SURVEYS

HYDRODYNAMICS AND HEAT TRANSFER IN SUPERFLUID HELIUM

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Utilization of the unique properties taken on by substances at low temperatures (chiefly superconductivity) involves a number of thermophysical problems, one of which is the cooling of various specimens to very low temperatures [1]. Many of the investigations along this line are devoted to heat exchange with He I, the nonsuperfluid phase of helium. However, recently there has been intensive study of the possibility of using superfluid helium, He II, as the coolant in cryogenic installations. This is due to a number of specific features of He II.

Perhaps the most important of them is that under the effect of a temperature drop, there appears a special kind of macroscopic motion (of the convection type), as a result of which large heat fluxes are observed even at low temperature gradients. The advantage from the purely design-oriented point of view is that this eliminates complex problems involved in setting up forced circulation of the flow.

Equally important is the behavior of He II in nonstationary conditions. Unlike the case of other media, in superfluid helium the propagation of thermal perturbations is of a wave nature. This, on the one hand, leads to high rates of heat removal and, on the other hand, offers new possibilities for controlling the heat exchange.

Furthermore, for the operation of some cryogenic systems — e.g., instruments used in astrophysical research — it may be necessary to reach temperatures T lower than 2°K, and thus He II is the only suitable coolant.

Being superfluid, the helium can penetrate into very narrow slits — e.g., into the interturn spaces of superconductive coils — and intensify the heat exchange there. Lastly, it is probable that other specific phenomena — e.g., the thermomechanical effect — may be used in cryogenic installations.

Today there are many studies devoted to heat transfer between solids immersed in superfluid helium (see, e.g., the surveys [2-4]). Researchers have accumulated large amounts of experimental data, relating chiefly to thermal resistance at the boundary (the Kapitsa jump), critical heat fluxes at which a phenomenon of the film-boiling type occurs, and also data on heat exchange in various regimes.

When it comes to theoretical generalizations, the situation is much worse. Up to now, not only was there no theory of heat transfer in superfluid helium, but there was not even any agreement regarding the physical processes taking place at the boundary and in the liquid. As a result of the lack of such a theory, new experimental results not only do not clarify the general situation but, on the contrary, make it even more confused.

There have, of course, been attempts to construct a theoretical justification for various experimental facts (for further details on this score, see the surveys [2-4]); here we must point out the following facts. In the study of specific problems of heat exchange with He II, generalizations have often been made within the framework of concepts taken from the theory of heat exchange in the boiling of classical liquids. In such cases the specific features of superfluidity have been taken into account only partially, and sometimes they have been totally ignored.

At the same time, the laws of hydrodynamics, including the propagation of heat in He II, are radically different from the behavior of ordinary systems. But as in the case of the boiling of classical liquids, the hydrodynamic processes taking place in a volume of superfluid helium play an extremely important role and represent the basis (together with the ideas of the theory of boiling) on which one can build an adequate theory of heat exchange in He II.

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Recently, the hydrodynamics of He II have been intensively studied, owing to the possibility of using helium as a refrigerant. At the same time, researchers in this discipline have obtained a number of classical results giving a unified picture of such a system.

This survey is intended to describe the present status of the hydrodynamics of a superfluid liquid. In view of the purpose of the article, attention has been concentrated mainly on questions related to heat transfer in a volume of He II.

1. Stationary Thermomechanics

1.1. The Two-Liquid Model. If we trace the history of the discovery of superfluidity, we see that He II might more properly be called super-heat-conductive. The transition to the superfluid state was first observed by Kamerlingh Onnes [5]. When temperatures lower than 2.17°K were reached, the surface of violently boiling helium suddenly became calm, and the boiling stopped. It was assumed (and subsequently confirmed by experiment) that the lack of bubble-type boiling was the result of the unusually high thermal conductivity of He II, which made impossible the overheating necessary for bubble formation. The thermal conductivity of He II was first measured by Keezom [5]. It was found that the quantity λ'_{eff}, defined as the ratio of the heat flux W to the temperature gradient ∇T in a capillary filled with helium, has a numerical value millions of times as high as the analogous quantity for He I and hundreds of times as high as the values for coppper and silver. This regime was established almost instantaneously; it appeared that He II was an ideal refrigerant capable of very rapidly transferring enormous heat fluxes.

Besides the phenomenon of superconduction of heat, other properties of He II were also discovered — e.g., the thermomechanical effect or the absence of viscosity — which do not fit at all into the framework of classical ideas.

All of these effects can be explained by using the two-liquid model which follows from Landau's theory of superconductivity [6]. From the viewpoint of hydrodynamics, He II can be viewed as a mixture of two components. One of them, a superfluid liquid with density $\rho_s(p, T)$, moves with velocity $\mathbf{v}_s$. The superfluid component has no shear viscosity, and therefore it cannot be subjected to torsion (rot $\mathbf{v}_s \equiv 0$), and also cannot absorb and carry heat. The other component is normal, with density $\rho_n(p, T)$ and velocity $\mathbf{v}_n$ and behaves like an ordinary viscous liquid. The motion of the two components is thermodynamically reversible and consequently independent.

The equations of motion of such a liquid can be obtained on the basis of the laws of conservation [6, 7]. We shall write out and explain these equations:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{j} = 0, \tag{1}$$

$$\frac{\partial j_i}{\partial t} + \frac{\partial\Pi_{ik}}{\partial r_k} = 0, \tag{2}$$

$$\frac{\partial S}{\partial t} + \operatorname{div} S\mathbf{v}_n = 0, \tag{3}$$

$$\frac{\partial\mathbf{v}_s}{\partial t} + \nabla\left(\mu + \frac{\mathbf{v}_s^2}{2}\right) = 0. \tag{4}$$

Equations (1), (2) are the usual laws of conservation of mass and impulse density. The impulse flux tensor Π_{ik} is equal to

$$\Pi_{ik} = \rho_s v_{si} v_{sk} + \rho_n v_{ni} v_{nk} + \delta_{ik} p. \tag{5}$$

The subscripts i, k denote the coordinates x, y, z; δ_{ik} is the unit tensor.

As can be seen from (5), the complete impulse-flux tensor can be decomposed into a normal part and a superfluid part, and Eq. (2) has an obvious structure. Equation (3) is also obvious. This is the law of conservation of entropy. Here we see reflected the fact that entropy is carried over only by the normal component. The expression (4) for the velocity of the superfluid component is new, in contrast with the expression for an ordinary liquid. It contains the information that the superfluid component cannot be subjected to torsion because it has no shear viscosity. Therefore, rot $\mathbf{v}_s = 0$, and consequently, the convection term is $(\mathbf{v}_s\nabla)v_s = \nabla\mathbf{v}_s^2/2$. The driving force for the superfluid part is the chemical potential $\mu(p,T)$.

The case of dissipative motion was analyzed by Khalatnikov [7]. It was found that, unlike the case of ordinary liquids, in helium there are not three but five independent kinetic coefficients. One of these, the shear-viscosity coefficient η, exists only for the normal component, as was to be expected. In addition, we find three coefficients of bulk viscosity and a coefficient of thermal conductivity χ.

