STABILITY OF STATIONARY FILTRATIONAL COMBUSTION WAVES

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It is well known that the combustion of gases, powders, and gas-free systems can proceed in a very nonstationary manner. Filtrational combustion, viz., propagation of the heterogeneous exothermic reaction zone in a porous medium accompanied by percolation of the oxidizer, in this respect, is not an exception. Sharp nonstationary effects in the form of spin propagation of the reaction zone have been observed with filtration regimes in propagation were discovered in front propagation while stimulating experimentally the processes accompanying in situ combustion of oil [3].

The nonstationary nature of the process with external conditions constant is a result of the instability of stationary regimes. The effect of the parameters and the arrangement of the combustion process on its stability can be understood by studying the nonstationary response of the reaction front to a small perturbation of its stationary structure.

The main result of such an analysis is the determination of the boundaries of the regions in which stationary and nonstationary propagation regimes of filtrational combustion waves are realized. A secondary result is a calculation of the growth increments and damping decrements of different perturbations in the region of instability of the stationary regime. Assuming that the perturbation mode with maximum increment predominantly develops, it is possible to make certain assumptions as to the nature of the nonstationary propagation of the combustion wave.

The stability of the filtrational combustion front relative to small scale deformations with wavelength much less than the thermal layer have been analyzed before in [4]. Without stopping to consider the details, we note that in the cases of greatest practical interest, when the percolation rate exceeds many times the velocity of the front, the shortwave asymptotics are not representative for studying the stable combustion. A reaction front that is stable to small-scale perturbations, as will be shown in what follows, can turn out to be unstable to planar and long wavelength perturbations.

FORMULATION OF THE PROBLEM

We are investigating the stability of stationary propagation regimes for an exothermal interaction zone for particles in a porous medium interacting with the oxidizer in a gas flow, percolating in the direction of motion of the front or opposite to it [5-8]. In the quasihomogeneous approximation (the scale of the heterogeneous medium is assumed to be small), the system of filtrational combustion equations, which express the balance of heat (1), oxidizer (2), gaseous (3), and condensed (4) components, and components in the reaction wave, as well as the law for percolation of a gas into a porous medium (5), has the following form:

$$C\left(\frac{\partial T}{\partial t}+u^0\frac{\partial T}{\partial x}\right)=\lambda\Delta T-c_g\vec{G}\,\mathrm{grad}\,T+Q\rho_0 w, \tag{1}$$

$$\frac{\partial(\rho_g a)}{\partial t}+u^0\frac{\partial(\rho_g a)}{\partial x}=-\mathrm{div}(\vec{G}a)+D\rho_g\Delta a-\mu\rho_0 w, \tag{2}$$

$$\frac{\partial\rho_g}{\partial t}+u^0\frac{\partial\rho_g}{\partial x}=-\mathrm{div}(\vec{G})+\mu_g\rho_0 w, \tag{3}$$

$$\frac{\partial\eta}{\partial t}+u^0\frac{\partial\eta}{\partial x}=w,\quad \rho_c=\rho_0(1-\eta),\quad \rho_p=\mu_p\rho_0\eta, \tag{4}$$

$$\begin{gathered}\vec{G}=\rho_g\vec{v},\ \vec{v}=-k_p\,\mathrm{grad}\,p,\\ C=c_c\rho_c+c_g\rho_g+c_p\rho_p,\ \mu_p=1-\mu_g,\ p=R\rho_g T\sigma^{-1}=R'\rho_g T.\end{gathered} \tag{5}$$

Equations (1)-(5) are written in a system of coordinates moving from right to left in the direction $-x$ with the velocity of stationary combustion u^0. Here t is the time; T,

