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Harmonic continuation and gridding effects on geoid height prediction

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Abstract: Least-squares collocation and Stokes integral formula, as implemented using the Fast Fourier Technique, handle the harmonic downward continuation problem quite differently. FFT furthermore requires gridded data, amplifying the difference of methods.

We have in this paper studied numerically the effects of downward continuation and gridding in a mountainous area in central Norway. Topographically smoothed data were used in order to reduce these effects. Despite the smoothing, it was found that the vertical gravity gradient had values up to -11 mgal/km. The corresponding differences between geoid heights and the height anomalies at altitude reached 12 cm.

The differences between geoid heights obtained using collocation or FFT with gravity data at terrain level or sea level showed differences between the values of up to 10 cm r.m.s. A part of this difference was a consequence of different data areas used in the FFT and collocation solution, though.

Major discrepancies between the solutions were found in areas where the topographic smoothing could not be applied (deep fjords with no depth information in the used DTM) or where there seemed to be gross errors in the data.

We conclude that proper handling of harmonic continuation is important, even when we as here have used a 1 km resolution DTM for the calculation of topographic effects. The effect of data gridding, required for the FFT method, seems not to be as serious as the need to limit the data distribution area, required when least squares collocation is used with randomly distributed data.

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1. Introduction

In flat or moderately hilly terrain it has been possible to execute geoid computations of very high precision. This conclusion has been reached by comparing geoid height differences obtained from GPS and levelling with computed geoid height values, see Engelis et al. (1984), Torge et al. (1989), Forsberg and Madsen (1990), Madsen and Tscherning (1990). Solutions using various methods have also been compared (Sideris and Forsberg, 1990, Forsberg and Solheim, 1988, Barzaghi et al. 1988, Mainville et al., 1991).

In order to speed up the computations, or to perform geoid computations over large areas, gridded data are usually used. The gridding generally implies that some information is lost when going from the original point data to the gridded data. On the other hand data gaps have to be filled by some suitable prediction method in advance, yielding a more homogeneous and stable computation of geoid heights, less prone to oscillations and pertubations due to the data gaps.

Gridded gravity data typically are associated with a corresponding mean height grid, illustrating that the gravity data by nature refer to a non-level surface. However, for many methods data have to be given at a common level. To derive such a grid directly means that we not only have to perform a horizontal interpolation, but also a kind of (generally) downward continuation. The downward continuation is mostly done using the hypothesis that there are no masses between the point and the surface on which the gridded data are located, i.e. harmonic continuation.

We will in the following study the gridding and downward continuation effects by comparing geoid heights computed using different methods between themselves, and by comparison with geoid height differences obtained from GPS and levelling.

2. Brief description of geoid computation methods used

We have used the least squares collocation (LSC) method and the Stokes/Molodensky integral method implemented using the Fast Fourier Transform (FFT). An extensive literature exist describing both methods, see e.g. Tscherning, (1985) and Schwarz et al. (1990).

Most important for the LSC method is that it may use randomly distributed data, with non-uniform noise associated with the data. It is taken into account that data may be located at different altitudes through the use of spatial covariance functions. In the presently used LSC implementation (GEOCOL, see Tscherning, 1989), a rotational invariant covariance function is used. The used covariance model implies that the associated approximation to the anomalous gravity potential is harmonic down to a sphere inside the earth, the so-called Bjerhammar sphere. The use of the method requires that a system of equations is solved with as many unknowns as the number of data used. However, if gridded data are used exclusively, the system of equations may be solved very fast taking advantage of its Toplitz structure.

The FFT method takes advantage of the simple spectral relationship between the gravity anomaly Fourier spectrum and the geoid height spectrum, see Schwarz et al. (1990). Data must be gridded and "reduced" to a common surface. This reduction may be performed iteratively or by series expansions using the Molodensky theory. A simple linear approximation is obtained by first computing the vertical gravity gradient T_{zz} from the gridded gravity anomaly values, and then computing gridded anomaly values at the reference level using the gradient for the downward reduction.

The computation of the geoid from gravity data requires in principle a global, continous data coverage. However, data gaps will in practice always occur due to lack of measurements or restriction. Apart from lacking data other sources of error include our limited ability to take into account enough data in actual computations, the use of approximations such as the spherical or planar approximation, and the effects of topography-induced aliasing in interpolating and averaging point gravity data in rugged topography. These errors may be reduced drastically by smoothing the data in a consistent manner so that harmonicity is preserved, followed by a "desmoothing" of the prediction result.

This smoothing or "remove-restore" technique have become a standard procedure in many applications. The smoothing consists of eliminating the influence of a high-degree spherical harmonic expansion (here OSU89B), taking care of the long wavelengths, and of removing effects of the residual topography relative to a mean elevation surface, taking into account short-wavelength gravity field variation.

The residual terrain model (RTM) reduction may in practice be computed by e.g. prism methods, selecting a mean height surface by filtering given DTM heights to e.g. 0.5° or 1° resolution. The resulting terrain reduced field still in principle refers to the surface of the *original* topography height level. A LSC height anomaly prediction is thus carried out using original heights of gravity points, and predicting geoid values at the surface of the original topography as well, with geoid terrain effects also computed at these points. A similar FFT prediction further involves a downward continuation of anomalies, followed by upward continuation of geoid heights, prior to the addition of terrain effects.

The above scheme with terrain reductions and upward/downward continuation applies for the prediction of height anomalies only. The geoid is formally defined as the equipotential surface coinciding with sea-level *inside* the actual masses of the topography. However, what is required in Stokes' formula is the *harmonic* continuation of gravity anomalies to sea level, and thus results are formally height anomalies at sea level. In this paper we have taken the liberty to use the term "geoid" loosely for the height anomalies at sea level, so the reader should be aware of the difference to the classical definition.

3. Test area and data

In order to study gridding and harmonic continuation effects on geoid height prediction, we have

(1) compared results of geoid calculations with precise geoid heights obtained in a mountainous area, and

(2) compared the results using different methods with each other.

By working with real data we have the drawback of small contination effects, difficult to recognize in the "background noise" of other errors. However, the frequently used alternative of using synthetic data with random noise did not appeal to us, as such models often tend to have unrealistic spectral signatures, see Tscherning (1983).