We write an expression for the energy flux **W** which follows from Eqs. (1)-(4) and which will be useful in our further discussion:

$$\mathbf{W} = \left(\mu + \frac{\mathbf{v}_s^2}{2}\right)\mathbf{j} + ST\mathbf{v}_n + \rho_n \mathbf{v}_n(\mathbf{v}_n - \mathbf{v}_s,\ \mathbf{v}_n) + \mathbf{W}_{\mathrm{irr}}. \tag{6}$$

Here $\mathbf{W}_{irr}$ stands for the irreversible fluxes caused by the dissipative effects, which are negligibly small for all real cases. It should be noted at once that we observe a macroscopic energy flux $\mathbf{W} = \mathrm{St}\mathbf{v}_n$ even in the case when the total mass flow **j** is equal to zero. (The quantity $\rho\mathbf{v}_n(\mathbf{v}_n - \mathbf{v}_s, \mathbf{v}_n)$ is much smaller than $\mathrm{ST}\mathbf{v}_n$ for all velocities that can actually be attained.)

The system of equations (1)-(4) is very complicated, and furthermore, it is not closed. Because there are two velocities, the thermodynamic quantities are functions of the relative velocity $\mathbf{w} = (\mathbf{v}_n - \mathbf{v}_s)$*, and determining this functional relation requires solving a complex problem in quantum mechanics. However, in the case of low velocities the functional relation can be obtained from thermodynamic considerations, as was done in [7], and thus the system can be closed.

We consider a special but very important case of the solution of these equations. Using the thermodynamic identity $\rho d\mu = -SdT + dp + (\rho n/2\rho)\,\mathbf{w}d\mathbf{w}$, we rewrite (4) as

$$\frac{D\mathbf{v}_s}{Dt} + \frac{1}{\rho}\nabla p - \sigma\nabla T + \frac{\rho_n}{\rho}\nabla \mathbf{w}^2 = 0,\ \sigma = \frac{S}{\rho}. \tag{7}$$

Here D/Dt denotes, as usual, the operator $\partial/\partial t + (\mathbf{v}_s\nabla)$. From this equation it can be seen that even when there is no pressure gradient in the system, there will be a motion caused by the temperature gradient. The superfluid component moves in the direction of the heater, and the normal component moves away from it, so that the total mass flow will be zero, in accordance with the fact that $\nabla p = 0$. The energy flux in this case is nonzero and is equal to $\mathrm{ST}\mathbf{v}_n$, since the heat is carried only by the normal component.

To this kind of motion, which is reminiscent of convection, we shall, following the terminology now widely used in non-Soviet literature, apply the term "counterflow," and we shall call the corresponding section "thermomechanics" [8]. As mentioned above, the capability of He II, when acted upon by a temperature gradient, to give rise to a motion with macroscopic heat fluxes makes it a very promising material for thermophysical applications.

As an example of one of the properties of counterflow, we shall consider Keezom's experiment [5]. For small velocities (and, in accordance with the formula $\mathbf{W} = \mathrm{ST}\mathbf{v}_n$, small heat fluxes), in Eqs. (1)-(4) we can disregard the variation of density, entropy,† and nonlinear terms. In this case the system (1)-(4) (with dissipative terms) can be subdivided into equations for the normal component and the superfluid component:

$$\rho_n\left(\frac{\partial \mathbf{v}_n}{\partial t} + (\mathbf{v}_n\nabla)\,\mathbf{v}_n\right) = -\frac{\rho_n}{\rho}\nabla p - \rho_s\sigma\nabla T + \eta\nabla^2\mathbf{v}_n, \tag{8}$$

$$\rho_s\left(\frac{\partial \mathbf{v}_s}{\partial t} + \nabla\frac{\mathbf{v}_s^2}{2}\right) = -\frac{\rho_s}{\rho}\nabla p + \rho_s\sigma\nabla T. \tag{9}$$

Here we have taken account of the shear viscosity η for the normal component. The bulk viscosity does not play a role in this type of motion, and the thermal conductivity contributes only a slight correction to the final answer.

*In ordinary hydrodynamics, owing to Galilean invariants, the internal characteristics are, of course, independent of the velocity.
†The condition for disregarding the variation in density is, as is known, the following: $v^2/c_1^2 \ll 1$. For entropy we require an analogous condition, v^2/c_2^2, $v = \max(v_n, v_s)$.

For (9) we can see that in the stationary case $\nabla p = S\nabla T$, the so-called London formula. The temperature drop is accompanied by a pressure drop, which forces the viscous normal component to flow. From (8) we have $\eta\nabla^2\mathbf{v}_n = (\rho_n/\rho)\nabla p + \rho_s\sigma(\nabla p/\rho\sigma) = \nabla p$. It follows from this that when there is motion along a capillary, $\mathbf{W} = ST\mathbf{v}_n$ is proportional to ∇p, and in accordance with London's formula, proportional to ∇T; the proportionality constant depends on the geometry of the capillary. For a tube with radius a we have

$$W = \frac{a^2 S^2 T}{8\eta}\nabla T = \lambda'_{\text{eff}}\nabla T. \tag{10}$$

For $a \approx 0.1$ cm (T = 2°K), we obtain λ'_{eff} 10^3 W/cm•°K. It should be noted that the thermal conductivity of He I is equal to $3\cdot10^{-3}$ W/cm•°K, and that of copper is 5 W/cm•°K.

Thus, we have shown how the high thermal conductivity of He II can be explained within the framework of the hydrodynamics of superfluidity. In principle, the system of equations (1)-(4) enables us to solve the problem of energy fluxes for a given temperature distribution and (or) the problem of temperature distribution for given fluxes. For motions of the counterflow type, where $\mathbf{j} = 0$, Eqs. (1) and (2) are identically satisfied, and a theory of this kind would be formally close to gasdynamics (with the substitution $\rho \to \sigma$, $p \to T$, where $\sigma = \sigma(T)$ is a condition of the barotropic-distribution type). If $\mathbf{j} \neq 0$ (forced flow), this analogy disappears and we then must deal with highly complex and specific problems. A discussion of the various questions connected with laminar flow of He II is contained in [5].

Unfortunately, the situation in reality is more complicated. Even when we reach some critical fluxes of $\mathbf{W} \approx 10^{-3}$-$10^{-2}$ W/cm^2 (which corresponds to $\mathbf{v}_n$, $\mathbf{v}_s \approx 1$ cm/sec), we already find that the function $W(\nabla T)$, which agrees so well with the theory, is no longer valid.* In the region of critical fluxes, the form of this function is affected by a great many factors, going back to the prehistory of the process. Obviously this is due to a change of regime of the transition-to-turbulence type. However, as the flux W increases, the function $W(\nabla T)$ takes on the universal form (see [4])

$$\nabla T \sim W^3 \tag{11}$$

(Figs. 1 and 2). Sometimes [2, 3] we find (11) written in the form $W = \tau_{\text{eff}}\nabla T$, where the expression proposed for λ_{eff} is the empirical formula

$$\lambda_{\text{eff}} = C(T)\,T\eta\rho^{2/3}\sigma^{4/3}(\nabla T)^{-2/3}. \tag{12}$$

In the supercritical regime the effective thermal conductivity of helium sharply decreases. For example, for $\nabla T = 0.01$°K/cm and T = 1.9°K, we have $\lambda_{\text{eff}} \approx 100$ W/cm•deg, which is lower by one and a half orders of magnitude than in the laminar regime (10).

