ELECTRON BREAKDOWN IN A DIELECTRIC WITH A NONELECTRICAL ENERGY SOURCE

I. G. Nekrashevich and I. A. Bakuto

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Consider a dielectric receiving power P from some nonelectrical source, e.g., ionizing radiation. The processes may be described via the general Eq. [2] for the energy balance:

$$\frac{dQ}{dt}=jE+P-(w_0-w)\frac{dn_-}{dt}, \tag{1}$$

in which Q is the heat produced in unit volume, t is time, j is the current density, w_0 is the energy needed to produce a charged particle from a neutral one in its lowest energy state, w is the mean energy of the neutral particles under the given conditions, and n is the number of negative particles produced in unit volume (the positive particles are, of course, equally numerous).

We transform (1) on the basis that heat is an additive quantity and that $j = j_- + j_+$:

$$j_-E\cdot\frac{dQ_-}{dt}\bigg|_{j_-E}+j_+E\frac{dQ_+}{dt}\bigg|_{j_+E}+P\frac{dQ'}{dt}\bigg|_P=$$
$$=j_-E+j_+E-w_0\left(1-\frac{w}{w_0}\right)\left|\frac{dn_-}{dt}\right|+P.$$

Symbols to be used are

$$\frac{dQ_-}{dt}\bigg|_{j_-E}=\mu;\quad \frac{dQ_+}{dt}\bigg|_{j_+E}=\eta;\quad \frac{dQ'}{dt}\bigg|_P=\zeta;\quad \frac{w}{w_0}=\delta,$$

in which $\frac{dQ}{dt}=\frac{dQ_-}{dt}+\frac{dQ_+}{dt}+\frac{dQ'}{dt}$, because $\frac{dQ_-}{dt}$, $\frac{dQ_+}{dt}$, $\frac{dQ'}{dt}$ are the thermal powers corresponding to the electron and ion currents and to the nonelectrical source.

Then

$$w_0\frac{dn_-}{dt}=\frac{1-\mu}{1-\delta}j_-E+\frac{1-\eta}{1-\delta}j_+E+\frac{1-\zeta}{1-\delta}P$$

or

$$w_0\frac{dn_-}{dt}=\alpha j_-E+\beta j_+E+\varkappa P, \tag{2}$$

in which $\alpha=\frac{1-\mu}{1-\delta};\quad \beta=\frac{1-\eta}{1-\delta};\quad \frac{1-\zeta}{1-\delta}=\varkappa.$

We are concerned with electron breakdown, so we must put $\beta = 0$ [2], which means that the ion current plays no active part in producing fresh charged particles. Then (2) becomes

$$\frac{w_0}{q}\cdot\frac{dj_-}{dx}=\alpha j_-E+\varkappa P, \tag{3}$$

because $n_-=\frac{j_-}{qv_-}$, in which q is the electronic charge and v_- is the mean drift velocity of the electrons, which we take to be constant along their path.

We may solve (2) subject to the assumption of constant coefficients, which gives us conclusions [3] analogous to those from Townsend's theory of breakdown in gases; we therefore consider (3) for the case where $\alpha = f(j_-E + P)$ and is less than one. We can then use a series expansion $j_-E + P = a_0 + a_1\alpha + a_2\alpha^2 + \ldots$

From this we take only $j_- E + P = a_0 + a_1\alpha$, from which we derive α and substitute in (3) to get

$$\frac{dj_-}{dx} = \frac{qE^2}{a_1 w_0} j_-^2 + \frac{P - a_0}{a_1} \cdot \frac{qE}{w_0} j_- + \frac{q\varkappa P}{w_0}$$

or

$$\frac{dj_-}{dx} = a j_-^2 + b j_- + c, \qquad (4)$$

in which $a = \frac{qE^2}{a_1 w_0}$, $b = \frac{P - a_0}{a_1} \cdot \frac{qE}{w_0}$, $c = \frac{q\varkappa P}{w_0}$.

Equation (4) has the following solutions if the coefficients are constant, $\Delta = b^2 - 4ac$ being the decisive parameter:

1) $\Delta < 0$; $\qquad \frac{2}{\sqrt{-\Delta}} \operatorname{arctg} \frac{2aj_- + b}{\sqrt{-\Delta}} = x + c'$,

2) $\Delta > 0$; $\qquad \frac{1}{\sqrt{\Delta}} \ln \left\{ \frac{2aj_- + b - \sqrt{\Delta}}{2aj_- + b + \sqrt{\Delta}} \right\} = x + \ln c''$,

3) $\Delta = 0$; $\qquad -\frac{4c/b^2}{j_- + 2c/b} = x + c'''$.

Here c', c", and c‴ are arbitrary constants, which may be derived for the very simple boundary conditions $j_- = j_0$ at $x = 0$ and $j_- = j$ at $x = l$, in which l is the distance between the electrodes and j is the total current. These boundary conditions are analogous to those used in Townsend's theory in the absence of the processes at the cathode represented by γ. Then we have

1) $$j = \frac{2aj_0 + \frac{b(2aj_0 + b)}{\sqrt{-\Delta}} \operatorname{tg} \frac{\sqrt{-\Delta}}{2} l}{2a\left[1 - \frac{2aj_0 + b}{\sqrt{-\Delta}} \operatorname{tg} \frac{\sqrt{-\Delta}}{2} l\right]},$$

2) $$j = \frac{\sqrt{\Delta} - b + (\sqrt{\Delta} + b) \frac{2aj_0 + b - \sqrt{\Delta}}{2aj_0 + b + \sqrt{\Delta}} e^{l\sqrt{\Delta}}}{2a\left[1 - \frac{2aj_0 + b - \sqrt{\Delta}}{2aj_0 + b + \sqrt{\Delta}} e^{l\sqrt{\Delta}}\right]},$$

3) $$j = \frac{4c/b^2 - \left(\frac{4c/b^2}{j_0 + 2c/b} - l\right)}{\left[\frac{4c}{b^2} \Big/ \left(j_0 + \frac{2c}{b}\right)\right] - l}.$$

As condition for onset of breakdown we take the rise to infinity in the current, which occurs when the denominator becomes zero; the onset occurs in the above three cases when

$$\frac{\sqrt{-\Delta}}{2a j_0 + b} = \operatorname{tg} \frac{\sqrt{-\Delta}}{2} l, \qquad (5)$$

$$\frac{2a j_0 + b + \sqrt{\Delta}}{2a j_0 + b - \sqrt{\Delta}} = e^{l\sqrt{\Delta}}, \qquad (6)$$

$$\frac{4c/b}{bj_0 + 2c} = l. \qquad (7)$$

These very different results reflect the great variety of forms of electrical breakdown in insulators. In fact, α and

$\varkappa$ are related by $\frac{\varkappa}{\alpha} = \frac{1-\zeta}{1-\mu} = \alpha'$, so (3) can be put as

$$\frac{w_0}{q}\frac{dj_-}{dx} = \alpha\,(j_-E + \alpha' P). \quad (8)$$

The above argument is applied to α with α' = constant and P = constant to get criteria analogous to (5)-(7) but with different coefficients.

The detailed results on any particular form of breakdown should enable one to establish which of the above theoretical results is most closely applicable.

REFERENCES

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Institute of Technical Physics
AS BSSR