We have in this paper used a characteristic area in central Norway. This area have also earlier been used for gravity field modelling studies (Sideris and Forsberg, 1990). The northern part of the european GPS geoid traverse runs through this area (Torge et al., 1989), and there exist furthermore a detailed GPS-levelling survey in the area carried out by G. Simensen, NTH,Trondheim. We have used the latter data for the comparisons of this paper, containing a total of 20 GPS levelling points.

Fig. 1 shows the generalized topographic heights in the area. Mountains range up to 2200 m, but unfortunately the NTH GPS geoid points are located in a more benign part of the area (fig. 2), so continuation effects are expected to be relatively small.

Many gravity values are available in the surrounding area. We have chosen to use exactly the same datapoints as used in the calculation of the NKG-89 nordic standard geoid (Forsberg, 1990), but restricted data to be within a 3^o x 6^o region containing totally 1953 points, see Fig. 3.

For land areas a 1 km resolution digital terrain model is available. This DTM has been used for the calculation of terrain effects on the gravity and on geoid heights for the production of the NKG-89 geoid, using a RTM reduction based on a 1^o x 2^o mean height surface. Exactly the same kind of terrain reduction have been used here.

The result of the NKG-89 geoid calculation is available in the form of values given on a UTM grid with spacing 5 km. It will be used for comparison purposes below.

From both GPS-levelling data and gravity data the effect of a global spherical harmonic expansion have been subtracted (and restored when appropriate). We used here the OSU89B model (Rapp and Pavlis, 1990). It only differs slightly in this area from the more recent OSU91A.

In order to apply LSC, the empirical covariance function must be estimated and an analytic representation determined. The empirical estimate was found using the program EMPCOV (Tscherning et al., 1991) and the analytic representation determined using COVFIT (Knudsen, 1987).

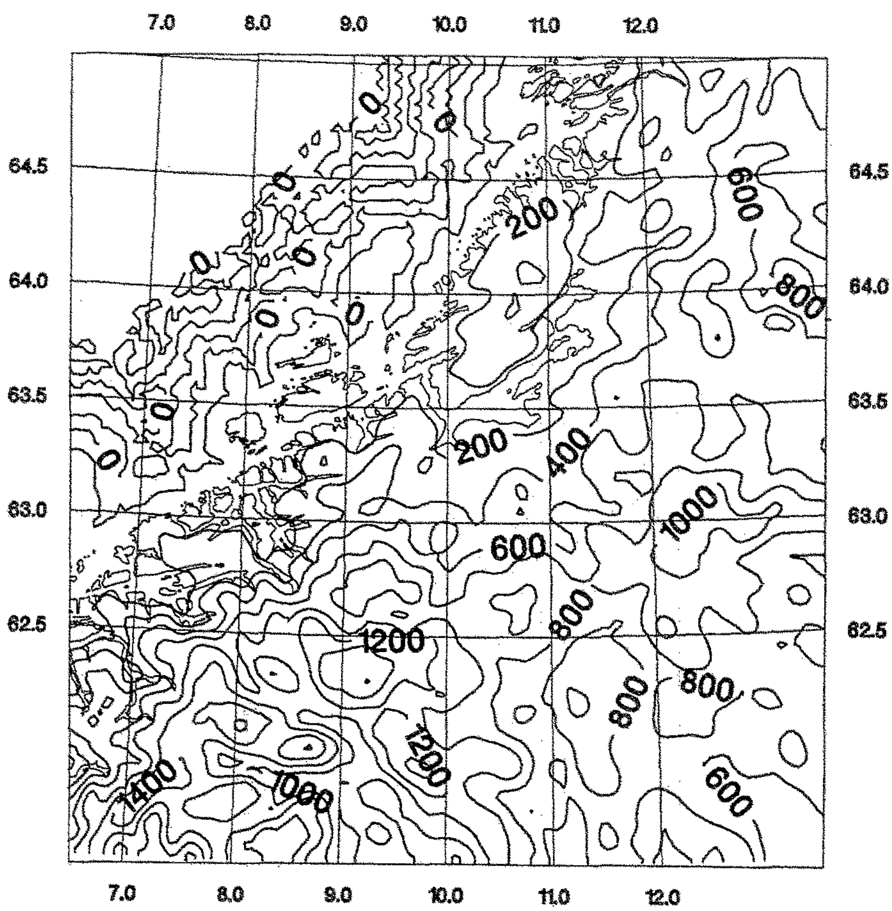

Fig. 1. Topographic heights from the used DTM.

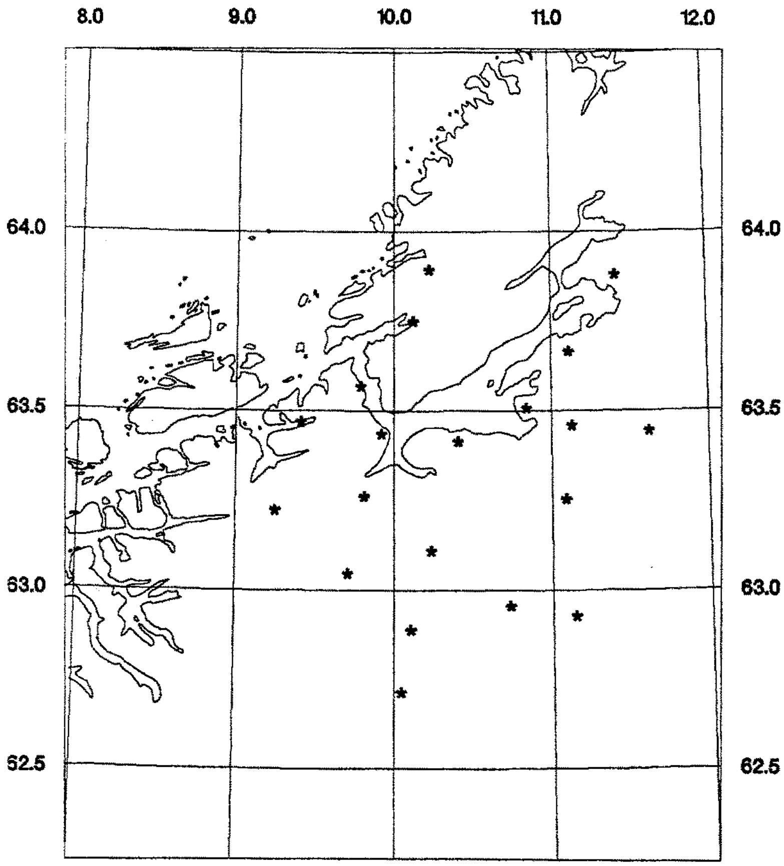

Fig. 2. Location of GPS-levelling points.