The relation (12), with experimentally determined coefficients C(T), is one of the most important achievements in the investigation of the thermal conductivity of He II. In this form it can be used for concrete calculations in cryogenic systems or serve as an auxiliary formula for considering some theoretical questions (e.g., the question of the critical heat fluxes that precede boiling).

Expression (12) is suitable from the standpoint of applications, but it has no great theoretical significance. The question of the value of C(T) and the question of the origin of (11) itself remains ultimately unexplained up to the present time.

The value of W (or v_n) increases at a rate which is less than linear as ∇T increases; this means that in the volume of helium there arises some mechanism which causes a retardation of the action on the two components. In 1949, Gorter and Mellink [9] assumed that in the supercritical regime the motion of the superfluid component and the normal component will not be independent but will take place with friction, and the frictional force (per unit volume) is equal (Fig. 3) to

$$\mathbf{F}_{sn} = A(T)\,\rho_s\rho_n(\mathbf{v}_n - \mathbf{v}_s)^2(\mathbf{v}_n - \mathbf{v}_s). \tag{13}$$

It is not difficult to see that such friction leads to formula (11). If in Eq. (8) we add a term $\mathbf{F}_{sn}$ and disregard ordinary viscosity, which is negligible in this case, then in the stationary regime we can obtain (remembering again that $\mathbf{W} = ST\mathbf{v}_n$):

*It should be recalled that we are talking about counterflow.

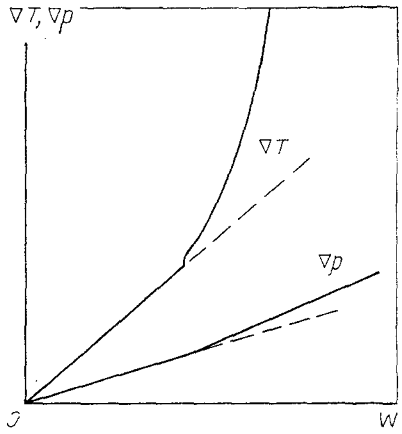

Fig. 1

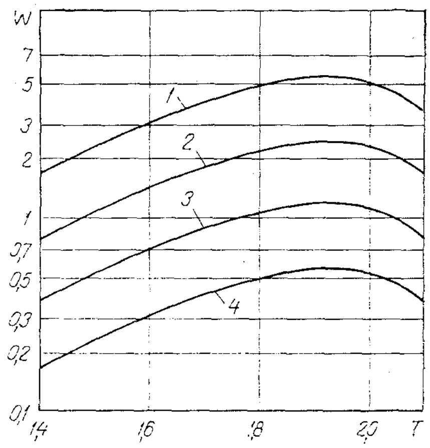

Fig. 2

Fig. 1. Typical behavior of ΔT(W), ∇p(W) in arbitrary units.

Fig. 2. Heat flux W (W/cm^2) as a function of temperature for various temperature gradients: 1) $\nabla T = 10^{-1}$•K/cm; 2) 10^{-2}; 3) 10^{-3}; 4) 10^{-4}.

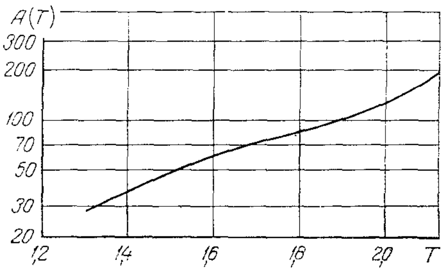

Fig. 3. Coefficient of mutual friction A(T) (cm•sec/g) as a function of temperature (from the measurements of [10]).

$$\nabla T = A(T)\frac{\rho_n}{\sigma^4\rho_s^3 T^3}W^3, \qquad (14)$$

which coincides with (11).

Even though it leads to agreement with experimental results, the Gorter–Mellink hypothesis of mutual friction is merely a reformulation of the problem. The appearance of frictional forces which violate the condition of nondissipative motion and are subject to formula (13) has been found to be a problem which is quite complex and has not been finally solved to the present day.

1.2. Critical Velocities; Quantum Vortices. Today there is no doubt that the mutual friction is due to so-called quantum turbulence, the presence of randomly oriented quantum vortex lines in the helium. In order to understand that this is so, we must turn to the foundations of two-velocity hydrodynamics [6].

Because of the very low temperature, there are substantial quantum effects in the helium. Therefore heat enters the helium in the form of a quantum-type collective motion of all the atoms, the so-called phonons and rotons, or elementary perturbations, as they are known. The collection of all phonons and rotons may drift as a unit through the liquid, carrying energy, momentum, and mass, i.e., behave like an ordinary gas (a gas of quasiparticles). Because of the interaction of the quasiparticles, this gas behaves like a viscous gas. According to Landau, the mass carried by this motion is associated with the mass of the normal component. The remainder of the mass (the background) is the superfluid component. The quantum nature is manifested in the fact that the phonons and rotons appear singly, possessing a momentum **p** and an energy $\varepsilon(|\mathbf{p}|)$. If the helium moves with velocity **V**, then by virtue of Galileo's transformation, we have (in the laboratory system)

$$\varepsilon' = \varepsilon(|\mathbf{p}|) + \mathbf{pV}. \tag{15}$$

If $\varepsilon' < 0$, then in the moving helium it is thermodynamically advantageous to have the formation of elementary perturbations, i.e., the kinetic energy can be converted into heat. The concrete form of $\varepsilon(|\mathbf{p}|)$ predicted by Landau is, however, not very significant to us; what is important is the fact that when $V > \min \varepsilon|\mathbf{p}|/|\mathbf{p}|$, ε' must be negative and the motion of the helium must be accompanied by dissipation. Thus, the existence of critical velocities follows from Landau's theory [6].

However, the value for $\mathbf{V}_{cr}$ obtained by this formula is equal to 60 m/sec, which is approximately 1000 times the experimentally observed value. This was one of the major defects of Landau's theory. Another was the question of the vortex-free motion of helium, its failure to go into rotational motion.

As noted earlier, the fact that the superfluid component does not interact with the walls means that it cannot be brought into rotation. In confirmation of this, an experiment [11] was devised for measuring the depth of the parabolic meniscus of rotating helium. It was expected that, since only the normal component rotates, the depth of the meniscus would be less by a factor of ρ_n/ρ than in an ordinary liquid. The experiment yielded a negative result. The meniscus was found to be the same as if the entire liquid were rotating.

