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Analysis of tropospheric delay prediction models: comparisons with ray-tracing and implications for GPS relative positioning

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ABSTRACT

Ray-tracing is used to examine the accuracy of several well known models for tropospheric delay prediction under varying atmospheric conditions. The models considered include the Hopfield zenith delay model and related mapping functions, the Saastamoinen zenith delay model and mapping function, and three empirical mapping functions based upon the Marini continued fraction form. Modelled delays are benchmarked against ray-tracing solutions for representative atmospheric profiles at various latitudes and seasons. Numerical results are presented in light of the approximations inherent in model formulation. The effect of approximations to the temperature, pressure and humidity structure of the neutral atmosphere are considered; the impact of surface layer anomalies (i.e., inversions) on prediction accuracy is examined; and errors resulting from the neglect of ray bending are illustrated. The influence of surface meteorological parameter measurement error is examined. Finally, model adaptability to local conditions is considered. Recommendations concerning the suitability of the models for GPS relative positioning and their optimal application are made based upon the results presented.

1. BACKGROUND

1.1 The Delay Equations

The refractive effect of the neutral atmosphere on satellite ranging is expressed in first approximation by the integral equation

$$D_t = \int_{r_s}^{r_a} [n(r) - 1] \sec\psi(r)\, dr + \left[\int_{r_s}^{r_a} \sec\psi(r)\, dr - \int_{r_s}^{r_a} \sec z(r)\, dr \right] \qquad (1)$$

wherein the "tropospheric delay" D_t is expressed as an excess path length, n is the refractive index, r is the geocentric radius, and ψ and z respectively denote the apparent and true satellite zenith angle. Equation (1) holds for a spherically symmetric atmosphere, wherein n varies solely as a function of geocentric radius, and is evaluated between the tracking station and the top of the neutral atmosphere. The first integral accounts for the difference in the electromagnetic and geometric lengths of the refracted transmission path. The bracketed integrals account for path curvature, that is, the difference in the geometric lengths of the electromagnetic and rectilinear paths relating the satellite and tracking station. The effect of curvature is generally significant only at zenith angles beyond 70-80 degrees. Consequently, the bracketed terms in (1) are often neglected in practice.

A solution to equation (1) can be attained by ray tracing, given knowledge of the actual refractive index profile, or by analytical approximation. In the latter case a closed-form or truncated-series approximation is sought based upon a simplified atmospheric model. Generally, water vapour and the dry gases are treated as separate contributing components. Each component is then written as the product of a zenith delay term, approximating the integral of the refractive index profile in the radial direction, and a mapping function, mapping the increase in delay with advancing zenith angle. In general form

$$\begin{aligned} D_t = {} & D^z_{dry}(P_s) \times \\ & MF_{dry}(\psi_s, P_s, T(h)) + \\ & D^z_{wv}(T_s, RH_s, T(h)) \times \\ & MF_{wv}(\psi_s, T_s, RH_s, T(h), RH(h)) \qquad (2) \end{aligned}$$

where

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D^z_{dry}	:	zenith delay due to dry gases
D^z_{wv}	:	zenith delay due to water vapour
MF_{dry}	:	dry gas mapping function
MF_{wv}	:	water vapour mapping function
ψ_s	:	refracted zenith angle at the tracking station
h	:	height above the tracking station
T_s	:	surface temperature
P_s	:	surface pressure
RH_s	:	surface relative humidity
T(h)	:	temperature profile above the tracking station
RH(h)	:	relative humidity profile above the tracking station

Equation (2) has been written in terms of the most basic meteorological information required to model each component. However, it should be noted that various derivative parameters are often used. These most often include the partial pressure of water vapour, and the dry gas and water vapour components of refractive index.

1.2 Refractivity Definitions

The refractivity $N=(n-1)\times10^6$ is related to temperature (T) and the partial pressures of the dry gases (P_d) and of water vapour (e) by the expression

$$N = K_1 (P_d/T) + K_2 (e/T) + K_3 (e/T^2) \tag{3}$$

where K_1, K_2, K_3 are empirically determined coefficients. In radio meteorology (3) is most often re-written in the form

$$N = K_1 (P/T) + K'_2 (e/T^2) \tag{4}$$

where $K'_2 = [(K_2-K_1)T + K_3]$. The first and second terms of (4) are commonly referred to as the "dry" and "wet" components of refractivity. Alternatively, (3) can be re-written as

$$N = K_1 \frac{M}{M_d}\frac{P}{T} - (K_1 \frac{M}{M_d} - K_2)\frac{e}{T} + K_3 \frac{e}{T^2} \tag{5}$$

$$\text{where } \frac{M}{M_d} = \frac{T}{T'} = (1 + 0.3780\frac{e}{P})^{-1} \tag{6}$$

In the above T' is the virtual temperature, and M and M_d denote the molar mass of moist and dry air respectively. The first term of (5) is sometimes referred to as the "hydrostatic" component of refractivity {Davis, 1986}.