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temperature; $\vec{v}$, and $\vec{G}$, velocity and mass flux of the gas; ρ_g, ρ_c, and ρ_p, gas content, starting condensed substance, and solid-phase product per unit volume of the medium; p, pressure; a, concentration of the oxidizer; ρ_0, starting content of condensed substance; k_p, D, and λ, coefficients of percolation, diffusion, and thermal conductivity in the porous medium; σ, porosity; R, gas constant; $R' = R\sigma^{-1}$, μ, μ_g, and μ_p, stoichiometric reaction coefficients with respect to the oxidizer, gas, and condensed reaction product; C, heat capacity (at constant pressure) per unit volume of the medium; c_g, c_c, and c_p, heat capacities per unit mass of gaseous reagent, starting substance, and condensed product, coupled as a result of the assumed constancy of the heat of reaction Q by the relation

$$c_c = \mu_g c_g + c_p \mu_p.$$

Taking into account the sharp increase in the reaction rate w with temperature, we approximate w by the generalized function (infinitely thin reaction zone model)

$$w = \Phi(T_p)\delta(T - T_p),\ T_p = T(\xi),$$

normalized to some quantity Φ, depending on temperature T_p in the reaction zone ($x = \xi$).

We neglect the gas in the pores and its change ($\partial\rho_g/\partial t$, $\partial\rho_g/\partial x$) in the mass balance, assuming that most of the oxidizer arrives in the reaction zone with the percolating flux, i.e., $G \gg \rho_g u$. The last inequality is practically almost always satisfied in filtrational combustion processes, since the velocity of propagation of the front is much less than the linear velocity of percolation [6, 7].

Equation (3), in this case, takes the form

$$\operatorname{div}(\vec{G}) = \mu_g \rho_0 w. \tag{3'}$$

Completing the formulation of the problem, we will write out the stationary distribution of parameters in the region of the starting substance (x < 0) and product (x > 0), obtained by integrating the system of stationary ($\partial/\partial t = 0$) combustion equations (1)-(5) [8]:

$$\begin{gathered}
x < 0:\ T = T_1^0 = T_0 + qe^{x/l},\quad G_x = G_1^0 = \text{const}_1,\quad G_y = 0,\\
\eta_1^0 = 0,\quad \rho_c^0 = \rho_0,\quad \rho_p^0 = 0,\quad l = \lambda/c^0,\quad q = Qm^0/c^0,\quad C^0 = u^0 C + c_g G^0,\\
x > 0:\ T = T_2^0 \equiv T_p^0 = T_0 + q,\quad G_x = G_2^0 = \text{const}_2,\quad G_y = 0,\\
\eta_2^0 \equiv \eta_p^0,\quad \rho_{c2}^0 = \rho_0(1 - \eta_p^0),\quad \rho_{p2}^0 = \mu_p \rho_p \eta_p^0
\end{gathered} \tag{6}$$

The mass velocity of the stationary combustion of matter m^0, proportional to the conversion depth in the reaction front η_p^0, is determined by the Zel'dovich–Frank–Kamenetskii integral

$$\rho_0 u^0 \eta_p^0 = m^0 = \sqrt{2\lambda\rho_0\Phi(T_p^0)/Q}. \tag{7}$$

The quantity C^0 remains continuous with the transition through the boundary x = 0 ($\rho_g \ll \rho_c$, $\rho_g \ll \rho_p$),

$$C^0 = \rho_0 c_1 u^0 + G_1^0 c_g = \rho_{c2}^0 c_1 u^0 + \rho_{p2}^0 c_2 u^0 + G_2^0 c_g,$$

while the flux of heat and matter, according to the model adopted for the source, are discontinuous in the reaction zone

$$G_2^0 - G_1^0 = \mu_g m^0,\quad \lambda \frac{dT_1^0}{dx}\Big|_{x=0} - \lambda \frac{dT_2^0}{dx}\Big|_{x=0} = Qm^0. \tag{8}$$

STABILITY ANALYSIS

In order to study the stability of stationary solutions of the system (1)-(5) with respect to small perturbations, we will define the distortion of the reaction front in the form

$$\xi = \varepsilon e^{\omega t + iky},$$

where y is the coordinate tangent to the front; ω, frequency; k, wave number; ε, amplitude of the perturbation.