In order to explain this situation, Feynmann and Onsager [12] assumed, justifying their assumption from the quantum-mechanics point of view, that the condition of nonvorticity, $\text{rot}\,\mathbf{v}_s = 0$, is satisfied in rotating helium everywhere except at one-dimensional singularities (vortex lines). Around the vortex lines the circulation is constant and equal to

$$\oint \mathbf{v}_s d\mathbf{l} = \frac{2\pi\hbar}{m_{\text{He}}}. \tag{16}$$

The velocity field of the unified vortex accordingly becomes $v_s(\mathbf{r}) = \frac{\hbar}{m_{\text{He}}}\frac{1}{|\mathbf{r}|}$. It should be noted that $\text{rot}\,\mathbf{v}_s = 0$ everywhere except at the vortex point, where the curl is undefined.

When a certain critical value of rotational velocity is exceeded, there will be formed in the vessel containing the helium a number of vortices which are uniformly distributed in the volume, and the average motion will imitate the rotation of a solid.

Here it is important to consider the following. Since the velocity increases near the axis of the vortex, at some value of r it will exceed the Landau critical velocity. A thin tube of the order of several angstroms will be formed, consisting entirely of the nonsuperfluid component. As a result, we find that the density of the normal component is nonuniform near the axis of the vortex.

When the normal component moves through the structure of such a vortex, the phonons and rotons are scattered over the vortex, transferring some of their momentum to it. Thus, when there are vortices, the relative motion of the two components will no longer be nondissipative, and we will find mutual friction. As was shown in [13], the force of mutual friction (per unit length of the vortex) is proportional to the relative velocity **w**. It should be noted that unlike Landau's critical velocities, the superfluid component does not disappear and the hydrodynamics retains its two-velocity character. There are quite a few experimental proofs of the existence of vortex lines, ranging all the way to the photographing of the vortex lattice (for more details, see [14]).

The foregoing relates to the case of helium which is rotating. Obviously, when helium moves in tubes, we must observe something similar, i.e., when certain velocities of motion (of the order of 1 cm/sec) are exceeded, there will appear in the volume a number of vortex lines (which, of course, are no longer straight lines) and vortex rings, whose existence leads to mutual friction.

Opinions differ concerning the mechanism of the occurrence of vortices in He II. The most widespread theory is the fluctuation theory of vortex-nucleus formation [15].

1.3. Superfluid Turbulence. In the preceding section we arrived at the conclusion that when the normal component moves with respect to a vortex, there arises a frictional force proportional to the length of the vortex line and to $\mathbf{v}_n - \mathbf{v}_s$. How does the presence of this force lead to the Gorter–Mellink formula, and thus to a cubic function?

The theory of supercritical counterflow in channels was first proposed by Vinen [13]. The following qualitative picture is assumed. When the critical velocities (or heat fluxes) are exceeded, vortex lines (no longer straight) or vortex rings are created in the channel. The total length of these per unit volume is L. As the counterflow velocity increases, the value of L will increase and the vortex lines will form a confused tangle. When L is large enough to make the intervortex distance $\delta = L^{-1/2}$ much smaller than the dimensions of the channel, the flow loses its individuality and the picture of the vortex lines becomes universal. Such a situation is called superfluid turbulence. As is known, a curved vortex line must move at some velocity which depends on the shape of the line and on its vortex intensity $\varkappa$. For example, a vortex ring of radius R moves with a velocity $v = (\varkappa/2\pi) \ln (R/a_0)$, where a_0 is the thickness of the vortex tube. However, if the vortex is not free but interacts with external forces (in the present case this means friction from the normal component), there arises a force which is perpendicular to the plane of the ring, the so-called Magnus force. Analysis shows that it must lead to an increase in the length of the vortex line, and L must increase. However, as L increases, the tangle becomes denser and vortex-line intersection effects come into play. As a result of the collisions, the rings will be broken up, forming rings of smaller dimension, and these in turn will again be subdivided, and so on, ending with the formation of circles of very small dimensions, known as rotons. Thus, the energy of the kinetic motion is converted into heat. This mechanism prevents the length L from increasing indefinitely under the action of the Magnus forces.

On the basis of these considerations, Vinen [13] obtained the balance equation for the quantity L:

$$\frac{dL}{dt} = \alpha |\mathbf{v}_n - \mathbf{v}_s| L^{3/2} - \beta L^2, \tag{17}$$

where α and β are empirical parameters.

Although this is a crude and in fact qualitative conclusion, Eq. (17) satisfactorily describes almost all the experiments conducted with supercritical flows. In particular, it yields formula (13). To see this, we note that in the stationary case ($dL/dt = 0$) we have

$$L_{\text{eq}} = \frac{\alpha^2}{\beta^2} (\mathbf{v}_n - \mathbf{v}_s)^2. \tag{18}$$

Furthermore, since the force per unit length is proportional to the relative velocity $\mathbf{f} \sim (\mathbf{v}_n - \mathbf{v}_s)$, the total force (for an isotropic distribution of the vortex lines) must be

$$\mathbf{F}_{sn} \sim \frac{\alpha^2}{\beta^2} (\mathbf{v}_n - \mathbf{v}_s)^2 (\mathbf{v}_n - \mathbf{v}_s). \tag{19}$$

Equation (17) was obtained more than twenty years ago. Recently, interest in superfluid turbulence has again increased. This is due precisely to the question of using He II as a refrigerant. A fairly large number of experimental studies have been published (see [16]). Among the theoretical investigations we may mention the study by Schwarz [16], who again considered the question of the dynamics of a vortex tangle. Schwarz introduces the distribution function $\lambda(\mathbf{v}_l, t)$, the length of the vortex lines per unit volume whose velocities lie in the interval $\mathbf{v}_l$, $\mathbf{v}_l + d\mathbf{v}_l$. The equation for the quantity $\lambda(\mathbf{v}_l, t)$ was obtained by Schwarz practically directly from the equations of the dynamics of vortex motion. The quantity $\lambda(\mathbf{v}_l, t)$ is of greater interest than L, and Schwarz's results are more numerous. It is interesting that for $L = \int \lambda(\mathbf{v}_l) d^3\mathbf{v}_l$ he obtains an equation of the type (17), although he gives it a somewhat different interpretation and does not include any adjusting parameters.

However, both in [16] and (indeed, even more) in [13] there are quite a few simplifications and assumptions which are not entirely justified. Therefore, in spite of the undoubted success of this theory, we may assume that the problem of superfluid turbulence is still far from being completely solved. Some difficulties arising in the use of this theory are described below.

In this connection it is useful to consider the question of ordinary turbulence developed in the normal component. In experiments in counterflow, in addition to the above-described critical flow there is another value at which the measured pressure drop is subject to the relation $\nabla p \sim W^{7/4} \sim v_n^{7/4}$ (Fig. 1). The exponent 7/4 indicates that we are dealing with classical turbulence. The onset of this regime was first described in [17] and is associated with a Reynolds number constructed as follows:

$$\mathrm{Re} = \frac{\rho D v_n}{\eta}. \tag{20}$$

It was found that the critical Reynolds number corresponding to the beginning of turbulence is strongly dependent on temperature, which is very strange. This gave rise to many discussions, and various kinds of corrections were proposed, but it appears that the problem of classical turbulence still has not been clarified. Construction of the Reynolds number on the basis of the total density corresponds, in the opinion of the authors of [17], to the fact that both components are turbulized simultaneously. However, such a view contradicts the ideas of two-velocity hydrodynamics. To see this, we note that until the appearance of quantum vortices, the motion of each component is independent of the other. The only mechanism of interaction is the mutual friction. We have already referred to the problem of the occurrence of quantum vortices. It may be that this has something to do with questions of stability, but it is doubtful that such a complex phenomenon (even if it relates only to the superfluid component) could be described by the criterion (20).