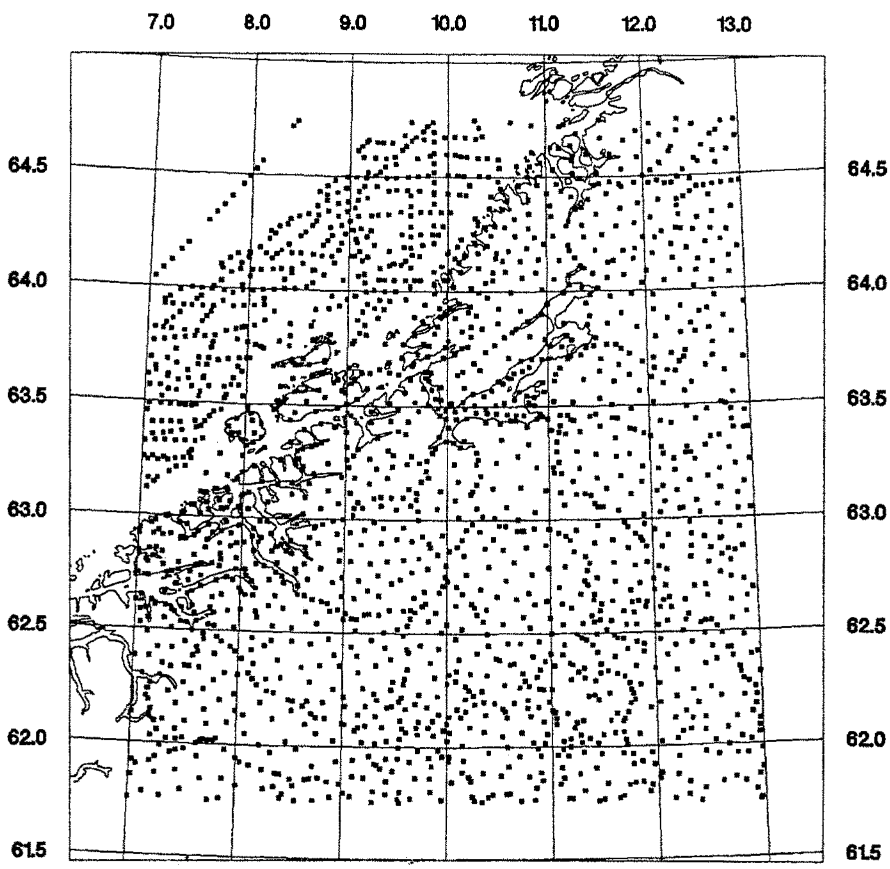

Fig. 3. Location of the used gravity data.

The analytic model fitted consisted of the error degree variances to degree 360 associated with the OSU89B spherical harmonic model multiplied with a scale factor of 0.7, while the degree variances for degrees higher than 360 were modelled by A/((i-1)*(i-2)*(i+4)), where $A*R^2$ = 58.8 $mgal^2$, and R = 6371.0 km. A depth to the Bjerhammar sphere of 1.0 km was used. The associated gravity standard deviation at sea level is 13.89 mgal and the correlation length 20 km. The associated geoid height standard deviation is 0.48 m.

4. Gridding effects

Gridding effects may be studied by
(1) comparing the original gravity values with values predicted from grids with varying spacing,
(2) by comparing geoid heights obtained using various grid spacings with geoid heights obtained using the original data, and
(3) by examining the error estimates obtained from collocation, providing the covariance function have been fitted to the actual data, as is the case here.

It is obvious, that the denser the gravity grid, the more accurate the recovery of the original values (the recovery is naturally limited if the data is assumed to have a high noise, implying filtering of the data). On the other hand, it is of no value to create a much finer grid than what the original data quality justifies. If we use the rule of thumb (Tscherning, 1985)

mean error =
*0.3*sigma*gridspacing/correlation distance*

then we have with sigma (gravity standard deviation) equal to 13.9 mgal, correlation distance 20 km, and data noise put equal to 1 mgal (= the wanted mean error), that we need a grid spacing of approximately 5 km in order not to loose information.

We have illustrated the gridding effects using grids with spacing 6', 5', 4' and 3', see Table 1. Figure 4 shows the contoured differences between original values and values computed using the 5' spacing. It is obvious from the figure that the result is very good in general, but there are some spots with large differences, e.g. the two spots at latitude 63.5 deg. The westernmost spot is associated with data containing gross errors. We will return to the reason for the occurrence of the easternmost spot in the next section.

Using the same spacings, we also computed geoid heights (at zero level) using the gridded data as input, and compared the predictions to a LSC computation using individual points at altitude, see Table 2. It is remarkable, that the geoid results are so close, illustrating that the accuracy of geoid calculations are only marginally improved with increasing data spacing due to the long-wavelength nature of the geoid.

Table 1. Difference between original values and values predicted using LSC. Results are shown for smoothed, gridded gravity data at zero level with variable grid spacing. For comparison the results of gridding unsmoothed free-air anomalies is also shown. Data noise in gridding assumed to be 1 mgal uniformly.

Original data		*Differences obtained using spacing*				
		6'	5'	4'	3'	Free air 3'
mean	-1.70	0.03	0.05	0.07	-0.05	-0.16
stdv.	13.89	4.92	4.31	3.47	3.50	7.87
min	-49.47	-30.63	-30.30	-27.72	-22.98	-44.96
max	42.31	38.39	35.72	42.85	38.35	54.55

Table 2. Differences between geoid heights obtained using LSC with the original data at altitude and gridded data using various spacings. All units meter.