1.3 The Gas Equations

Various profile forms and zenith delay expressions used in modelling the refractive effect of the dry gases have been derived through suitable approximations to the gas equations for moist air. The equation of state of moist air is written as

$$\rho = \frac{PM}{RT} \tag{7}$$

where ρ denotes density, M is the molecular mass of moist air, and R is the gas constant. Substitution of (7) into the hydrostatic equation

$$\frac{dP}{dh} = -\rho g \tag{8}$$

and integrating then yields the atmospheric pressure profile

$$P(h) = P_s \exp [\frac{-1}{R} \int_0^h \frac{Mg}{T} dh] \tag{9}$$

where h is the geopotential height above the surface. In evaluating (9), g is generally replaced by a value taken at the height of the atmospheric centroid for the latitude in question, and M by the molar mass of dry air M_d. The integral then depends solely upon the temperature profile, which can be represented by a suitable combination of isothermal and polytropic layers. For an isothermal layer the pressure profile takes the form

$$P(h) = P_b \exp[\frac{-gM}{RT} h] \tag{10}$$

where P and h are now referenced to the base of the layer. Substitution of (10) into the dry or hydrostatic terms of the refractivity equations yields the refractivity profile

$$N_1(h) = N_{1b} \exp[\frac{-gM}{RT} h] \tag{11}$$

where the subscript 1 refers to either the dry or hydrostatic component as appropriate. For a polytropic layer the corresponding pressure and refractivity profiles are

$$P(h) = P_b (\frac{T}{T_b})^{[\frac{gM}{R\alpha}]} \tag{12}$$

$$N_1(h) = N_{1b} \left(\frac{T}{T_b}\right)^{\left[\frac{gM}{R\alpha} - 1\right]} \tag{13}$$

where α = -dT/dh is the temperature lapse rate.

The shape of the refractivity profile is a critical factor in mapping function accuracy at large zenith angles. Conversely, the zenith effect of the dry gases is independent of the profile shape. This can be shown by substituting the equation of state (7) into the hydrostatic term of the refractivity equation (5) to obtain

$$N_d = K_1 \frac{M}{M_d} \left(\frac{P}{T}\right) = K_1 \rho \frac{R}{M_d} \tag{14}$$

Integrating in the zenith direction yields

$$D^z{}_d = 10^{-6} \int_{r_s}^{r_a} N_d(r)\, dr = 10^{-6} \frac{K_1 R}{M_d} \int_{r_s}^{r_a} \rho(r)\, dr \tag{15}$$

From the hydrostatic equation

$$P_s = g \int_{r_s}^{r_a} \rho(r)\, dr \tag{16}$$

yielding

$$D^z{}_d = 10^{-6} K_1 \frac{R}{M_d\, g} P_s \tag{17}$$

Following a similar procedure for the dry component definition (4) yields an identical result for the zenith delay, but in which M_d is replaced by M, the molar mass of moist air. As a consequence, the dry zenith effect is indirectly influenced by water vapour content, whereas the hydrostatic zenith delay is not.

Unlike the dry gases water vapour does not conform to the hydrostatic equation, and is subject to wide variation in concentration, including condensation, under normal atmospheric conditions. Consequently there exists no counterpart to the developments given above for the wet or non-hydrostatic components of refractivity. Models describing the distribution of water vapour are hence largely empirical in nature.

2. MODEL DESCRIPTIONS

The theoretical development of the models considered in this paper has been described in detail in an earlier work {Janes, Langley & Newby, 1989}. For the sake of continuity a review of this material is provided below. Three additional models for the zenith effect of water vapour, not covered in the above paper, are also described.

2.1 The Hopfield Model & Related Mapping Functions

The Hopfield model employs the dry and wet refractivity component definitions (4) and is based upon a single-layer polytropic model atmosphere extending from the surface to an altitude of approximately 40 km. The relevant expressions as given in Hopfield{1972} are:

$$N_i(h) = N_{is} \left(\frac{H_i - h}{H_i}\right)^{\mu} \tag{18}$$

with i = 1, 2 denoting the dry and wet components respectively, and

$$\mu = 4\,;$$
$$H_1 = 40136\text{m} + 148.72\text{m/C}^\text{o} \times T_c\,; \tag{19}$$
$$H_2 = 11000\text{m}$$

$$D^z{}_t = \frac{10^{-6}}{5} \left[N_{1s} H_1 + N_{2s} H_2\right] \tag{20}$$

For the dry gases (18) is equivalent to (13) with $\mu = (gM/R\alpha) - 1$, H_1 = To/α and α = 6.8 o/km. The wet component refractivity profile is assumed to follow a similar "quartic" form. The nominal values quoted in (19) for the exponent μ and the dry and wet term scale heights H_1 and H_2 were derived by fitting one-year spans of radiosonde data at 14 sites distributed over the western hemisphere. Equation (20) is obtained by integrating (18) for each component from the surface to the appropriate scale height, and summing the results. It should be noted that the scale heights are referenced to the tracking station, not sea level.