We see the following situation. As a result of the action of several mechanisms, quantum vortices appear in the helium. The conditions for their occurrence are less rigid than the conditions for the occurrence of classical turbulence in the normal component. For example, at a velocity of 1 cm/sec (it is known with certainty that vortex formation occurs at such a velocity) and a channel width of 0.1 cm, we have Re = 300, which is much lower than the critical value. Nevertheless, the tangle of vortex lines, moving chaotically, perturbs the normal component, "shakes it up," and turbulizes it. As a result we find a picture of interaction between two types of turbulence (similar to the interaction between acoustic noise and turbulence). To describe such a complex cascade process by a single criterion of the type (20) makes no sense.

2. Nonstationary Processes

2.1. Acoustics of Superfluid Helium.

A special place in the investigation of heat exchange in cryogenic systems is occupied by the problems of nonstationarity. This is associated chiefly with the problems of reliability and stability. Of primary importance in this connection is the problem of rapid and effective heat removal. In order to clarify the possibilities of helium as a refrigerant in such circumstances, we must consider the various manifestations of nonstationarity in the questions discussed above. It should be noted that a study of the nonstationary thermomechanics of He II is necessary for constructing a theory of heat exchange with a solid. In fact, the various heat-exchange crises and changes of regime result from the development of hydrodynamic processes taking place in the volume of the liquid. In this sense, an understanding of the hydrodynamic processes taking place in helium plays an auxiliary but necessary role. An example of such an approach is Kutateladze's theory of boiling crises [18]. It should be emphasized once again that the formal application of this theory to He II yields no results, since the superfluid liquid is governed by its own hydrodynamic laws.

In our study of nonstationary thermomechanical motions in He II, we may distinguish three regimes: 1) The evolution of thermal disturbances is subject to the usual two-velocity hydrodynamics (1)-(4); 2) quantum vortices appear in the volume of the helium; 3) the possibility of phase transitions is realized both in the gas and in the nonsuperfluid helium. The onset of these regimes cannot be unambiguously associated with the value of the heat flux. The rate of change of the flow and the time of action of the thermal load are equally important.

In what follows, we shall consider the case of small flows and small velocities. As is known, in ordinary hydrodynamics, small perturbations in density and pressure are propagated as sound waves. Heat, on the other hand, is propagated in accordance with the heat-conduction equation. If the entropy equation is taken into account, the sound will be damped.

What is the situation in He II? It appears that the heat is again propagated in the helium in the form of waves. This can be obtained in the usual manner. We linearized Eqs. (1)-(4) on the basis of their deviations from the equilibrium values. As is known, the system of linear equations has a solution in the form $\exp i(\omega t - \mathbf{kr})$. The condition for the existence of nontrivial solutions of this system gives the relation between the frequency ω and the wave number $\mathbf{k}$ — the dispersion formula. Omitting some simple calculations, we find that $\omega = c_{1,2}|\mathbf{k}|$, where

$$c_1^2 = \left(\frac{\partial\rho}{\partial p}\right)_T^{-1}, \quad c_2^2 = \frac{\rho_s}{\rho_n}\,\sigma^2\left(\frac{\partial\sigma}{\partial T}\right)_p^{-1}. \tag{21}$$

This means that two kinds of sound waves are possible in He II. The kind corresponding to the first solution is ordinary sound — oscillations in density, where $\mathbf{v}_n = \mathbf{v}_s$. The second acoustic branch — the second kind of sound — describes the nonstationary variant of counterflow. The total flow has a mass $\mathbf{j} = 0$. In the second type of sound, the values of temperature and entropy oscillate about their equilibrium values, and $\delta\rho = 0$. Thus, the second sound carries heat waves, and therefore it is sometimes referred to by the term "temperature waves." The velocity of the first sound is practically constant (with respect to temperature) and is equal to $c_1 = 240$ m/sec. The velocity of the second sound, c_2, is strongly dependent on temperature (Fig. 4).

The possibility of transmitting thermal disturbances by a wave method is not only unique but of great practical interest. A number of circumstances contribute to this. First of all, since all the quantities in the wave depend on $x - c_2 t$, the energy flux is proportional to c_2 and may become very large. Secondly, in sound, as experiments show, the critical velocity of vortex formation is not reached until large values, up to several meters per second, corresponding to fluxes of 50-100 W/cm^2, are reached (see [19]). Furthermore, as important as the heat flux is a characteristic of nonstationary heat exchange such as the typical time of temperature smoothing, τ_{typ}. In ordinary heat exchange $\tau_{typ} = D^2/\chi \cdot 10^3$ sec ($D \approx 1$ cm), and for boiling and convection $\tau \gtrsim 10^{-2}$-10^{-1} sec; on the other hand, for He II, $\tau_{typ} = D/c_2 \approx 10^{-4}$ sec. Lastly, the wave mechanism provides excellent opportunities for controlling the heat exchange. Such phenomena as diffraction, interference, shock waves, etc., can all be utilized in applications.

2.2. Nonlinear Acoustics of Superfluid Helium. A number of effects associated with the wave mechanism of heat transfer, as well as the possibility of their practical use, may be seen by considering the example of the nonlinear acoustics of He II. We associate with this a number of problems based on Eqs. (1)-(4), taking account of nonlinearity and viscosity. Vortex formation and phase transitions are not considered here.

One of the problems in [7, 20] is devoted to the investigations of the evolution of strong temperature dependence in He II. Without going into details of the calculations, we give below the equations for the evolution of a heat pulse $\delta T(x, t)$ which is produced, say, by means of a heat flux $W(t)$ at the boundary ($x = 0$):

$$\frac{\partial\delta T}{\partial t} + \left(c_2 + \alpha(T)\frac{\sigma\rho_s}{2c_2\rho_n}\,\delta T\right)\frac{\partial\delta T}{\partial x} = \zeta\,\frac{\partial^2\delta T}{\partial x^2}. \tag{22}$$

The coefficient of nonlinearity $\alpha(T)$ is shown in Fig. 5. The absorption coefficient ζ either is made up of the ordinary kinetic quantities or [20] represents relaxation damping near the phase transition point.

Equation (22) can be reduced to the so-called Burgers form and solved exactly. We shall not do so here but shall consider some qualitative effects. If the amplitude δT is small, we can disregard the second term within the parentheses and obtain the usual wave equation, whose solution is $\delta T = \delta T(x - c_2 t)$. Therefore the term $\sim\alpha(T)\delta T$ is, in a sense, an increment added to the velocity; however, it is not constaint but depends on the local value of δT. In other words, different segments of the wave profile move at different velocities. For example, when $\alpha(T) > 0$, the crest of the wave rushes against the "trough," forming what is called a shock front. When $\alpha(T) < 0$, the situation changes and the shock front is formed far away. For pulse with a negative sign, when $\alpha(T) < 0$, we may have the formation of a shock wave of "cold" — a phenomenon which is specific to helium and can be used for pulsed cooling.

