POSSIBILITIES FOR A UNIFIED THEORY

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1. Physical reality may be classified in the following categories; 1) space (R), 2) time (T), 3) ordinary matter (M), and 4) gravitation (G). To these we may add the cosmological setting, which covers phenomena specifically related to the whole of the known universe.

Space and time raise no doubts, whereas gravitation may be considered a purely phenomenological concept, of the type of the earlier division of matter (we omit ordinary) into particles that have rest mass and those that do not. A maximally unified theory (MUT) must strive to give not only a unified theory of elementary particles (including the excited states of these, which make up the nucleus, atoms, molecules, planets, and so on) but also one of gravitation, with the closest possible relation to space and time; it must also give a nonlocal but natural world-view corresponding to the cosmological facts, in particular the expansion of the universe.

There have recently been many attempts to set up a partly unified theory, on account of the numerous particles and resonons now known, together with the extensive evidence (optical, radio-astronomical, and so on) on the cosmos; but only the first steps have been made towards a maximally unified theory. My object here is to survey these attempts and to indicate likely prospects for the near future.

R and T have been combined in the special theory of relativity, while Einstein's general relativity (GR) interprets gravitation as a curvature in R and T caused by any form of M and by the gravitational field itself. All the attempts made in the 1920's to give a unified geometrical theory of gravitation and electromagnetism (universe of five dimensions, remote parallelism, unsymmetrical metric, etc.) were unsuccessful, as were all attempts to derive quantum theory from such formally generalized geometries (Weil, Eddington, and so on). The basic cosmological fact (expansion) is a possible consequence of GR, but the latter would also allow contraction and gives no information about the actual known universe. This resembles the position in relativistic quantum field theory, which describes the motion and interaction of particles with great precision but takes the charges and other constants from experiment.

M is now accepted as being undoubtedly distributed over groups of leptons, mesons, baryons, the parallel antiparticles, and (more recently) the families of resonons containing eight or ten members, which point directly to SU_3 unitary symmetry (from which the omega-particle was predicted) or generalizations of this of SU_6 type, which take account of the relation between ordinary space and isospace.

2. The most recent and most promising trends towards a MUT may be considered first by reference to the picture of ordinary matter. The argument follows these main lines in field theory: 1) axiomatics, 2) dispersion analysis, 3) group theory; then there are some dynamical trends: 4) compensation fields, 5) nonlinear spinor theory, 6) dynamical models of particles. All of these, except 1, use matter in the unified picture and overlap to a certain extent. Approach 2 treats particles as poles in the plane of the complex variable moment and has had some success, but it seems too formal, and here I consider the dynamical treatments.

The most radical of these is the proposal to use as basis a nonlinear spinar field that interacts with itself and whose excited states give particles and resonons. The basic feature is Dirac's equation modified by a nonlinear term of some form (Heisenberg, Ivanenko). Here the particles fully acquire the half-integral de Broglie spin; the basic field must have the half-integral spin $S = (1/2)(h/2\pi)$ while the higher states may have any other spin. The more empirical Fermi-Yang and Sakata models (which have, for instance, the π-meson represented as constructed from a nucleon and antinucleon) have stimulated the successful group approach via SU_3 but are incorrect. But Gell-Mann and Zweig consider that the basis may be taken not as p, n, and Λ but as subparticles (quarks) p_0, n_0, Λ_0 (fractional charge), which make up all real particles (strongly interacting ones, at least).

The nonlinear theory can be combined with the quark hypothesis by using a unitary spinor with three components, each of these being an ordinary spinor. The algebra of unitary spinors then gives (Naumov and Kurdgelaidze) the basic invariants, which give a nonlinear equation of Dirac type.

Naumov recently proposed the simplest method of deriving the mass of the basic baryon, which has a relation to the Nambu and Jona-Lazinio method for superconduction. These two methods are much simpler than the old Heisenberg one, which also required additional hypotheses about the structure of Hilbert space.

Heisenberg and Durre, and Marshak and Okubo (Dubna conference, 1964), have given a somewhat different approach to the inclusion of quarks in the nonlinear theory. The best formulation of the old Thomson-Lorentz-Abragam approach (for the field mass of a particle) has been given recently by de Broglie, Vigier, and Yukawa, and also by Loskutov and de la Penne, in which the baryon is surrounded by two clouds bearing the isospin and strangeness, which can be excited. This has given good ratios for the masses, which agree to a large extent with Zweig's quark formulas and the presently popular Okubo formulas.