Original data		*Differences obtained using grid spacing*			
		6'	5'	4'	(4' - 5')
mean	-0.19	-0.01	-0.01	-0.01	0.00
stdv.	0.46	0.02	0.01	0.01	0.01
min	-1.41	-0.07	-0.08	-0.06	-0.04
max	0.59	0.08	0.05	0.01	0.10

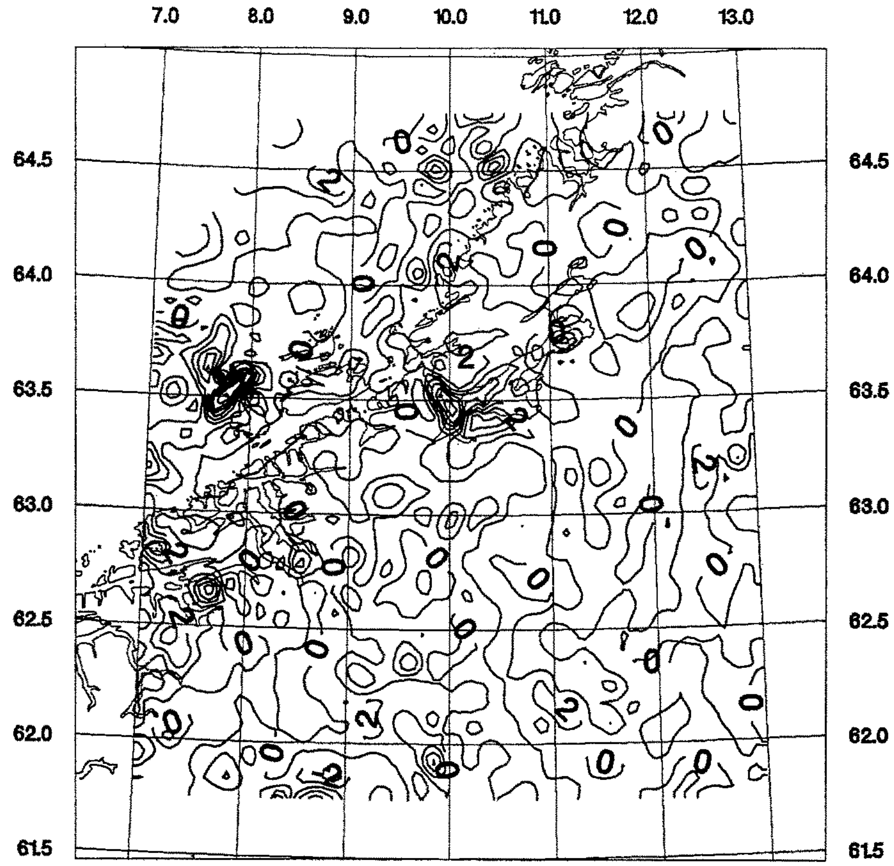

Fig. 4. Differences between gravity anomalies at terrain level and predicted anomalies from grid with 5' spacing at zero level. Contour interval 2 mgal.

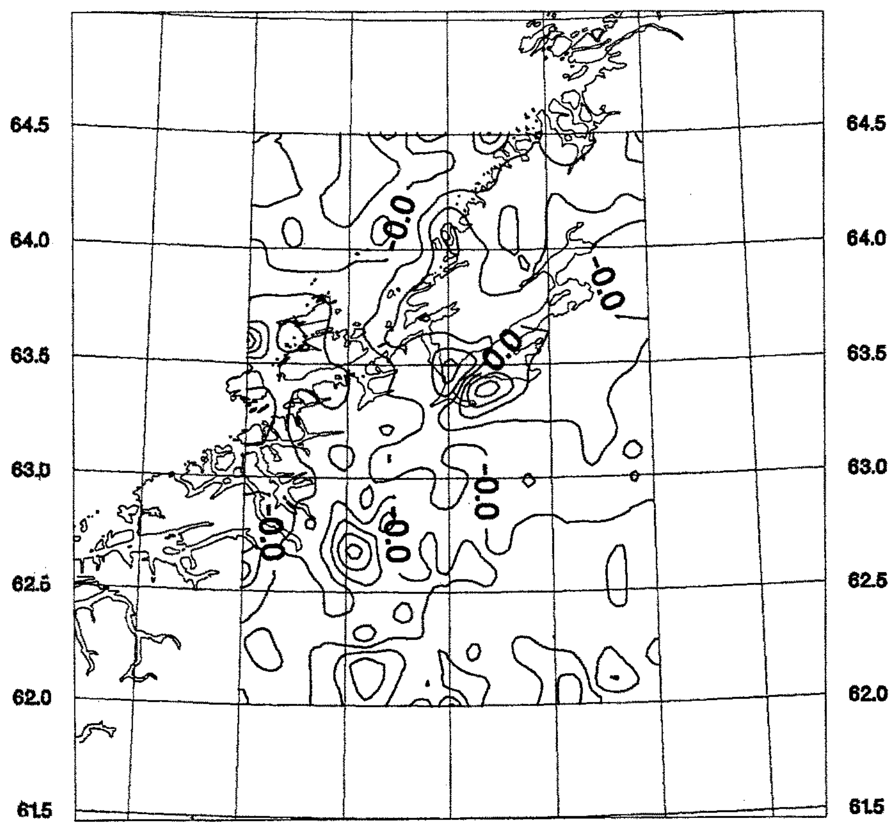

Fig. 5. Difference between geoid heights computed from gravity at altitude and from 5' gravity grid at 0. Contour interval 0.01 m.

5. Harmonic continuation effects

Continuation effects have been studied both globally and locally by numerous authors (e.g. Wichiencharoen, 1982; Sideris 1987; Hipkin, 1988; Wang, 1988; Martzell and Harrison, 1989). It is our opinion that there has not been put enough emphasis on the fact that if the attraction of a topography of uniform density is removed, and not shifted, then the various correction terms becomes at least 2 to 10 times smaller.

