found from the formulas

$$\alpha_{1,2} = 0{,}5\left(p_3 p_5 \pm \sqrt{(p_3 p_5)^2 + 4 p_1 p_5 (p_1 p_5 (p_1 p_4 - p_2 p_3))}\right) / (p_1 p_4 - p_2 p_3);$$
$$p_1 = (r^l, r^l),\ p_2 = (Bu^l, r^l),\ p_3 = (Ar^l - \mu^l B r^l, r^l),\ p_4 = (Br^l, r^l),\ p_5 = (Bu^l, u^l). \quad (2.3)$$

The plus or minus sign is chosen in (2.3) for each iteration on the basis of a comparison of μ^l and μ^{l-1}.

Calculations were performed on an ES 10 — 66 computer at the Institute of Mechanics of the Ukrainian Academy of Sciences. We used an application package written in the language PL/1. The programs included the following steps: a) introduction of a grid with a nonuniform mesh in both directions; b) determination, by numerical calculation, of the horizontal dimensions of the grid region so as to satisfy conditions (1.5); c) direct solution of the problem on a sequence of grids. In steps (b) (with an increase in the horizontal dimensions of the region by a factor of two) and (c) (with a doubling of the density of the grid), the process was continued until the results agreed to within 1 + 2%.

3. Numerical Results for a Composite with a Longitudinal-Transverse Orientation of the Layers. As an example of a material with orthotropic layers, we will examine a composite composed of reinforced layers arranged in a longitudinal-transverse pattern. Each alternating layer represents a matrix reinforced by continuous parallel fibers in the direction of the ox_1 axis (layers of thickness h^+) or in the direction perpendicular to the x_1ox_2 plane (layers of thickness h^-). We will assume that the dimensions of the structural elements inside each layer are such that, in a continuum approximation, each element is a uniform orthotropic body with elastically equivalent directions that coincide with the coordinate axes ox_1 and ox_2. Knowing the elastic constants of the fibers (elastic modulus E_f and Poisson's ratio ν_f) and the matrix (E_m, ν_m) for the actual materials [12, 14], we can use equations in [13] to calculate effective values of the elastic constants for the layer as a whole. We should point out that in the space coordinates the layers are transversely isotropic bodies with an isotropy axis that coincides with the direction of reinforcement.

A numerical solution was found for buckling modes with a period equal to one, two, or three periods of the structure ($k = 1, 2, 3$ in (1.3)). It turned out that the critical strain was less at $k = 1$ than at other k for all of the layer elastic constants and thicknesses we examined. In light of this, when we subsequently speak of the critical strain in compression along the periodic system of cracks, it will be understood that this strain corresponds to buckling at $k = 1$.

We should also note that the agreement required in step (c) (see section 2) was achieved on grids with 2145 and 8385 nodes. Computing time was as long as 60 min. The horizontal dimensions that ensured satisfaction of conditions (1.3) were (3-6)a. (This data is for the case $k = 1$. For $k > 1$, the time and the number of nodes increase in proportion to k).

Table 1 and Figs. 2-5 shows values of critical strain for the above-described composite when the layers are made of boron fibers or Tornel−300 carbon fibers (the values of E_f and ν_f for these fibers are shown in Table 1) and the composite has an epoxy matrix. The results were obtained for different h^+/a and h^-/a and a volume concentration of fibers in each layer c_f. For the matrix, in every case we took $E_m = 2$ GPa and $\nu_m = 0.4$. In addition to the critical strain in the compression of the composite along the periodic system of macrocracks (ε_{cr}) which we found from our calculations, we show values of critical strain ε_{cr}^{s} and ε_{cr}^{m} for the same material determined in other investigations. The values of ε_{cr}^{s} correspond to the case of two parallel structural cracks of length a, with a layer of thickness h^+ between them [7, 10]. The values of ε_{cr}^{m} correspond to the critical strain in compression along a microcrack [5, 6, 9].