Yionoulis{1970}, Goad & Goodman {1974}, Black {1978}, and Black & Eisner {1984} have all developed mapping functions based upon the Hopfield single layer polytropic model. These generally proceed from the approximation

$$\sec \psi(r) \cong \sec z(r) = \left[\left(1 - \left(\frac{r_s^2}{r^2}\right) \sin^2 z_s\right) \right]^{-1/2} \tag{21}$$

Equation (21) is Snell's law for a spherical atmosphere of uniform refractivity. Under this assumption the bracketed terms in (1) collapse and path curvature is neglected. Further simplification of the first integral of (1) is then accomplished by expanding (21) in series or through geometrical approximation of the denominator. Generally, the dry and wet components are mapped separately using similar functions.

2.2 The Saastamoinen Model

The Saastamoinen {1973} model employs the hydrostatic component definitions of equation (5). The dry atmosphere is modelled by a polytropic troposphere extending from the surface to an altitude of 11-12 km, surmounted by an isothermal stratosphere extending from the tropopause to approximately 60 km. The corresponding profile and zenith delay expressions are as given by equations (10) through (13) and (17) of section 1.3. The specification of the layer parameters in these expressions is left to the user. Atmospheric water vapour is constrained to the troposphere and assumed to follow a form similar to that of the polytropic pressure profile (12)

$$e(h) = e_s \left(\frac{T}{T_s}\right)^{\nu \left[\frac{gM}{R\alpha}\right]} \tag{22}$$

where ν typically assumes a value between 2 and 5. The zenith effect of water vapour is obtained by substituting (22) into the latter two terms of (5) and integrating the result to the height of the tropopause, yielding

$$D^z_{wv} = \frac{10^{-6}}{C\alpha} \left\{ \left[K_2 - K_1 \frac{M_w}{M_d}\right] e_s + \left[\frac{K_3}{1 + \frac{1}{C}}\right] \frac{e_s}{T_s} \right\} \tag{23}$$

where C is the exponent in equation (22). The combined effect of the hydrostatic and water vapour terms under nominal mid-latitude conditions is given by the standard formula

$$D^z_t = \frac{0.002277}{g'} \left\{P_s + \left[\frac{1255}{T_s} + 0.05\right] e_s\right\} \tag{24}$$

where g' is a correction to standard gravity determined for station latitude and orthometric height. The resulting delay is in metres for pressure in millibars, temperature in kelvins, and gravity in metres/sec-sec.

Mapping of the zenith delay is based upon a truncated binomial series expansion of Snell's law for a spherical atmosphere

$$\sec \psi(r) = \left[\left(1 - \left(\frac{n_s^2\, r_s^2}{n^2\, r^2}\right) \sin^2 \psi_s\right) \right]^{-1/2} \tag{25}$$

Separate series expansions are developed to account for the contributions arising from each of the dry gas and water vapour layers. Corrections are also developed to account for the influence of path curvature. All terms are explicitly parameterized in terms of the tropospheric temperature lapse rate, tropopause altitude, and water vapour lapse rate parameter ν. Typical values for mid-latitude conditions have been incorporated into the standard formula as correction coefficients tabulated for various zenith angles and station elevations.

2.3 Mapping Functions of the Marini Continued Fraction Type

Various authors have developed mapping functions of the continued fraction form first employed by Marini {1972}. The mapping functions of Marini & Murray {Davis, 1986}, Chao {1972}, and Davis {1986} fall within this category. Mapping functions of this type can generally be written in the form

$$MF(E_s) = \frac{D_t}{D^z_t} \cong \frac{a}{\sin E_s + \cfrac{b}{\sin E_s + \cfrac{c}{\sin E_s + \;\ldots}}} \tag{26}$$

where E is the unrefracted satellite elevation angle and a, b, c, . . . are profile dependent shape coefficients. Marini {1972} developed expressions which allow computation of the required coefficients for various layer types. However, empirical fitting of the coefficients to ray-traced radiosonde profile data is more common. The empirical approach leads to a computationally simple function and inherently accounts for path curvature at low elevation angles. However, the resulting coefficients do not always lend themselves to clear physical interpretation, and in some cases are not easily adapted to suit local or seasonal variations from the representative profiles from which they are derived.