This is because some terms effectively include a term proportional to the mean density of the topography. If the density of the "remaining" topography varies around some ±0.2 g/cm^3 rather than 2.67 g/cm^3 then it is obvious that the part of the vertical gravity gradient which has its origin in the topography becomes much smaller. Other correction terms are computed as integrals of the gravity anomaly times powers of height differences. Here it is our experience that topographically smoothed gravity anomalies have a standard deviation which is often less than half the standard deviation of free-air anomalies, and the correction terms again become much smaller, and may frequently be altogether neglected.

This fact was clearly pointed out in Sideris and Forsberg (1990), where it was shown how much smaller the Molodensky term (g_1) was in the case of "combined" use of terrain reduction and harmonic continuation. The lack of attention to the use of the terrain-reduced Molodensky terms (g_1^c) so far is probably related to the large computational effort to do the necessary calculations. However, with FFT methods the handling of large DTM's become manegeable, see e.g. Forsberg (1985) or Sideris (1987).

In the following we will as in section 4 use data from which we have subtracted the contribution of the OSU89B spherical harmonic expansion and the residual topography effects. The continuation effects will therefore be substantially smaller than if free-air data alone had been used.

Now, using LSC or FFT iteratively, calculating the vertical gravity gradient implicitly or explicitly, we may obtain gravity values at zero level or at any height as preferred. If height anomalies are the ultimate goal of the computations, then there is no need of using zero as the reference level for the calculations, and a constant mean height level is more suitable. This

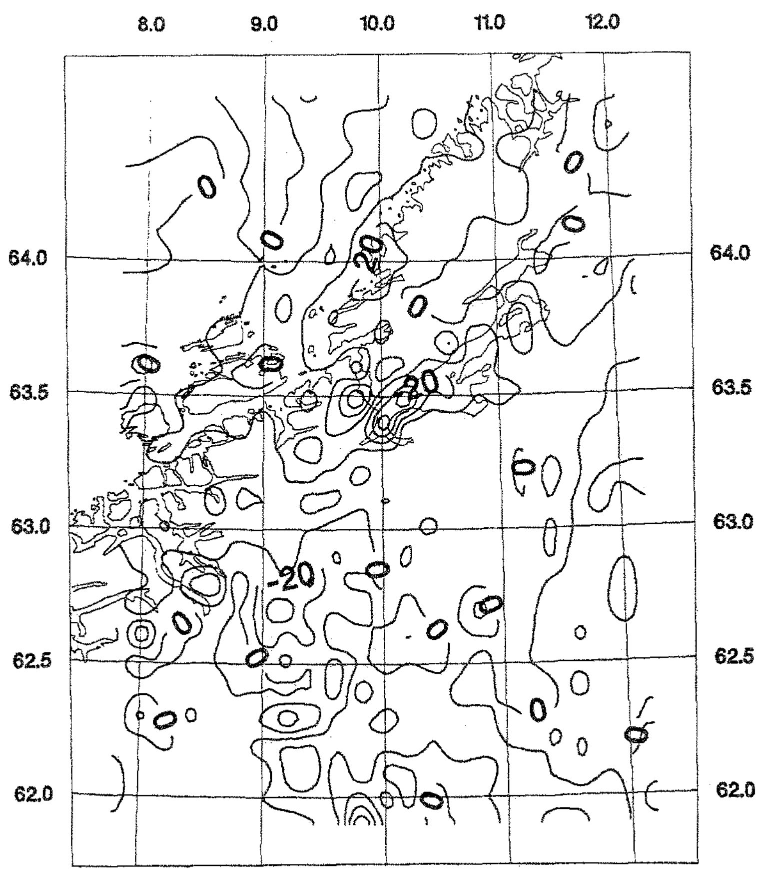

Fig. 6. Vertical gravity gradient T_{zz} at 0-level. Cont. interval 20 E.

has been implemented in the FFT method.

In our test area we have as mentioned a 1 km DTM available, but for land areas only. We are however, in a coastal area with deep fjords, which will then not be taken into account. Using LSC we calculated the vertical anomalous gravity gradient, Tzz, see Figure 6. These gradients must primarily be due to mass anomalies below sea level since we use terrain-reduced data. This is clearly illustrated if one inspects the centre of figure 6, where a large gradient of 50 E (1E = $10^{-9}s^{-2}$) is located. It corresponds with the Trondheim Fjord, the depth of which is not represented in our DTM. So the gravity anomalies have not been smoothed appropriately, see Fig. 7. It is at exactly the same location where also the gridding results were the worst, i.e. where the horizontal gradients were the largest.

The differences between gravity at terrain level, and gravity computed using first downward continuation to zero level and then upward continuation again, are illustrated in Figure 8. Some statistics of the results are found in Table 3. Note the difference between using the true (generally higher) value for the data noise, and the uniform noise of 1 mgal.

FFT was also used for downward continuation to the mean terrain level (425 m) rather than 0. The difference between the gravity anomalies at terrain level and at mean level had a standard deviation of 0.46 mgal.

But how different are geoid heights computed from data at terrain and data continued to zero level? This is illustrated in Figure 9, and statistics are given in Table 4. We see that the results are