All of the relations shown in Figs. 2-5 were obtained for Tornel−300 fibers. Let us analyze them. Figure 2 shows the dependence of the critical strain ε_{cr} on the value of h^+/a (solid line) with fixed $h^-/a = 1$ and 50% — the concentration of fibers in the layers. The dashed curve corresponds to ε_{cr}^{s} for the same values of h^+/a. The dot−dash line shows the value of ε_{cr}^{m}. It can be seen that if one of the layers is sufficiently thick ($h^+/a > 2$), the values of ε_{cr}, ε_{cr}^{s}, and ε_{cr}^{m} are nearly the same. In this case, all three types of cracks behave as a microcrack, i.e., the critical strain is determined only by the properties of the adjacent layers [4, 8]. Figure 3 shows the dependence of the critical strain on h^-/a with fixed $h^+/a = 0.2$, $c_f = 50\%$, and the same notation for ε_{cr}, ε_{cr}^{s}, and ε_{cr}^{m}. At $h^{-2}/a > 2$, the values of ε_{cr} and ε_{cr}^{s} nearly coincide. In other words, when the rigid layer is thin and the soft layer is relatively thick, the behavior (stability) of the periodic system of parallel cracks is determined by the stability of a pair of parallel cracks (structural cracks, to use the classification in [4, 8]). These findings are consistent with the physical premises by which cracks were originally classified as macrocracks, structural cracks, and microcracks. One other relation that follows from physical considerations is also satisfied for Figs. 2 and 3

$$\varepsilon_{cr} \leqslant \varepsilon_{cr}^{s} \leqslant \varepsilon_{cr}^{m}. \quad (3.1)$$

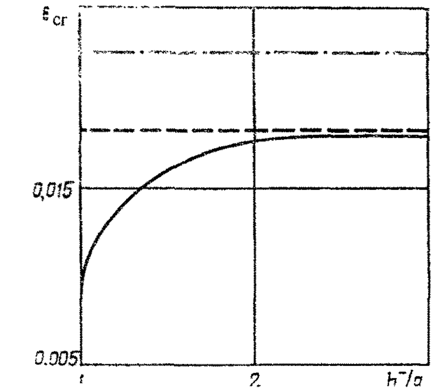

Fig. 3

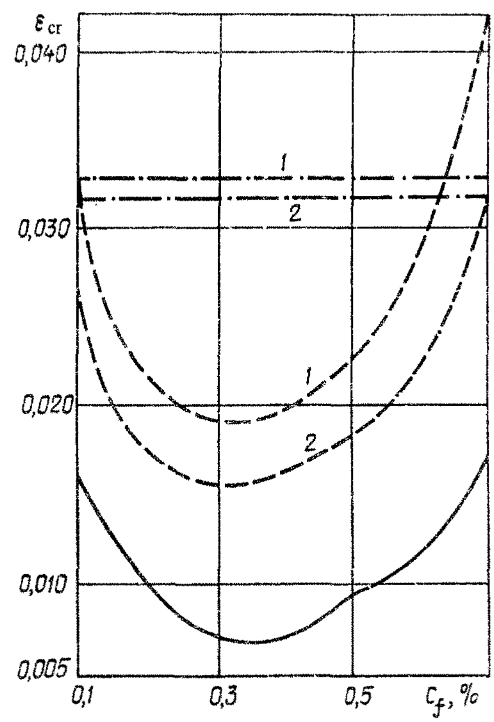

Fig. 4

Inequalities (3.1) have been proven correct, the results being shown in Figs. 4 and 5. In Fig. 4, we fixed the values of h^+/a = 0.2 and h^-/a = 1. The solid line shows the dependence of ε_{cr} on the volume concentration of fibers in the layers. The dashed line shows the relations for ε_{cr}^{m} (denoted by 1) and ε_{cr}^{s} (denoted by 2). For Fig. 5, we took h^+/a = 0.5 and h^-/a = 1. The solid line corresponds to ε_{cr}, while the dashed line corresponds to $-\varepsilon_{cr}^{m}$.

We should also point out that while buckling that occurs with compression along microcracks or structural cracks may lead only to local fracture near the cracks rather than failure of the material as a whole, buckling that occurs with compression along a periodic system of cracks should obviously result in the material's failure. In the latter case, we observe a phenomenon similar to that of the plastic hinge.