There have thus been considerable advances in the unified theory of matter as regards the families of particles and resonons; in addition, it seems now to be clear that one can hope to calculate even the masses and coupling constants, although only preliminary results are available as yet.

3. I consider that the framework of Einstein's GR is too narrow for us to consider further the place of gravitation; we need to try various major extensions of GR. It is clear that the Riemann-Einstein metrical potentials $g_{\mu\nu}$ are suitable only for the description of the interaction between gravitation and various systems (macroscopic and atomic ones of boson type), because fermions (which are described by spinors) interact with the tetrad quantities $h_\nu(a)$, which are the roots of $g_{\mu\nu}$ (Fok and Ivanenko, Weil). At each point we set up (apart from the vector characterizing the curvilinear coordinates h_ν) a vector characterizing the orthogonal tetrad components $h(a)$; from the mutual projections we construct a metrical tensor $g_{\mu\nu} = h_\mu(a)\, h_\nu(a)$. The usual covariance of GR is then accompanied by invariance with respect to local rotations of the tetrad, so it is necessary to rework the whole of GR from the point of view of the tetrads, which is not merely a simple rewriting, on account of the need for additional and boundary conditions. Three types of additional condition are presently under discussion:

1) $$\frac{1}{\Lambda}\frac{\partial(\Lambda h_\nu(a))}{\partial x_\nu} = 0 \qquad (\Lambda = \det|h_\mu(a)|).$$

This quasiharmonic condition corresponds to a certain continuity of the tetrad under transforms; it goes over to de Donder's harmonic condition for holonomic systems. Four components are eliminated, whereupon others are eliminated in accordance with the character of the particular problem;

2) Meller's additional conditions, which are chosen for the construction of an energy complex free from the difficulties of the Bauer-Meller one; and

3) Schwinger's additional conditions (here not further specified). The existing uncertainty in the choice of additional conditions recently led Meller to abandon these (perhaps prematurely, to judge from his closing comment at the Galileo Conference of September, 1964) and to use instead boundary conditions that give the tetrads constant values. Meller still supports the tetrad formulation of GR, because he sees this as the only possible way to construct a rational expression for the energy, because the old Einstein formulas, or the Meller-Mitskevich ones (derived by variation from the terminated G), or the total invariant of R, have led to difficulties (as Bauer and Meller have pointed out). This feature is a second major argument in favor of the tetrad theory of gravitation.

It is clear that the energy complex should be derived by varying G or R with respect to the $h_\mu(a)$; in particular, we may use Rodichev's condition to get from $R = -\Delta_{abc}\, C^{abc}$ (in which the Δ are Ricci coefficients, as are the C, because $\Delta_{abc} = C_{abc} - C_{cab} - C_{bca}$ are dependent on the first derivatives of the tetrad components) the equation for the gravitational field as

$$\Delta^{\sigma\rho}_{m;\rho} = t^\sigma_m + \kappa T^\sigma_m,$$

in which t is the tensor for that field and T is the energy tensor for ordinary matter. The tetrad energy complexes are readily found from G and R by variation without imposing additional conditions or by isolating the divergence in the field equations and transferring the other items to the right as a source of gravitational field. These expressions may be found via the superpotentials. We apply Neter's theorem to the local 4-rotations to get the spin momentum of the field, which in the case of Rodichev's additional conditions takes the form

$$S(ab)^\sigma = \frac{c^4}{2\pi\kappa} C(ab)^\sigma.$$

The tetrad extension of GR is required in the compensation treatment of the gravitational field, which brings it into line with other boson fields in this respect. Invariance with respect to any group of transforms in ordinary or isotropic space (simple or Lorentz rotations, gauge transforms, and so on) gives rise to conservation laws. The parameters of the transform may be taken as localized (functions of the four coordinates) instead of constant, in which case the additional terms arising from the derivatives of these parameters are compensated by introducing a new compensating field that transforms in an appropriate fashion. Localization of the gauge transform of a phase of a wave function gives rise to an electromagnetic compensating field; Yang and Mills indicate that localization of gauge transforms in isospace related to conservation of hypercharge and isospin gives rise to compensating fields whose quanta are the recently discovered meson resonons. Localization of a Lorentz group (when the parameters of the rotation are functions of the coordinates) or, more precisely, of a complete inhomogeneous Poincaré-Lorentz group that includes shears, produces a compensating derivative that coincides in the main with the covariant derivative of Riemann-Einstein *GR* (Utiyama, Brodskii-Ivanenko-Sokolik-Frolov, Kibble, Schwinger). The theory may be substantially extended via tetrads whose rotations correspond to Lorentz transforms.

