EFFECT OF THE ENERGY DEPENDENCE OF THE IONIZATION LOSS ON THE EQUILIBRIUM SPECTRUM OF SHOWER ELECTRONS

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The equilibrium electron spectrum $P^{(\pi)}(E)$ of the shower formed by a primary electron having an energy E_0 is described by the equation [1]

$$LP^{(\pi)}(E) - \frac{\partial}{\partial E}\{\beta(E)P^{(\pi)}(E)\} = \delta(E - E_0). \tag{1}$$

Here L is the integral operator associated with bremsstrahlung processes, the Compton effect, the photoelectric effect, and pair formation; and $\beta(E)$ is the ionization energy loss per radiation length.

Tamm and Belen'kii [2] found the solution $P_0^{(\pi)}(E)$ of Eq. (1) for the case of constant loss $\beta(E) = \beta_0$, but at secondary-particle energies below the critical energy it is not correct to assume a constant ionization loss. The correction due to the energy dependence of β can be found by a perturbation method.

It was shown in [3] that

$$P^{(\pi)}(E) = P_0^{(\pi)}(E) + \int_0^\infty dE' g(E, E') \frac{\partial}{\partial E'}\{\Delta\beta(E')P_0^{(\pi)}(E')\}, \tag{2}$$

where $g(E, E')$ is the Green's function of Eq. (1) and where we have $\Delta\beta(E) = \beta(E) - \beta_0$.

The derivative $(\partial/\partial E)\{\Delta\beta(E)P_0^{(\pi)}(E)\}$ is nonvanishing only at small E, for which the Green's function can be found by neglecting cascade transitions, from the equation

$$-\frac{\partial}{\partial E}\{\beta(E)g(E, E')\} = \delta(E - E'),$$

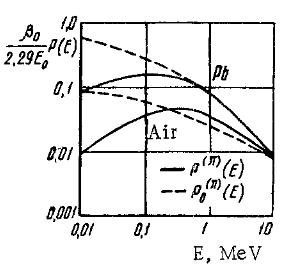
whose solution is

$$g(E, E') = \frac{1}{\beta(E)} e(E' - E), \tag{3}$$

where $e(E' - E)$ is the unit step function.

Substituting Eq. (3) into Eq. (2), we find

$$P^{(\pi)}(E) = \frac{\beta_0}{\beta(E)} P_0^{(\pi)}(E). \tag{4}$$



As $E \to 0$, we have $\beta(E) \sim \ln E$, so account of the energy dependence of the ionization loss eliminates the logarithmic divergence of the equilibrium spectrum.

Figure 1 shows the effect of this correction on the equilibrium spectra in lead and air.

LITERATURE CITED

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