The value of ε_{cr} can also be used as a lower estimate for critical strain in the case of compression along a finite number of parallel macrocracks. In fact, in accordance with a general principle of mechanics, freeing of the material from some of the internal bonds will not necessarily lead to an increase in the critical loading parameters. The formation of an interlaminar crack is in essence the release of the material from a bond between the layers. Thus, for the critical strain in compression along a

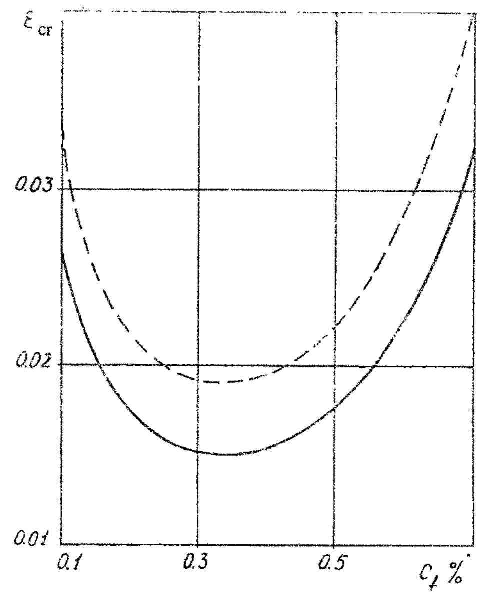

Fig. 5

finite number of parallel macrocracks (ε_{cr}')

$$\varepsilon_{cr}' \leqslant \varepsilon_{cr}. \tag{3.2}$$

If the number of parallel macrocracks is greater than one, then by virtue of the above we can also propose a lower estimate for ε_{cr} and take ε_{cr}^{s} (the critical strain for two parallel structural cracks) as this estimate:

$$\varepsilon_{cr}^{s} \leqslant \varepsilon_{cr}' \leqslant \varepsilon_{cr}. \tag{3.3}$$

With allowance for (3.1)-(3.3), we have estimates for ε_{cr}' in the case of one macrocrack

$$\varepsilon_{cr}^{m} \leqslant \varepsilon_{cr}' \leqslant \varepsilon_{cr} \tag{3.4}$$

and in the case of a finite number of parallel macrocracks

$$\varepsilon_{cr}^{m} \leqslant \varepsilon_{cr}^{s} \leqslant \varepsilon_{cr}' \leqslant \varepsilon_{cr}. \tag{3.5}$$

As noted above, the formulation of the problem used in this investigation (the model of a piecewise-uniform medium and the basic equations of the TLTSDB [1]) is the most exact of the existing formulations. The given phenomenon cannot be described even qualitatively by using applied schemes based on calculation of the critical strain for compression along a crack by replacing the layer between the cracks by a plate with different support conditions at the ends. To illustrate, the dot–dash line in Fig. 4 shows the dependence of the critical strain on the concentration of fibers in the layer. The results were obtained by the given scheme for a hinged plate in accordance with Euler's method (ε_e) (designated by the number 1 in the figure) and three-dimensional theory [1] (ε_{th}) (designated by the number 2). Here

$$\varepsilon_e = \frac{1}{12}\left(\frac{\pi h^+}{a}\right)^2, \tag{3.6}$$

$$\varepsilon_{th} = \varepsilon_e \left[1 - \frac{1}{3}\left(\frac{\pi h^+}{a}\right)^2 \left(\frac{2}{5(1-\nu_{13}\nu_{31})}\left(\frac{3E_1}{G_{12}} - \nu_{21} - \nu_{23}\nu_{31}\right) + 1\right)\right].$$

The dot–dash curves in Fig. 4 give an error up to 45% compared to the solid lines (obtained using the exact formulation). This shows that the other theoretical schemes cannot be used to study such a fine-scale phenomenon in the structure of a material.

It is also interesting to compare our results with certain experimental data from [4]. The empirical ultimate compressive strength of a composite with layers composed of an epoxy matrix reinforced with Tornel–300 fibers and arranged in a longitudinal-tranverse pattern is $\varepsilon_{eml} = 1.34\%$ for a 50% volume concentration of fibers in each layer. When we compare this value to the results in Table 1 and Figs. 2-5 (with allowance for (3.3), we see that the presence of macrocracks between the layers of the composite can significantly reduce its strength.

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