We consider that Planck's formula should be considered as a particular case of Wien's thermodynamic formula, especially in view of the simple derivation given below.

The energy density of equilibrium radiation is dependent solely on the frequency ν and temperature T:

$$du(\nu) = u(T,\nu)\,d\nu. \tag{2}$$

There are only three dimensional universal constants (Planck's constant h, Boltzmann's constant k, and the velocity of light c), so (2) may be put as

$$du(\nu) = v\left(\frac{h\nu}{kT},\ \nu\right)d(h\nu) \tag{3}$$

with one dimensionless argument; the dependence of v(x, y) on its second (dimensional) argument may be established by comparing the dimensions on the two sides of (3). We isolate on the right explicitly the factor $(\nu/c)^3$:

$$du(\nu) = \left(\frac{\nu}{c}\right)^3 g\left(\frac{h\nu}{kT}\right)d(h\nu). \tag{4}$$

Function g is dimensionless and cannot be dependent on the dimensional argument ν, which constitutes the content of Wien's theorem.

This is not a thermodynamic derivation of Wien's formula, but its generality is not in doubt. An important consequence is that Wien's law retains its form for other types of equilibrium radiation, e. g., gravitational.

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DETERMINATION OF THE ATTENUATION OF LIGHT BY PARTICLES

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The physical aspects of the definition of attenuation coefficient have been discussed in detail for scalar waves and for particles [1-4]. Here I extend the treatment to the scattering of a plane elliptically polarized electromagnetic wave (propagating parallel to a locus q) at a particle of any shape. I also discuss the conditions for the attenuation coefficient to coincide with the screening coefficient.

The components of the scattered field in the wave zone take the form

$$E_j = \frac{e^{ikr}}{ikr}\, a_{j,p}(\Theta,\varphi)\,E_{op};\quad \boldsymbol{H} = m_a\,[\boldsymbol{n},\boldsymbol{E}]; \tag{1}$$

E, H, E_0 and H_0 are the vectors of the scattered and incident waves, k is wave number, n is a locus in the direction of observation of the scattered light, and m_a is the refractive index of the medium outside the particle; r, Θ, φ are the polar coordinates of the point of observation (the origin lies at the center of the particle, while Θ is reckoned from the direction of q, so n = q for $\Theta = \pi$) and the $a_{j,p}$ are certain complex functions, with j, p = 1, 2, subscripts 1 and 2 referring to components perpendicular and parallel to the reference plane respectively.

The screening coefficient σ is [5] defined by

$$\sigma = I_0^{-1}\int \boldsymbol{\Pi}^a\boldsymbol{n}\,ds - I_0^{-1}\int \boldsymbol{\Pi}\boldsymbol{n}\,ds, \tag{2}$$

in which $\Pi = (c/8\pi)\,\mathrm{Re}[E + E_0,\ H^* + H_0^*]$, $\Pi^a = (c/8\pi)\,\mathrm{Re}[E,\ H]$ are the Poynting vectors of the total and scattered fields, c is the velocity of light, and I_0 is the intensity of the incident wave. The integration is taken over a sphere of radius r. The first term in (2) represents the scattered light; the second, the absorbed light.

We substitute (1) into (2) and use the above assumption about E_0 and H_0 with $I_0 = (c/8\pi)m_a(E_0,\ E_0^*)$; in addition, the integral with respect to Θ is calculated by the stationary-phase method to give

$$\sigma = -\frac{1}{k^2}\,\mathrm{Re}\int_0^{2\pi}\{(a_{11}+a_{22}) + (a_{22}-a_{11})\cos 2\beta\cos 2\varphi + (a_{12}+a_{21})\cos 2\beta\sin 2\varphi + i\,(a_{21}-a_{12})\sin 2\beta\}\,d\varphi. \tag{3}$$

Here β is the ellipticity of the incident wave; all the $a_{j,p}$ have arguments π and φ.

For a spherical particle $a_{12} = a_{21} = 0$, a_{11} and a_{22} being independent of φ and $a_{11}(\pi) = a_{22}(\pi) \equiv a(\pi)$, so

$$\sigma = -\frac{4\pi}{k^2} \operatorname{Re} a(\pi), \tag{4}$$

which is not dependent on the state of polarization of the incident wave.

The attenuation coefficient is defined by

$$\eta = \frac{I_0 z - \int \Pi q \, dz}{I_0}. \tag{5}$$

The integration is carried over the receiving area z of the detector, which lies perpendicular to q at a distance $e \gg z^{1/2}$ from the particle. The numerator in (5) is the difference of the energies recorded by the receiver when the particle is absent from the incident flux and present in it.

We assume that $z \gg \lambda e$ (λ is the wavelength of the incident light) and that the detector does not record the scattered light; then we get by substitution into (5) for Π and use of (1) with E_0 and H_0, together with the Θ integral calculated as above, that $\eta = \sigma$.

If the detector does record the scattered light, we have from (5)

$$\eta = \sigma - \Delta \equiv \sigma - \frac{1}{2}\int (F_1 + F_2 \cos 2\beta \cos 2\varphi + F_3 \cos 2\beta \sin 2\varphi + F_4 \sin 2\beta)\frac{|\cos \Theta|}{k^2 r^2} dz,$$
$$F_{1,2} = \pm |a_{11}|^2 + |a_{22}|^2 + |a_{12}|^2 \pm |a_{21}|^2, \quad F_3 = 2 \operatorname{Re} (a_{11}a^*_{12} + a_{22}a^*_{21}), \; F_4 = 2i \operatorname{Im} (a_{11}a^*_{12} - a_{22}a^*_{21}). \tag{6}$$

For a spherical particle, $\Delta = (1/2) \int F_1 |\cos \Theta| \, dz \, |k^2 r^2|$ is independent of the state of polarization; the second term in (6) is a correction significant when λ is much less than the size of the particle.

The condition $z \gg \lambda e$ indicates that the detector is much larger than the first Fresnel zone; if this is not so, but $e \gg z^{1/2}$, we have $\eta \neq \sigma$. In particular, for a spherical particle and a detector in the form of a square of side 2b, subject to $\operatorname{Re} a(\pi) \gg \operatorname{Im} a(\pi)$ [6], we have

$$\eta = \sigma 4C\left(b\sqrt{\frac{k}{\pi e}}\right)\cdot S\left(b\sqrt{\frac{k}{\pi e}}\right), \tag{7}$$

in which C and S are Fresnel integrals.

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