The Chao {1972} mapping function is based upon the fit to an average refractivity profile derived from radiosonde observations taken over the period of one year at Edwards AFB, California. The dry and wet refractivity profiles were fitted separately. The continued fraction series for each component is truncated after three coefficients (a, b, c), of which the first (a) is set to unity. The terminating sine function is replaced by the tangent of the elevation angle to ensure the series goes to unity at zenith. The coefficients are expressed as numeric constants.

The Marini & Murray mapping function {Davis, 1986} maps the total zenith delay. The continued fraction series is again truncated after three coefficients. However, in this case the first two coefficients in the series are expressed as functions of the Saastamoinen zenith delay (24) and tracking station elevation. The third is a fitted numeric constant.

The Davis mapping function {Davis, 1986} maps the hydrostatic component of the zenith delay. The continued fraction series contains four coefficients, the first of which is set to unity, and the last of which is a fitted numeric constant. The remaining two coefficients (b and c) are expanded as functions of the departure from standard values of five meteorological parameters: surface pressure, temperature, and water vapour pressure, tropospheric temperature lapse, and tropopause altitude. The coefficient functions were fitted to a Saastamoinen-type, two-layer dry atmospheric model in which all five parameters were assigned a typical range of values.

2.4 The Chao Water Vapour Zenith Delay Model

The Chao {1972A} water vapour model is based upon application of the hydrostatic law to water vapour. However, Chao replaces the equation of state (7) with the adiabatic approximation

$$e = k^{\gamma} \rho^{\gamma}{}_{wv} \tag{27}$$

where γ is the specific heat of water vapour. The Chao zenith delay formula is written as

$$D^z{}_w = 1.63 \frac{e_s{}^{1.23}}{T_s{}^2} + 2.05\,\alpha \left[\frac{e_s{}^{1.46}}{T_s{}^3}\right] \tag{28}$$

2.5 The Berman Models for Zenith Delay due to Water Vapour

Berman {1976} describes two models for predicting the zenith effect of water vapour. The Berman 1970 model is based upon integration of the hygrometric profile

$$e(h) = 0.061\ \mathrm{RH} \exp\left[\frac{AT(h) - B}{T(h) - C}\right] \tag{29}$$

to the height of the tropopause under uniform relative humidity and a linear temperature lapse. In the above, $A = 17.1485$, $B = 4684.1$, and $C = 38.45$ are hygrometric constants. The resulting zenith delay expression is written as

$$D^z{}_w = \frac{3.73 \times 10^4}{\alpha\,(B - AC)} \left[1 - \frac{C}{T_s}\right]^2 e_s \tag{30}$$

The Berman 1976 (day/night) model is based upon the correlation between the dry gas and water vapour components of the zenith delay and their surface refractivities. The Berman 1976 model is written as

$$\frac{D^z{}_w}{D^z{}_d} = K \left[\frac{N_{wo}}{N_{do}}\right] \tag{31}$$

where

$K_{day} = 0.2896$,
$K_{night} = 0.3773$,
$K_{mixed} = 0.3224$

The above values for K were empirically determined by fitting 10 seasonally distributed day/night radiosonde profiles located at Edwards AFB, California.

3. REFERENCE ATMOSPHERES

The US Standard Atmosphere and the US Standard Atmosphere Supplements provide a consistent family of reference atmospheres for the testing of model performance. The US Standard Atmosphere {NOAA, 1976} is an idealized, steady state representation of the earth's atmosphere from sea level to 1000 km, approximating median mid-latitude conditions for dry air. Below 80 km the standard is represented by various defining constants and gas law relations, together with a linearly-segmented median temperature profile. Tabulated values of the physical properties of dry air are listed to an altitude of 1000 km, and include representative temperature, pressure, and density profiles.

The US Standard Atmosphere Supplements {NOAA, 1966} tabulate latitudinal and seasonal departures from the US Standard Atmosphere. Supplemental reference atmospheres are provided for the following latitudes and times of the year:

Tropic	15N	Annual
Subtropic	30N	January & July
Mid-latitude	45N	January & July
Subarctic	60N	January & July
Arctic	75N	January & July

Representative temperature and pressure profiles are tabulated to 80 km. Relative humidity is tabulated to approximately the height of the tropopause (8 - 12 km).

Refractive index profiles were generated for each of these reference atmospheres. These were then ray traced to provide benchmark delay values for the various dry gas and water vapour components of the tropospheric delay. Representative values for the hydrostatic and water vapour components are summarized in Table 1. Variability in the hydrostatic delay is of the order of 3 cm at zenith, 9 cm at 70°, 17 cm at 80°, and 30 cm at 85°. Variation of the corresponding water vapour component is of the order of 25 cm at zenith, 72 cm at 70°, 1.4 m at 80°, and 2.7 m at 85°. Note that diurnal variations in temperature, pressure, and humidity would be superimposed upon these latitudinal and seasonal values.