The new theory is still incomplete, but we may speak of the tetrad extension of Einstein's GR as necessary for the description of spinor fermions and which requires compensation treatment; it can throw light on the difficult problem of gravitational energy and assist in the interpretation of inertia.

The tetrad formalism links gravitation to other fields via the compensation treatment and involves the use of the $h_\nu(a)$ as the basic primary quantities together with some appropriately chosen spinor for the unified theory of matter. Penrose's spinor form for Einstein's equations here acquires additional significance.

Torsion has a tensor that is expressed in the compensation treatment directly in terms of the field spin; moreover, a term containing torsion may be added to the compensating derivative. The most striking result related to torsion (Rodichev; see also Finkelstein and Peres) indicates that the covariant derivative of a spinor in a twisted space necessarily gives rise to a nonlinear term (of pseudovector type) in Dirac's equation. This may well be a geometrical interpretation of this important nonlinear equation that forms the basis of a future unified field theory. It may be that these results mean that space is twisted at very small distances (e.g., within elementary particles). Torsion prepares the ground for a fuller treatment and it would be desirable to quantize it (via torsons).

Matter and gravitation may be related to another aspect, namely that quantum theory of necessity implies the possible mutual transmutation of interacting fields. This has been confirmed by experiment, but it is a commonplace to say that the special geometrical character of the gravitational field implies that any conversion (of positrons, neutrinos, photons, and so on) into gravitons requires careful analysis and must give rise to philosophical discussion. Of course, we do not envisage the conversion of matter into space, but the conversion of matter into space curvature and the reverse. If gravitons actually carry energy, there can be no doubt that such transmutations are possible in principle. This hypothesis of mine has since been applied to several cases by various workers.

Gravitational radiation is of quadrupole type, and the gravitational constant is small, so the probability of such conversion is vastly smaller than for the usual transmutations involving electromagnetic energy and so on. However, qualitative extrapolation to high energies shows that gravitational and ordinary transmutations may become comparable in probability at extremely high energies and hence may be important in cosmology. These transmutations (with their new quantum aspect) relate matter to gravitation and space-time; they persistently require a maximally unified theory.

It is also possible that space is quantized as well as twisted inside particles (Abartsumyan-Ivanenko, Sneider, Cauchy, Shapiro), e.g., on account of Wheeler fluctuations in the metric.

4. So far I have not considered the picture for our part of the universe, where we find expansion as well as all the local properties considered above, and also a high concentration of particles (protons, neutrons, electrons) relative to antiparticles, together with a relatively low density of matter (about 10^{-30} g/cm^3). We may wonder whether these features are related one to another and to fundamental quantum aspects of elementary particles. The maximally unified theory must answer these questions.

Eddington's attempts to relate cosmology to quantum physics were not very successful, though they stimulated some useful work. Wheeler's views are similar, and he has tried in his geometrodynamics to devise a unified theory on the basis of topology and quantum mechanics (though this at present appears powerless as regards the inclusion of spinors).

On the other hand, it is possible that a contracting space would contain preferentially antiparticles. It is also possible to express Hubble's expansion constant λ in terms of atomic constants:

$$H = \sqrt{\lambda} \simeq \frac{m_g c^2}{h},$$

in which m_g is the mass of a graviton (10^{-66} g). Kurdgelaidze considers that the known proportionality (of the proton and electron masses to the constants for the strong and electromagnetic interactions) may be extended to the weak and ultraweak gravitational ones to give a mass for the graviton.

Here we have something more than mere play with constants; we have the first indications of a relation between such facts as the expansion of the universe and the behavior of elementary particles. The expansion indicates the arrow of time, so there should not be strict reversibility in elementary processes. Some such cosmological force may account for the difference in behavior between particles and antiparticles, and hence it may be responsible for parity nonconservation, in particular the anomalous decay of a K_2° meson to two pions instead of three. Kurdgelaidze's treatment relates the proportion of anomalous decays to the expansion factor, which itself is governed by the ratio of the average actual density of matter to the critical density. Quasars and other astronomical explosions show some resemblance to this expansion (Hoyle, Novikov); Sokolov's treatment of time parity nonconservation is notable here. This may be why the unified theory of modern physics should be the maximally unified one dealt with above; it may also be why a unified theory of matter is impossible.

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