We seek the nonstationary solutions for T, p, G, and the combustion velocity m(t) in the form of a sum of stationary distributions $T^{\circ}(x)$, $p^{\circ}(x)$, $G^{\circ}(x)$, m°, and small corrections, arising from the perturbation of the front:

$$T = T^0(x) + T'(x)\exp(\omega t + iky), \quad p = p_1^0 + p'(x)\exp(\omega t + iky),$$
$$G = G^0 + G'(x)\exp(\omega t + iky), \; m = m^0 + m'\exp(\omega t + iky).$$

The temperature, pressure, and flux fields must satisfy the following conditions on the surface of the nonstationary front (the derivatives along the normal coincide with the derivatives along x to within the squares of the amplitudes of the front distortions)*:

$$x = \xi, \; p_1 = p_2, \; G_{2x} = G_{1x} + \mu_g m, \tag{9}$$
$$T_1 = T_2, \; \lambda \cdot \partial T_1/\partial x - \lambda \cdot \partial T_2/\partial x = Qm. \tag{10}$$

Linearizing (3'), (4), (5), and (9) relative to small corrections and neglecting, as in the thermal diffusion theory of flame stability [9], the dependence of the transport coefficients (in this case, the permeability $f = k_p/2R\sigma T$) on temperature, it is possible to obtain the distribution of perturbations of the flux and a relation between the amplitudes of the pulsations in flow rate G_1', mass combustion rate m', and the displacement of the front ε:

$$G'_{x1} = G'_1\exp(kx + \omega t + iky), \tag{11}$$
$$k\left[G_2^0 - G_1^0 f_2/f_1\right]\varepsilon = \mu_g m' + G'_1(1 + f_2/f_1),$$
$$f_i = k_{pi}/2R\sigma_i T_i^0, \quad i = 1, 2. \tag{12}$$

Relations (11) and (12) are valid only for front distortions with wavelength much less than the dimensions of the uncombusted (l_1) and combusted (l_2) regions, i.e., when the inequalities $kl_1 \gg 1$ and $kl_2 \gg 1$ are satisfied.

In the case of one-dimensional perturbations, i.e., planar deformations of the heated layer, it follows from the equations of continuity (3') and (9) ($G_y = 0$) that

$$G_{x1} = \text{const}_1(-l_1 < x < 0), \; G_{x2} = \text{const}_2 \; (0 < x < l_2),$$
$$G_{x2} - G_{x1} = \mu_r m, \quad G'_{x2} - G'_{x1} = \mu_g m'.$$

The last of the relations written out, taking into account boundary conditions, determined by the filtrational combustion scheme, establishes a relation between the amplitudes of the perturbations of the combustion velocity and the flow rate, replacing Eq. (12) in the case of planar deformations of the front:

$$k = 0, \quad G'_1 = g_0 m' \mu_g. \tag{13}$$

The coefficient g_0 for the main filtrational combustion schemes equals, respectively:

1) Forced accompanying percolation (the gas flux is given behind the combustion front)

$$G_{x2} \equiv G_2^0, \quad G'_{x2} = 0, \quad g_{01} = -1,$$

2) Forced counter percolation (the gas flux is given in a direction counter to the motion of the combustion front)

$$G_{x1} \equiv G_1^0, \quad G'_{x1} = 0, \quad g_{02} = 0,$$

3) Natural accompanying percolation (the pressure is given on the outer boundary of the layer of product region, and the boundary of the starting matter is impermeable to gas)

$$G_{x1} = G'_{x1} \equiv 0, \quad g_{03} = 0,$$

4) The natural encountering percolation

$$G_{x2} = G'_{x2} \equiv 0, \quad g_{04} = -1,$$

5) Percolation under conditions with a given pressure differential

$$G'_{xi} = \text{const}, \quad p'_1(x = -l_1) = p'_2(x = l_2) = 0, \quad g_{05} = -1/(1 + r_1/r_2),$$
$$r_1 = l_1/f_1, \; r_2 = l_2/f_2.$$

*The continuity of T and p in the reaction zone follows from the finiteness of the heat and matter fluxes; the discontinuity of the gradients on the front is determined by simultaneous integration of (1), (3), and (4) in the reaction zone.