In the linear case, the first and second sounds are practically independent.* In the nonlinear case there is an interaction between the wave modes. We may find such a curious effect as nonlinear conversion of one sound into another [21]. In connection with this, it is also of interest to consider the question of the nonlinear conversion of "noise" of the first sound into noise of the second sound. There are often reasons (vibrations, instabilities) leading to the appearance of the noise of ordinary sound. As a result of the nonlinear

*They are related to the value $\partial\ln\rho/\partial\ln T \approx 10^{-3}$-$10^{-4}$, which is negligibly small for helium.

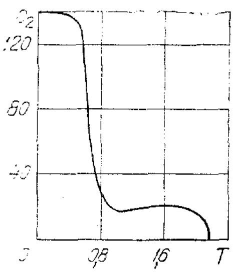

Fig. 4. Velocity of the second sound, c_2 (m/sec), as a function of temperature.

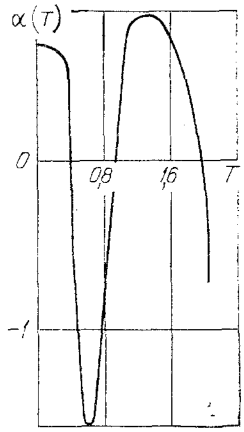

Fig. 5. Dimensionless coefficient α(T) as a function of temperature.

transformation of the first sound into the second, noise of the second sound appears in the system. Therefore, if we wish to solve the problem of the evolution of thermal disturbances — the fundamental problem of nonstationary heat exchange — we must take account of the fact that this disturbance is propagated not in a liquid at rest but against a background of strong noise, the so-called phenomenon of acoustic turbulence. As is known this leads to additional damping and dispersion, and both are expressed by the correlation characteristics of the noise. In [22] these spectral characteristics were found on the basis of the resulting kinetic equations for chaotic waves.

It is interesting to consider nonlinear effects in problems which are more than one-dimensional, e.g., in the propagation of broad bundles of the second sound. One such problem was investigated in [23], in which it was shown that at temperatures close to T_α (T_α is defined as the temperature at which the nonlinear increment to the velocity of the second sound vanishes: $\alpha(T_\alpha) = 0$), the initially broad bundle of monochromatic thermal waves undergoes a self-focusing action as a result of nonlinearity and is compressed to dimensions of the order of one wavelength. This interesting phenomenon may be used for the directed transfer of thermal energy in superfluid helium.

This area also includes the still unsolved problem of a broad bundle of thermal waves at temperatures other than T_α. Here the behavior of the wave is complicated by the formation of a shock front. However, it is qualitatively clear that when $\alpha(T) > 0$, the segments with large values of δT move forward, the front becomes convex, and this leads to a defocusing action. On the other hand, when $\delta T < 0$, such a wave will be focused (see [24]). The focusing of the wave means that on the axis of the bundle δT may reach very large negative values. This phenomenon can undoubtedly be used for cooling local segments of specimens.

Here we may also mention a study [25] of a method for measuring the thermal resistance of the boundary (the Kapitsa jump) by the methods of nonlinear acoustics.

2.3. Nonstationary Vortex Formations. The existence of critical rates of vortex formation, discussed above, is extremely important in connection with heat transfer in He II. Indeed, the transition from laminar to turbulent flow, as we have seen, is accompanied by a sharp increase in the temperature drop. As a result, there are areas of overheating and an increased danger of transition to boiling, after which the high thermal conductivity of He II "fades" against the background of the high thermal resistance of the vapor film.

As mentioned earlier, under stationary conditions the heat flux may take on very high values (up to 100 W/cm^2) [19], while the stationary critical fluxes are $W \approx 10^{-3}$–10^{-2} W/cm^2. The reason for this phenomenon is that the mechanism of formation of quantum turbulence requires a finite time τ_{turb} in the aforementioned experiments, where the signals lasted no more than 30–50 μsec, this time value was not reached. Two important questions arise in this connection: a) how superfluid turbulence develops in He II; b) in what manner the thermal wave is propagated if there are quantum vortices in its front.

The first question has been studied to some extent. Here we should mention, first of all, Vinen's classic study [13]. In his experiments (on the basis of which he developed his theory, discussed above) the turbulence was probed by means of the second sound, propagated across the counterflow. When the vortex tangle was formed, the amplitude of the second sound was reduced as a result of the mutual friction, and as it varied, it was possible to assess the nature of the growth of L. The law obtained on the basis of these experiments for the growth of the tangle agrees with Eq. (17), but not for small values of L. As $L \to 0$, this equation leads to a strong contradiction. We define the time of development of the turbulence τ_{turb} as the time necessary for the tangle to increase to half of its ultimate value. From (17) it follows that

$$\tau_{turb} = \int_0^{0.5L_{eq}} \frac{dL}{\alpha\,|\mathbf{v}_n - \mathbf{v}_s|\,L^{3/2} - \beta L^2}. \tag{23}$$

It can be seen at once that the integral diverges at the lower limit, $\tau_{turb} \to \infty$. The origin of this divergence is obvious. Formula (17) is the balance equation for the growth and annihilation of the vortex lines which already exist. When there are no such lines, there is nothing to grow. In other words, this equation does not include the mechanism of spontaneous generation of the vortex lines in He II.

Actually, of course, Vinen observed finite values of τ_{turb}. It was found that τ_{turb} depends substantially on the heat flux density W. Vinen [13] proposed the following empirical formula:

$$\tau_{turb} = a(T)\,W^{-3/2}, \tag{24}$$

where $a(T)$ is a quantity of the order of 0.1 $W^{3/2}\cdot sec/cm^3$ and depends slightly on the geometry.

In order to correct the theoretical situation, Vinen introduced into Eq. (17) an additional term of the form $\xi|\mathbf{v}_n - \mathbf{v}_s|^{5/2}$ (here ξ is an adjusting coefficient). Then the integral (23) does not diverge but yields an expression which coincides with (24). However, the argument used in introducing such a term is highly dubious. For example, in (23) we could "cut off" the integral at the lower limit in an appropriate manner, which physically means that there are "virtual" vortices in the helium.

Formula (24), as mentioned before, was obtained experimentally. The heat fluxes W did not exceed 1 W/cm^2, which yields $\tau_{turb} \approx 0.1$ sec. The investigation of the kinetics of the turbulence is extremely important, in particular that of $\tau_{turb}(W)$ at flux values which may occur in cryogenic systems (up to 100 W/cm^2).