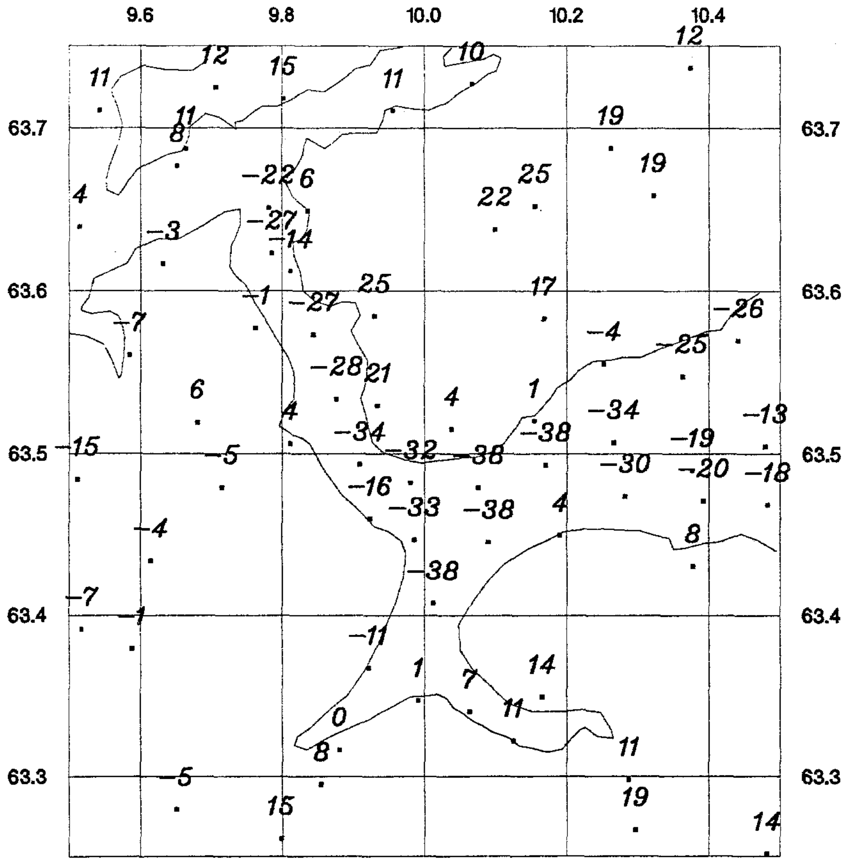

Fig. 7. Gravity anomaly point values. Note the lack of smoothing of the anomalies in the fjord due to lack of depth information in the used DTM. Units mgal.

Table 3. Difference between gravity data at terrain level and downward and upward continued gravity data from zero level, using LSC used for downward continuation. Units mgal.

		With true noise		*With noise fixed to 1 mgal*			
	Original values	*Downward contin.*	*Diff.*	*Downward contin.*	*Diff.*	*Upward contin.*	*Diff.*
mean	-1.70	-1.85	0.15	-1.79	0.10	-1.68	-0.01
stdv.	13.89	14.21	2.51	14.53	1.54	13.69	0.62

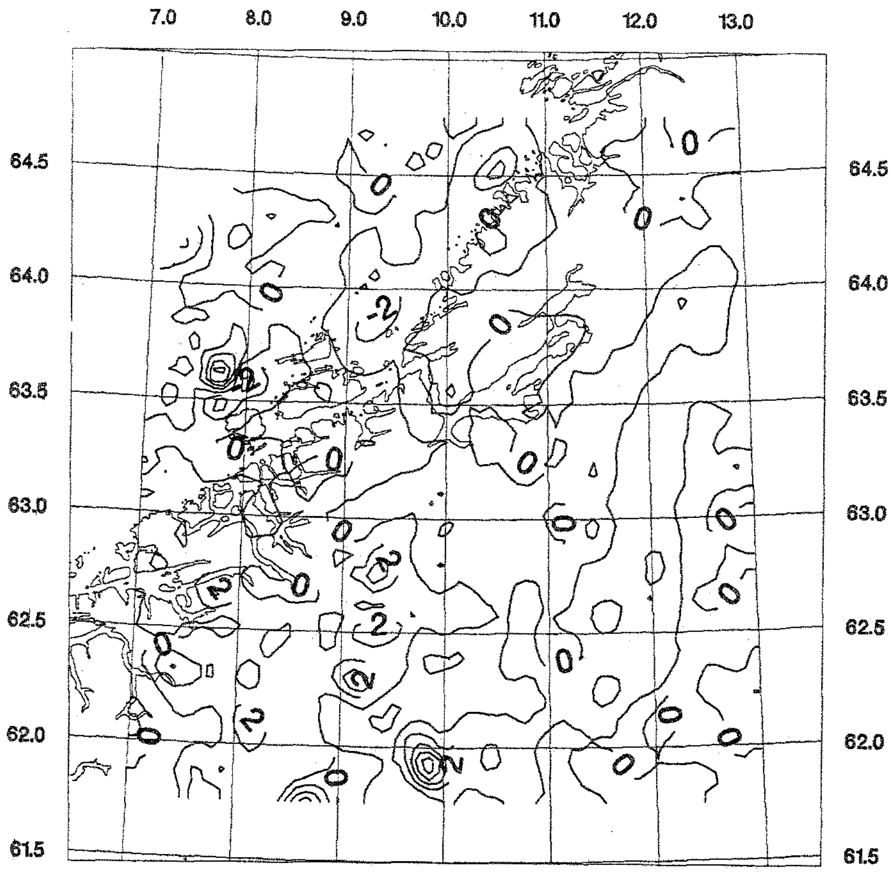

Fig. 8. Differences between gravity at terrain level and downward continued gravity to 0-level using LSC. Contour interval 2 mgal.

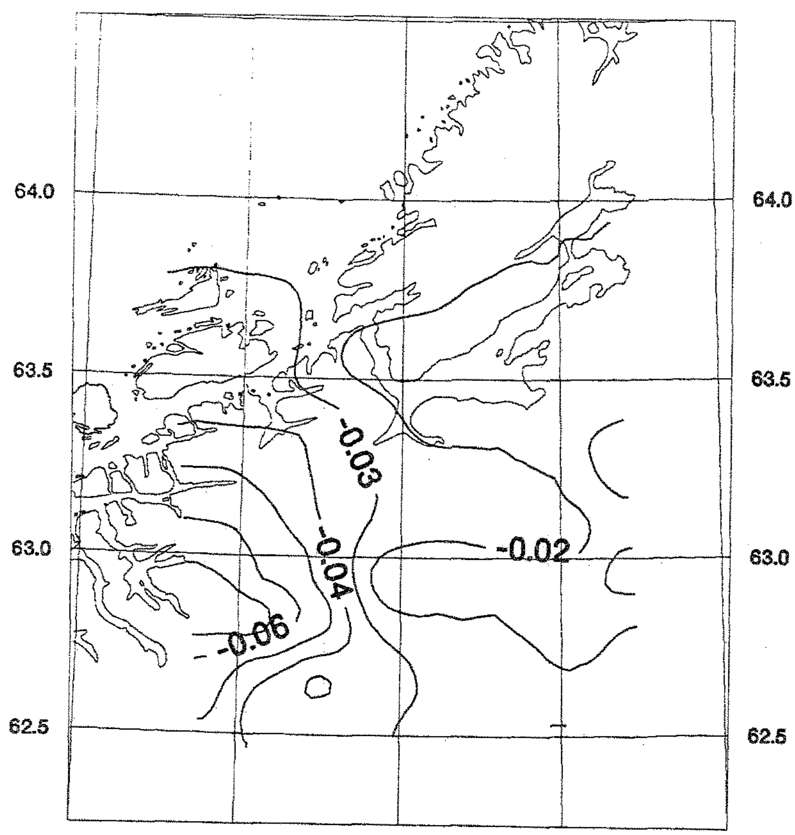

Fig. 9. Difference between geoid heights computed from gravity at terrain level and from gravity continued to zero level. Contour interval 0.01 m.