Table 1
US Standard Atmosphere Supplements 1966
Tropospheric Delay (m) vs Zenith Angle (deg)

Zenith Angle	15°N Annual	30°N July	30°N January	45°N July	45°N January	60°N July	60°N January	75°N July	75°N January
				Hydrostatic Component					
.0	2.313	2.313	2.328	2.308	2.321	2.297	2.305	2.300	2.301
70.0	6.702	6.703	6.747	6.688	6.727	6.658	6.683	6.667	6.674
80.0	12.841	12.842	12.936	12.817	12.910	12.766	12.835	12.786	12.823
85.0	23.381	23.379	23.600	23.356	23.612	23.300	23.530	23.341	23.523
				Water Vapour Component					
.0	.245	.263	.132	.182	.057	.134	.029	.097	.015
70.0	.714	.767	.385	.531	.166	.392	.086	.283	.045
80.0	1.396	1.499	.753	1.037	.324	.766	.167	.552	.088
85.0	2.707	2.900	1.460	2.009	.627	1.481	.323	1.068	.171

Refractive index profiles were also generated for the dry gas component of the Hopfield and Saastamoinen model atmospheres, using standard atmosphere surface values and the temperature profiles specified by these authors. The ray tracing solutions generated for these profiles were then benchmarked against the US Standard Atmosphere. The results are abstracted in Table 2. Delays computed for the Hopfield single-layer polytropic model differ from those of the standard profile by 2 mm at 70°, 2 cm at 80°, and 5 cm at 85°. The Saastamoinen two-layer model provides agreement to within 1 mm to 80°, and 6 mm to 85°. In both cases the more simplified models lead to an increasing over estimation of the delay with advancing zenith angle.

TABLE 2
Model Dry Atmosphere
Troposheric Delay (M) VS Zenith Angle (DEG)

Zenith Angle	US Standard	Hopfield	Saastamoinen
.0	2.313	2.313	2.313
20.0	2.461	2.461	2.461
40.0	3.017	3.017	3.017
60.0	4.609	4.610	4.609
70.0	6.703	6.705	6.703
80.0	12.855	12.867	12.856
85.0	23.468	23.513	23.474

Table 3 summarizes the effect of path curvature on delay for each of the supplemental atmospheres. The curvature effect was determined by comparing ray trace solutions computed along the refracted path according to Snells law (equation 25) with solutions evaluated along a rectilinear chord to the path (equation 21). In the absence of curvature the bracketed terms in equation (1) collapse thereby reducing the delay. However, since the chord path also travels through a denser portion of the atmosphere the first term in (1) is increased. This latter effect predominates, thereby leading to an overall over-estimation of the delay with advancing zenith angle. The resulting error due to the neglect of curvature is of the order of a 2-3 mm at 70°, 2 cm at 80°, and 17 cm at 85°, and arises primarily from the influence of the dry gases. The contribution arising from water vapour is negligible to 80°, and accounts for only 5-10 mm of the effect at 85°.

TABLE 3
US Supplemental Atmospheres 1966
Error (MM) Due to Neglect of Curvature

Zenith Angle	15°N Annual	30°N July	30°N Jan	45°N July	45°N Jan	60°N July	60°N Jan	75°N July	75°N Jan
				Hydrostatic Component					
70.0	2	3	2	2	3	3	2	3	3
80.0	20	21	21	20	20	21	19	26	26
85.0	168	169	169	167	169	166	165	197	201
				Water Vapour Component					
70.0	0	1	0	0	0	0	0	0	0
80.0	1	1	0	1	0	1	0	1	0
85.0	10	8	4	6	2	4	1	3	0

4. MODEL COMPARISONS

Each of the zenith delay models and mapping functions described in section (2) was implemented in the standard form specified by its author. In addition, a full-blown version of the Saastamoinen model was implemented using the explicit theoretical expressions provided in Saastamoinen {1973}. Delays were generated for each algorithm using surface parameters derived from the supplemental profiles. Model predictions were then benchmarked against the ray trace solutions for each reference atmosphere. Care was taken to ensure that mapping functions were matched with the parent zenith delay formula for which they were designed, and that model comparisons with ray tracing were consistent with respect to refractivity component definition.

Representative upper air parameters were obtained by combining values derived by fitting the supplemental profiles with those reported in various other sources. Additional temperature lapse rate and tropopause altitude data was abstracted from the World Atlas of Radio Refractivity {Bean et al., 1966} and the US AFGL Air Force Reference Atmospheres {Cole & Kantor, 1978}. The water vapour lapse exponent parameter ν was obtained by combining values derived by fitting the supplemental profiles with those reported by Smith {1966}. These approximate upper air parameter values were then employed as a common data set for all algorithms requiring them.