In addition to the question of the time τ_{turb} there is another very important question concerning the possibility of using Eq. (7) to describe such a clearly nonequilibrium situation as the evolution of a supercritical thermal disturbance [question b), see above]. The fact is that the derivation of this equation is based to a large extent on the calculation of the forces acting on the vortex lines. In the quasiequilibrium case these forces are proportional to $\mathbf{v}_n(t) - \mathbf{v}_s(t)$. In the general case they may depend on the time derivatives and possibly on the gradients of $\mathbf{v}_n(t)$ and $\mathbf{v}_s(t)$. This, of course, must affect the form of Eq. (17). However, it seems that these nonequilibrium effects have only a slight influence. This conclusion is based on the fact that in experiments on the propagation of the second sound across a supercritical counterflow, the variation of its velocity (and precisely this effect must result from the presence of $d\mathbf{v}_n/dt$ and $d\mathbf{v}_s/dt$ in the equation for L) is very slight, $\Delta c_2/c_2 \sim 10^{-4}$. Moreover, this variation can be explained within the framework of the Vinen–Schwarz equation (17) [26].

If we omit these two difficulties from consideration for the time being (and also other, less important defects of the Vinen–Schwarz theory), it becomes clear that the problem of the evolution of supercritical thermal disturbances must be solved on the basis of a system of equations which combines the hydrodynamic equations (1)-(4) with (17) in the proper manner. Such a combination was carried out in [26], yielding a closed system of equations for the collection of quantities ρ, $\mathbf{j}$, S, $\mathbf{v}_s$, L, which completely describe the state of the turbulent helium.

It should be emphasized once more that these equations were obtained by disregarding the defects of the Vinen–Schwarz theory and may be regarded as a first, crude approximation to reality.

Summing up, we can say that we do not yet have an apparatus for solving the problem of the development of thermal disturbances with a turbulent front. What is more, the construction of such an apparatus is significantly limited by the inadequacy of the experimental data.

Among other studies devoted to the kinetics of quantum turbulence, we should mention [27] on the development of turbulence in long tubes. However, the results of that study can hardly be used for constructing the necessary theory.

2.4. Phase Transitions. In the preceding two sections we have formulated problems of heat transfer in the volume of the helium, which can, at least in theory, be solved in the framework of two-velocity hydrodynamics.

As the thermal load increases further, or as its duration increases, the above-described processes will take place only during the first few moments. For example, in the transition to a turbulent regime, there will be a sharp increase in the temperature drops, after which there may be phase transitions both in the helium vapor and in the nonsuperfluid He I.

The dynamics of the formation of such transitions is the area which has been least studied. Although, as mentioned in the introduction, there are a large number of studies on the boiling of He II, they are restricted essentially to measurements of critical fluxes (at which phenomena of the boiling type occur) and of the heat-transfer coefficient in the boiling regime.

Continuing along our logical line, we would like to turn our attention to the following formulation of the problem. What is the manner of development in superfluid helium of a thermal disturbance described by supercritical parameters, i.e., parameters such that the temperature and pressure within such a disturbance satisfy the condition for phase transition (either He II–He I or He II–vapor)?

The problem of the He II–vapor transition, i.e., the problem of the formation of film boiling, is very close to the classical problem. Here hydrodynamics plays an auxiliary role, and it appears that the general theory* in this case can be set up on the basis of ideas concerning the boiling of ordinary liquids. Somewhat more complicated (although perhaps it might be better to call it more unusual) will be the problem of the development of a phase transition of the second kind, such as the transition He II–He I. The solution of problems of this kind involves some very subtle and complicated questions in the dynamics of fluctuations and belongs to the realm of basic rather than applied physics. There are very few experimental studies available. To be sure, we should note that among the above-mentioned studies on heat exchange the authors have sometimes observed some features of a transition to film boiling (e.g., the process of growth of a vapor film observed by Caspi and Frederking [28]). If we do not count these relatively uninformative references, we may perhaps list only two useful studies.

In [29] Peshkov observed that as the heat flux in He II increases, there appears an He I zone which, depending on the value of W, moves with different velocities, even coming to rest at some flux value. The existence of this zone was recorded by optical methods based on the increase in density and temperature. The temperature and density jump at the boundary indicates that when there is a heat flux, the transition between He II and He I becomes a phase transition of the first kind, and the boundary has finite heat resistance. Unfortunately, this study is the only one of its kind, even though this problem is of unquestionable theoretical and practical interest.

*Here we are talking about the theory of boiling in the thermophysical sense, i.e., the calculation of boiling crises, the heat-transfer coefficients, etc. However, it is also possible to solve a problem analogous to Stefan's problem on the dynamics of the interface boundary.

The second group includes a study by Van Sciver [30] on the kinetics of film boiling, in which he obtained an interesting relation for the time of formation of the helium film as a function of the heat flux:

$$\tau_{boil} = BW^{-4}, \qquad (25)$$

where B is a value of the order of 100 $W^4/cm^8 \cdot sec$. The time τ_{boil} was fixed on the basis of a sharp increase in the temperature of the heater, and therefore it was impossible to guarantee that Van Sciver observed film boiling. The formation of a film of nonsuperfluid helium, for example, should also lead to this result.

This is perhaps all we can list in the way of studies devoted to the dynamics of phase transitions. An understanding of these phenomena is of the first importance for the construction of a theroy of heat exchange in He II. We may hope that more studies along this line will come in the future.

Thus, as we have seen, the theory of heat exchange in He II is far from complete. There have been definite achievements, such as the Vinen—Schwarz theory or studies on nonlinear acoustics, but these are only isolated fragments. There remain many problmes of varying degrees of complexity and importance, some of which are closely linked to unsolved classical problems (such as turbulence and kinetics of boiling). These are mainly problems in basic physics, and their solution will require a great deal of time and effort.

Nevertheless, it appears that even today some results, including those mentioned in this survey, can be used for practical calculations of cryogenic installations using superfluid helium.

In our view, the study of heat exchange with superfluid helium should be developed along two main lines. One of these is the investigation of problems associated with direct technological applications, such as the determination of boiling crises and heat-transfer coefficients, the effect of various factors under conditions most closely resembling real cases. As mentioned before, a great many studies of this kind have been published recently (see [2-4]). The second line of investigation includes those studies which would most rapidly bring about the construction of a general theory of heat exchange in He II. Among them are, for example, studies on the development of superfluid turbulence, the formation of quantum vortices, or, say, the investigation of the dynamics of the transition He II—He I. These two lines of research need not, of course, be independent. Furthermore, each is necessary for the other. For, on the one hand, applied experimental investigations can establish the adequacy of the theoretical models, and on the other hand, theoretical investigations will facilitate more purposeful experimentation.