Table 4. Difference between geoid heights obtained from terrain level and from downward continued gravity data at zero level. LSC used locally. All units meter.

	From gravity at terrain level	*Downward cont. of gravity at 0-level*	*Difference*
Mean	-0.189	-0.171	-0.018
Stdv.	0.464	0.465	0.010
Min	-1.410	-1.380	0.060
Max	0.590	0.630	0.000

Table 5. Differences between geoid heights at zero level and height anomalies (H.A.) at terrain level from LSC, local FFT, and the NKG FFT height anomalies. All units m.

Method:	*LSC*			*Local FFT*			*NKG*	*Diff.*
	Geoid	*H.A.*	*Diff.*	*Geoid*	*H.A.*	*Diff.*	*H.A.*	*LSC-NKG*
mean	-0.05	-0.03	0.00	0.001	0.001	-0.001	0.58	0.63
stdv.	0.36	0.36	0.01	0.350	0.350	0.003	0.43	0.08
min	-1.26	-1.29	-0.04		-1.301	-0.018		
max	0.48	0.47	0.12		1.058	0.022		

nearly identical, which is in agreement with the result in Table 3, which shows how close the gravity anomalies are.

It is also of interest to see the magnitude of the difference between geoid heights and height anomalies (still for terrain reduced data). Here we have computed results using LSC, local FFT and also compared with the NKG-89 geoid (which is really a grid of height anomalies). The result is found in Table 5.

The differences between height anomalies (at terrain level) and geoid heights (at 0) are shown

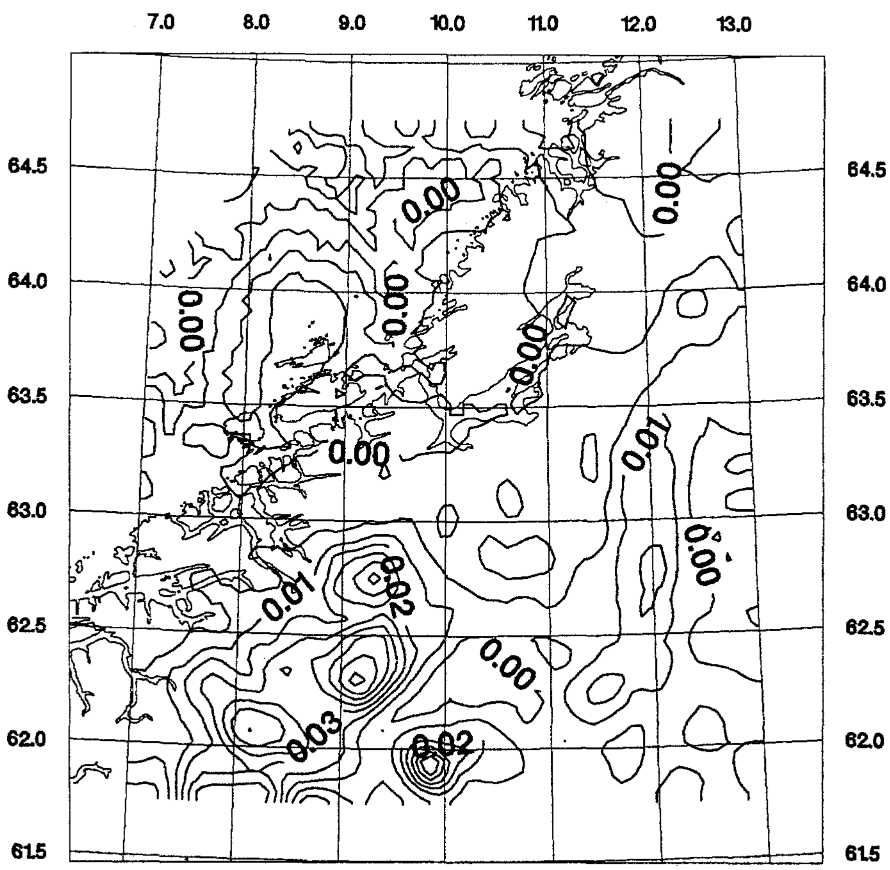

Fig. 10. Differences between geoid heights and height anomalies evaluated at terrain level. Contour interval 0.01 m.