4.1 Zenith Delay Model Comparisons

The results of our zenith delay model comparisons are summarized in Table 4. Both the Hopfield and Saastamoinen models for the zenith effect of the dry gases agree at the sub-centimetre level with our ray tracing results. The Saastamoinen estimates are consistently within a few millimetres of ray trace values. The standard and expanded versions of Saastamoinen provided equivalent results for the hydrostatic zenith delay. The Hopfield model tends to show a smaller range of variation in dry zenith delay than indicated by ray tracing, under estimating the delay at low latitudes, and over estimating at high latitudes, reflecting perhaps the residual indirect influence of water vapour content (through the molar mass of moist air) on the Hopfield formulation, and a predominance of mid-latitude sites in the Hopfield {1972} data set used in deriving the scale height relation (19).

This can be understood by considering the effect of variations in the molar mass of moist air on equation (17). The molar mass varies between 28.9644 kg/kmole (dry air) and 18.9644 kg/kmole (water vapour) depending upon vapour concentration. Assuming that vapour content, on average, decreases with latitude, leads to an under estimation of the delay at low latitudes, and an over estimation at high latitudes, for a model calibrated for temperate climates. The Hopfield dry term zenith expressions, although formulated differently, are equivalent to (17). The molar mass of moist air enters through the exponent μ of the Hopfield expression (18). Hopfield arbitrarily sets this value to 4. The variation in its actual value due to changes in vapour concentration then influences the empirical determination of representative scale heights.

As expected, the water vapour models exhibit departures from ray tracing significantly larger and more variable than their dry gas counterparts. Virtually all the models tested tend to over estimate the zenith delay at high vapour pressures, and under estimate the delay at low vapour pressures. The largest departures occur for the July reference atmospheres at lower latitudes. The Chao and Berman models appear to be the most sensitive to water vapour content, exhibiting the largest range of variation in zenith delay. They also provide the largest departures from ray tracing at high vapour pressures. One contributing factor may be the data base upon which the fitted parameters in these models were based; since all three were calibrated using radiosonde data from Edwards AFB, California. The best agreement

TABLE 4
Zenith Delay Error (mm)
Model - Ray Trace
US Standard Atmosphere Supplements 1966

	15°N	30°N		45°N		60°N		75°N	
Model	Annual	July	Jan	July	Jan	July	Jan	July	Jan
			Dry Gas Component						
Saastamoinen Precise	-1	-3	0	-1	-3	-1	-1	0	1
Hopfield	-5	-5	-1	2	0	5	5	8	7
			Water Vapour Component						
Saastamoinen Precise	6	20	-6	6	-3	3	-6	0	-1
Saastamoinen Standard	35	61	10	21	-6	-1	-12	-13	-6
Hopfield	0	18	-2	-1	-8	-13	-12	-17	-6
Chao	100	149	21	51	-13	6	-17	-15	-10
Berman 70	44	73	17	37	-1	5	-11	-5	-5
Berman 76	70	101	29	48	1	16	-10	-1	-5

with ray tracing is exhibited by the Hopfield model at low to middle latitudes, and by the full-blown Saastamoinen model at middle to high latitudes. Both these models exhibit relatively low sensitivity to water vapour content, resulting in the smallest dynamic range in delay for the reference atmospheres tested.

4.2 Mapping Function Comparisons

Table 5 summarizes the results of our mapping function comparisons. Comparisons relative to ray tracing for each reference atmosphere are tabulated for zenith angles of 70, 80, and 85 degrees. The quantities tabulated correspond to an error model of the form

$$\delta D_t(\psi) = \delta D_t^z \, MF(\psi) + D_t^z \, \delta MF(\psi) \quad (33)$$

where D_t^z is the zenith delay, ψ denotes the zenith angle, and δ denotes model - ray traced. The first term of (33) describes the effect of the zenith delay prediction error as mapped to the satellite zenith angle. The second, tabulated in Table 5, represents the mapping function error scaled by the zenith delay. Six of the ten mapping functions implemented map both the dry gas and water vapour components of the delay. The Saastamoinen Standard, Marini & Murray, and Black & Eisner mapping functions map the total delay. The Davis model maps only the hydrostatic component.

Errors in mapping the zenith effect of the dry gases are typically under a centimetre for zenith angles less than 70 degrees. Beyond 70 degrees the Hopfield-based functions exhibit a consistent tendency to over-estimate the increase in delay with advancing zenith angle, reflecting the effect of neglected path curvature and the polytropic single-layer dry air approximation common to these functions. Typical departures are of the order of 3-5 cm at 80 degrees, and 15-25 cm at 85 degrees. The Chao dry term function provides slightly better agreement between 70 and 80 degrees, but comparable agreement at greater zenith angles. The full Saastamoinen model provides still closer (1-2 cm) agreement to 80 degrees, but rapidly and radically breaks down at lower zenith angles. The Davis mapping function exhibits the smallest dry gas mapping errors, typically providing agreement at the sub-centimetre level to 80 degrees, and at the 2-3 cm level to 85 degrees.