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NOTATION

ρ, density of He II; T, temperature; $\mathbf{W}$, heat flux density; p, pressure; ρ_s, ρ_n, densities and $\mathbf{v}_s$, $\mathbf{v}_n$, velocities of superfluid and normal components, respectively; $\mathbf{j}$, $\mathbf{j}_0$, mass flux densities in the laboratory system of coordinates and in the case $\mathbf{v}_s = 0$, respectively; S, σ, entropy per unit volume and unit mass, respectively; E, energy per unit volume; μ, chemical potential, equal to $(\partial E/\partial \rho)_S$; η, coefficient of shear viscosity; λ, thermal conductivity; D, a, dimension of the system (radius of capillary); C(T), coefficient in the formula for the effective thermal conductivity in supercritical regime; A(T), coefficient in the formula for the Gorter—Mellink force; ε, $\mathbf{p}$, energy and momentum of elementary excitation; ε', energy of elementary excitation in laboratory system; $\mathbf{V}$, velocity of motion of the helium as a whole; $\hbar$, Planck's constant; m_{He}, mass of helium atom; L, total length of vortex lines per unit volume; $\delta = L^{-1/2}$, characteristic intervortex distance; α, β, coefficients in the Vinen—Schwarz equation; $\mathbf{f}$, frictional force per unit vortex length; $\mathbf{F}_{sn}$, force of mutual friction (per unit volume); $\lambda(\mathbf{v}_l, t)$, distribution function for length of vortex lines; c_1 and c_2, velocities of first and second sound, respectively; χ, thermal diffusivity; $\alpha(T)$, coefficient of nonlinearity (see 2.2); ζ, damping coefficient of second sound; τ_{turb}, characteristic time of formation of superfluid turbulence; $a(T)$, coefficient for time of vortex formation (see (2.3); τ_{boil}, characteristic time for boiling.

LITERATURE CITED

1. S. S. Kutateladze, Some Problems in Heat Exchange at Helium Temperatures [in Russian], Novosibirsk (1976). (Preprint, Thermophysics Institute of the Siberian Branch of the Academy of Sciences of the USSR, pp. 10-76.)
2. V. A. Grigor'ev, Yu. M. Pavlov, and E. V. Ametistov, Boiling of Cryogenic Liquids [in Russian], Energiya, Moscow (1977).
3. R. Airy, "Heat transfer in liquid helium," in: Heat Transfer at Low Temperatures [Russian translation], Mir, Moscow (1977), pp. 346-380.
4. V. Arp, "Heat Transport through helium II," Cryogenics, 10, No. 2, 96-105 (1970).
5. S. Paterman, Hydrodynamics of Superfluid [Russian translation], Mir, Moscow (1978).
6. L. D. Landau, Collected Works, Vol. I [in Russian], Nauka, Moscow (1969).
7. I. M. Khalatnikov, Theory of Superfluidity [in Russian], Nauka, Moscow (1971).
8. S. C. Soloski and T. H. K. Frederking, "Dimensional analysis and equation for axial heat flow of Gorter-Mellink convection," Int. J. Heat Mass Transfer, 23, No. 4, 437-441 (1980).
9. C.-J. Gorter and I. H. Mellink, "On the irreversible processes in liquid helium II," Physica, 15, Nos. 3-4, 285-304 (1949).
10. D. F. Brew and D. O. Edwards, "Heat conduction by liquid helium," Philos. Mag., 7, 721-735 (1962).
11. D. V. Osborne, "The rotation of liquid helium II," Proc. Phys. Soc., A63, 909-912 (1950).
12. R. P. Feynman, "Application of quantum mechanics to liquid helium," in: Progress in Low Temperature Physics, Amsterdam, Vol. 2, (1957), pp. 17-53.
13. W. F. Vinen, "Vortex lines in liquid helium," in: Progress in Low Temperature Physics, Amsterdam, Vol. 3, (1961), pp. 1-57.
14. E. L. Andronikashvili, Yu. G. Mamaladze, and Dzh. S. Tsakadze, Physics of Superfluid Helium [in Russian], Metsniereba, Tbilisi (1978).
15. S. V. Iordanskii, "Vortex formation in superfluid helium," Zh. Eksp. Tekh. Fiz., 48, No. 2, 708-714 (1965).
16. K. W. Schwarz, "Turbulence in superfluid helium," Phys. Rev., 18, No. 1, 245-262 (1978).
17. F. A. Staas, K. W. Taconis, and W. M. Van Alphen, "Experiments on laminar and turbulent flow of He II in wide capillaries, Physica, 27, 893-929 (1961).
18. S. S. Kutateladze, Fundamentals of the Theory of Heat Exchange [in Russian], Nauka, Novosibirsk (1970).
19. S. K. Nemirovskii, "Nonlinear wave processes in superfluid helium," Candidate's Dissertation, Physical and Mathematical Sciences, Novosibirsk (1979).
20. S. K. Nemirovskii, Nonlinear Waves of the Second Sound in He II in the Vicinity of T_λ [in Russian], Novosibirsk (1976). (Preprint, Thermophysics Institute of the Siberian Branch of the Academy of Sciences of the USSR; 08-78).
21. V. L. Pokrovskii and I. M. Khalatnikov, "Transformation of the first sound to the second sound in superfluid helium," Zh. Eksp. Teor. Fiz., 71, No. 5, (1974-1976).
22. S. K. Nemirovskii, "Acoustic turbulence in a superfluid liquid," Fiz. Nizk. Temp., 4, No. 1, 11-15 (1978).
23. S. K. Nemirovskii, "Cubically nonlinear effects in the propagation of the second sound," in: Materials of the Twentieth All-Union Conference on Low-Temperature Physics, NT-20: Theses of Reports [in Russian], Moscow (1978), pp. 219-221.
24. E. A. Zabolotskaya and R. V. Khokhlov, "Convergent and divergent sound bundles in nonlinear media," Akust. Zh., 16, No. 1, 49-53 (1970).
25. A. N. Tsoi and S. K. Nemirovskii, "Measurement of Kapitsa resistance using the second sound in He II," Fiz. Nizk. Temp., 6, No. 5, 671-673 (1980).
26. S. K. Nemirovskii, "Interaction of the second sound with quantum turbulence," in: Twenty-first All-Union Conference on Low-Temperature Physics: Theses of Reports [in Russian], Kharkov (1980), Part 4, pp. 53-55.
27. V. P. Peshkov and V. K. Tkachenko, "Kinetics of the violation of superfluidity in helium," Zh. Eksp. Teor. Fiz., 41, No. 5, 1427-1432 (1961).
28. S. Caspi and T. H. K. Frederking, "Triple-phase phenomenon during quenches of superconductors cooled by pressurized superfluid helium II," Cryogenics, 19, No. 9, 513-516 (1979).
29. V. P. Peshkov, "Conversion of λ-transition in helium in the presence of a heat flux into an ordinary transition of the first kind," Zh. Eksp. Teor. Fiz., 3, 581-582 (1956).
30. S. W. Van Sciver, "Transient heat transport in He II, " Cryogenics 19, No. 7, 385-392 (1979).

Remark at Press Time. In Sec. 2.3 we stated the view that the high heat fluxes observed in experiments on nonlinear acoustics were achieved because of the finiteness of the time of vortex formation, τ_{turb}, which was not exceeded in the studies cited. This view was, to some extent, confirmed in a recently published study by S. K. Nemirovskii and A. N. Tsoi entitled "Generation of vortices in He II by a large heat pulse," [Pis'ma Zh. Eksp. Tekh. Fiz., 35, No. 6, 229-231 (1982)]. In an experiment conducted according to Vinen's scheme it was found that there is strong damping of the second sound in the wake of a sufficiently long powerful heat pulse, which indicates the presence of quantum vortices.