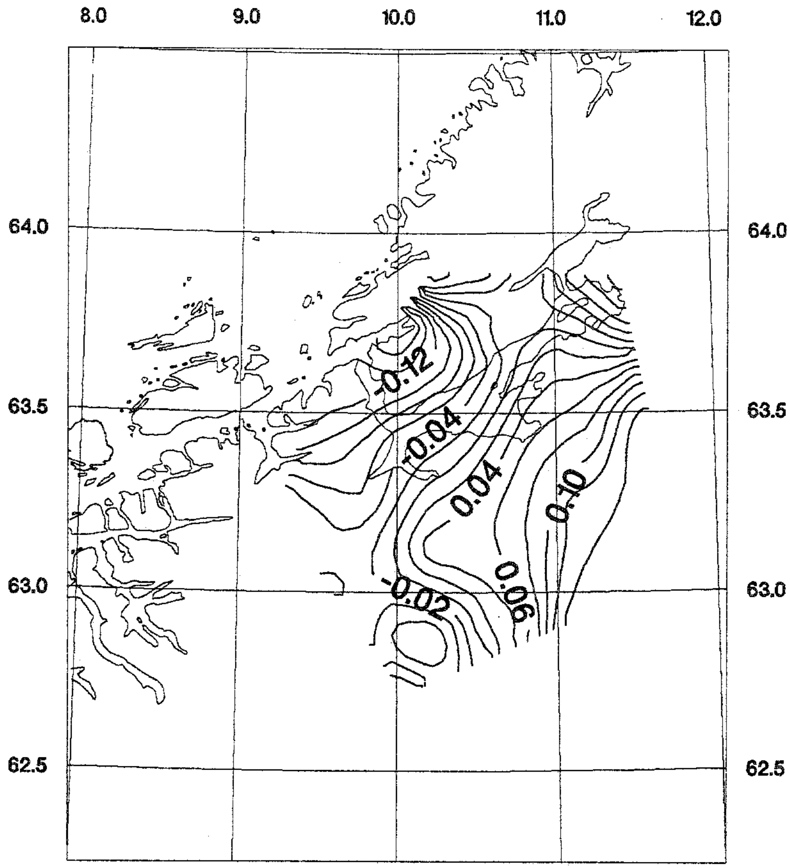

Fig. 11. Difference between GPS-levelling geoid heights and geoid heights from LSC. Contour interval 0.02 m.

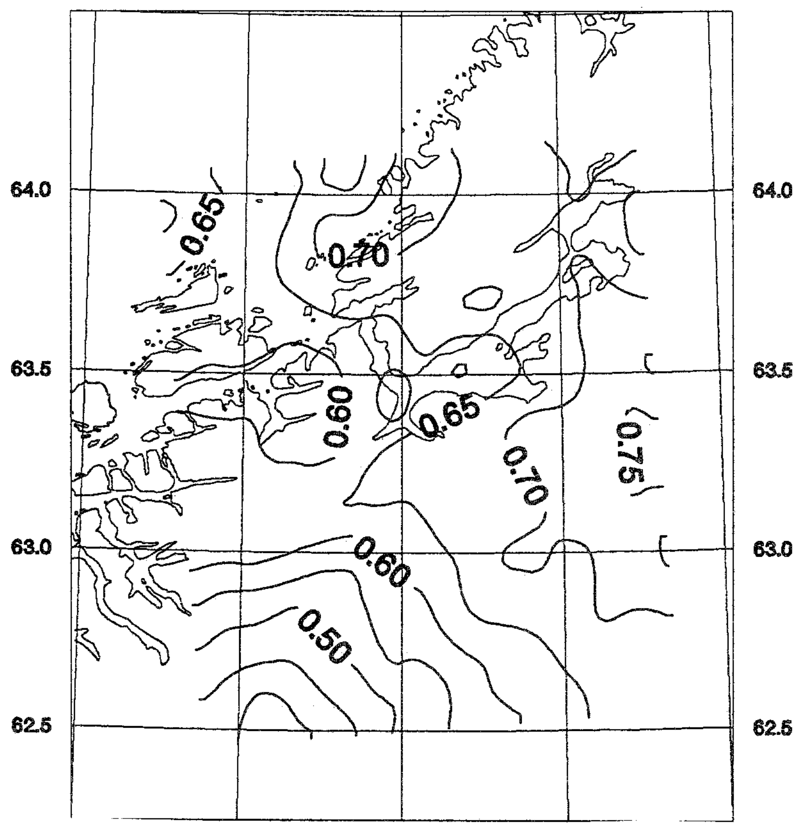

Fig. 12. Difference between height anomalies at terrain level and NKG FFT height anomalies. Contour interval 0.05 m.

in Figure 10. They are approximately equal to the residual gravity anomaly multiplied by the altitude.

We have finally compared various local LSC and FFT geoids with the GPS-levelling results, see Figure 11 and Table 6. The standard deviation of the differences is approximately 9 cm (mean removed). The magnitude of the differences reflects both errors in the GPS-levelling points, and errors in the calculated geoid heights. LSC error estimates indicates that the errors of the calculated geoid height differences should in general be around 4 cm. The accuracy of the GPS-derived geoid undulations is probably at a comparable level (or worse), with GPS point heights in part based on trigonometric levelling.

A comparison of the GPS-levelling values with the NKG-89 geoid heights shows also a 9 cm agreement. However, we see that there is a certain systematic difference of the same order (Figure 12), implying that the extend of the gravity data used (as expected) plays a major role.

Table 6. Comparison of NKG, LSC, and FFT geoid heights with GPS-levelling heights. Number of GPS points: 20. Absolute bias of GPS geoid heights adjusted to match LSC solution.

	GPS	*Differences:*		*LSC at*	
	geoid values	*NKG*	*LSC at 0*	*altitude*	*Local FFT*
mean	0.08	0.77	0.00	0.00	
stdv.	0.23	0.08	0.10	0.09	0.09

6. Conclusion

We have illustrated that the smoothing of the data is extremely important in order to reduce the information loss when gridding data, and when performing harmonic continuation.

It may be of advantage to use grid spacings smaller than the mean data spacing in order to limit the information loss in gridding, but if the grid is too dense the improvements in geoid precision may be marginal. Most important is the size of the area covered by data, since errors in the reference spherical harmonic models still are considerable in mountainous regions, even in regions of good mean anomaly coverage.

The consistency of the harmonic downward continuation for both gravity and geoid heights is very satisfactory. From gravity at zero level, appropriately downward continued, we get geoid results for which the differences have a standard deviation of only a few cm. However, the largest differences have a magnitude which can not be disregarded. The upward continued gravity anomalies are in agreement with the observed values within the noise level.

A further reduction of the differences should be possible using higher resolution terrain models for both land and sea areas. Also the use of known geological information concerning density changes should be considered.

As a final remark we note, that we are well aware that examples do not prove anything. The magnitudes of the phenomena will probably be larger in higher mountains. Unfortunately we will probably never be able to control the quality of computed height anomalies in higher mountains, because very few precise levelling lines are available at high altitudes. It could, however, be interesting to establish a "geoid" test area at high altitudes. This could be done using local precise levelling not necessarily connected to a regional or national control network.

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