Virtually all functions mapping the zenith effect of water vapour show sub-centimetre level agreement with ray tracing to 80 degrees, and 1-2 cm level agreement to 85 degrees, indicating that zenith delay prediction error, and the magnification of this error with advancing zenith angle, is the principal concern in calibrating the tropospheric delay due to water vapour. The Goad & Goodman function provides the closest overall agreement with ray tracing of the wet component mapping functions, departures generally falling within 3-5 mm of ray tracing to 85 degrees.

Of those functions which map the total delay, the Saastamoinen Standard algorithm exhibits the same break down at zenith angles greater than 80 degrees as the dry gas component of our full-blown Saastamoinen implementation. Both strongly under-estimate the increase in delay. However, the departures of the standard model are more severe, reaching 1.25-1.50 m at 85 degrees. The Marini & Murray function provides a level of agreement comparable to that of the Hopfield based mapping functions. Suprisingly, the Hopfield-based Black & Eisner function provides the best agreement with ray tracing in mapping the total delay at zenith angles greater than 80 degrees.

5. IMPLICATIONS FOR GPS RELATIVE POSITIONING

In a laterally homogeneous atmosphere the tropospheric delay varies only as a function of time and zenith angle. Differences in the delay for simultaneous measurements by multiple receivers are then due solely to the different zenith angles with which the co-observing stations see the common satellites. For a uniform distribution of satellites Beutler et al. {1988} have shown that the zenith angle effect results in a scale error of the order of

$$\frac{\Delta S}{S} = \frac{D^z_t}{r_s} \sec(\psi_{max}) \quad (34)$$

wherein D^z_t is the zenith delay, r_s is the geocentric radius of the tracking station, and ψ_{max} denotes the maximum zenith angle observed. Sea level values of the zenith delay are typically of the order of 2.3-2.6 m, implying a total scale effect of the order of 1-2 ppm for zenith angle maximums between 70 and 80 degrees. The excess path length at zenith can generally be further reduced to under 10 cm by appropriate modelling methods. Consequently, provided that the assumption of lateral homogeneity holds, a residual scale error of the order of 0.1 ppm or better would appear to be achievable.

In the actual atmosphere, the decorrelation of signal paths for co-observing stations is also governed by lateral gradients in atmospheric pressure, temperature and humidity, and by differences in station elevation. Beutler et al. {1988} have again shown that the effect of the differential troposphere can be written in first approximation for local networks as

Table 5
Mapping Function Error (mm)
Model Ray Trace
US Standard Atmosphere Supplements 1966

Zenith Angle = 70 Degrees

Model	15°N	30°N		45°N		60°N		75°N	
	Annual	July	Jan	July	Jan	July	Jan	July	Jan
Dry Gas Component									
Saastamoinen Precise	-1	-1	-3	0	-1	-1	1	-2	-2
Davis	0	-4	-3	-4	-2	-5	-3	-3	-6
Goad & Goodman	4	1	3	3	7	3	7	6	5
Yionoulis	2	2	5	6	5	4	5	4	8
Black	2	2	5	3	5	4	5	4	8
Chao	-8	-8	-8	-9	-12	-9	-13	-11	-13
Water Vapour Component									
Saastamoinen Precise	0	0	-1	0	0	-2	-2	1	0
Goad & Goodman	1	0	1	2	1	-1	-1	1	-2
Yionoulis	-2	-1	0	2	2	2	5	2	3
Black	-2	-1	0	1	2	2	3	2	1
Chao	-1	-2	-2	1	1	1	-2	0	1
Total Delay									
Saastamoinen Standard	-2	-6	-3	-1	-6	-2	-7	-5	-7
Marini & Murray	9	6	6	8	1	6	0	3	0
Black & Eisner	7	7	8	7	5	8	6	7	4

Zenith Angle = 80 Degrees

Model	15°N	30°N		45°N		60°N		75°N	
	Annual	July	Jan	July	Jan	July	Jan	July	Jan
Dry Gas Component									
Saastamoinen Precise	-12	-13	-20	-13	-20	-18	-12	-16	-18
Davis	9	-2	-5	-4	-7	-12	-8	-5	-15
Goad & Goodman	24	17	30	28	42	30	48	43	49
Yionoulis	29	27	36	36	43	34	48	42	57
Black	26	24	33	28	39	30	44	38	53
Chao	0	0	-11	-7	-26	-15	-43	-19	-46
Water Vapour Component									
Saastamoinen Precise	2	2	2	4	1	-1	1	4	0
Goad & Goodman	1	-1	1	3	0	-1	0	5	-3
Yionoulis	-3	-1	0	4	4	5	5	5	1
Black	-3	-2	0	3	4	5	5	5	1
Chao	-4	-3	-4	1	1	3	-2	-1	3
Total Delay									
Saastamoinen Standard	-24	-35	-23	-21	-35	-25	-48	-28	-54
Marini & Murray	63	61	43	53	19	37	3	29	0
Black & Eisner	40	39	44	43	35	45	28	44	24

Zenith Angle = 85 Degrees

Model	15°N	30°N		45°N		60°N		75°N	
	Annual	July	Jan	July	Jan	July	Jan	July	Jan
Dry Gas Component									
Saastamoinen Precise	-62	-652	-682	-675	-672	-714	-587	-654	-573
Davis	64	35	9	14	-19	-26	-17	9	-29
Goad & Goodman	159	144	201	187	250	197	293	270	326
Yionoulis	177	173	216	207	249	205	288	265	334
Black	166	162	202	185	234	193	272	251	318
Chao	219	223	157	190	68	140	-15	127	-36
Water Vapour Component									
Saastamoinen Precise	17	22	23	24	12	14	2	18	-1
Goad & Goodman	-6	-1	1	5	3	1	3	11	-6
Yionoulis	-4	9	3	12	11	19	12	14	5
Black	2	15	6	16	13	21	12	15	5
Chao	-10	1	-8	5	3	10	-5	1	7
Total Delay									
Saastamoinen Standard	-1247	-1259	-1251	-1241	-1333	-1286	-1431	-1294	-1458
Marini & Murray	272	297	164	214	26	120	-74	90	-94
Black & Eisner	47	41	79	75	51	75	1	94	-15

$$\Delta h_e = \Delta D^z{}_t \sec(\psi_{max}) \tag{35}$$

where $\Delta D^z{}_t$ denotes the difference in zenith delay between co-observing stations. Equation (35) indicates that neglect of the differential troposphere leads to approximately 3-5 mm of relative height error for every millimetre change in zenith delay between stations (ψ_{max} = 70-80^o).

Under nominal conditions (75% RH, 0-30^o C) zenith delay sensitivity to surface pressure, temperature and relative humidity respectively is of the order of 2 mm/mbar, 5-20 mm/C^o, and 1-3 mm/%. Hence, the detection of sub-centimetre changes in zenith delay is at the level of field met instrument precision. Moreover, instrument calibration or measurement errors, particularly in temperature, can introduce significant biases in relative height. Temperature and humidity measurements are also subject to external microclimatic influences owing to differences in surface albedo and local shading. These latter effects produce gradients within the surface layer which cannot be considered as indicative of changes in the tropospheric profile as a whole.

Consequently, modelling of the differential troposphere is advisable only where the meteorological gradients between co-observing stations clearly exceeds the accuracy to which these parameters can be measured with typical field instrumentation, plus the influence of any "surface layer noise" likely to be introduced by microclimatic effects. Note that pressure gradients also arise from elevation differences. Where horizontal gradients or vertical elevation differences between stations are significant, careful measurement of surface meteorology is essential to proper modelling of the resulting differential delay. Careful attention should also be shown to the possible existence of temperature and/or humidity profile anomalies, and to the selection of upper air profile lapse rate parameters which are representative of local conditions.

Conversely, where gradients and elevation differences are slight it is generally preferable to assume a laterally homogeneous atmosphere based either upon standard conditions (scaled to elevation) or upon averaged local meteorological measurements. Alternatively, one might employ the pressure measurements from each station to calibrate the dry atmosphere, but combine these with standard or averaged temperature and relative humidity data.

6. CONCLUSION

Based upon the analysis presented, the explicit form of the Saastamoinen zenith delay expressions, in combination with the Davis (hydrostatic) and Goad & Goodman (water vapour) mapping functions are recommended. It should be noted that our comparisons with ray tracing reflect optimal conditions in a number of respects. First, in many instances the upper air parameters employed in the models were derived, in part, from the reference profiles themselves, although in combination with data from other sources. Second, anomalous conditions involving temperature inversion or humidity irregularities are not represented in the reference profiles. Third and finally, the effects of instrument calibration and measurement error in the determination of the surface met parameters are neglected. Owing to these factors, the overall level of accuracy achievable in the prediction of tropospheric delay is somewhat less than our comparison results would indicate. Nevertheless, given that all algorithms benefited from our "perfect" knowledge to the extent that their formulations permitted, the comparisons made represent a valid assessment of the relative merits of the algorithms tested. To the extent that the supplemental profiles considered are indicative of median conditions for the latitude zones and seasons described it is expected that the Saastamoinen-Davis-Goad & Goodman combination would provide superior performance to the other models under most conditions.

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