The perturbation of the temperature field T'(x) is determined the linearized equation (1)

$$\lambda\frac{d^2T_i'}{dx^2}-C^0\frac{dT_i'}{dx}-(k^2\lambda+c_1\omega)\,T_i'=c_gG_{xi}'\frac{dT_i}{dx},\quad i=1,2,$$

whose solutions taking into account (11) are decreasing functions at infinity

$$T_1'=A_1e^{z_1x}\frac{c_gG_1'q}{C^0(\Omega-s/2)}\,e^{\frac{x}{l}(1+s/2)},\quad T_2'=A_2e^{z_2x},$$

$$z_1=\frac{1}{2l}\left[1-\sqrt{1+4\Omega+s^2}\right],\quad z_2=\frac{1}{2l}\left[1-\sqrt{1+4\Omega(1-\delta_g)+s^2}\right],\quad (14) \tag{14}$$

$$\Omega=\omega c_1\lambda/(C^0)^2,\quad s=2kl,\quad \delta_g=\mu_gc_g\eta_p^0/c_c,\quad c_1=c_c\rho_0.$$

Satisfying relations (10) in the reaction zone and linearizing them relative to small quantities, we have the equations for determining the constants A_1, A_2, ε, G_1', and m':

$$\frac{A_2}{q}=\frac{A_1}{q}+\frac{\varepsilon}{l}-\frac{c_gG_1'}{C^0(\Omega-s/2)}, \tag{15}$$

$$\frac{A_1}{q}lz_2-\frac{c_gG_1'(1+s/2)}{C^0(\Omega-s/2)}+\frac{\varepsilon}{l}=\frac{A_2}{q}z_2l+Qm'. \tag{16}$$

One more equation, which relates the unknown constants, can be obtained by noting that the main terms in Eq. (1) in the reaction zone are the rate of heat liberation Qw and the derivative $\partial^2T/\partial x^2$. The integrals of these terms over the reaction zone are finite, while the results of integrating the remaining terms does not exceed the square of the amplitude of the perturbation.

The solution of the equation

$$-\lambda\cdot\partial^2T/\partial x^2=Q\rho_0\Phi(T_p)\delta(T-T_p) \tag{17}$$

determines in the linear approximation the distribution of the gradient $\partial T/\partial x$ in the heat liberation zone.* Using the formal rule for substitution of variables

$$\delta(T-T_p)=\delta(x-\xi)/|\partial T/\partial x|,\ T_p=T(\xi),$$

let us integrate the upper equation over the reaction zone

$$x=\xi:\ \lambda\left[\left(\frac{\partial T_1}{\partial x}\right)^2-\frac{\partial T_2}{\partial x}\left|\frac{\partial T_2}{\partial x}\right|\right]=2Q\rho_0\Phi(T_p). \tag{18}$$

The second term on the left side of the equality is a second order infinitesimal and can be dropped. Relation (18) replaces the assumption usually used as to the quasistationary nature of the combustion velocity of matter in a nonstationary front, which does not take into account the perturbations of the source, owing to the nonstationary nature of the gradients in the reaction zone.†

Substituting the temperature distributions (14) into Eq. (18) obtained above and taking into account the stationary relations (6)-(8), we have:

$$\frac{\varepsilon}{l}+\frac{A_1}{q}z_1l-\frac{c_gG_1'}{C^0}\frac{1+s/2}{\Omega-s/2}=k\frac{A_2}{q},\quad k=\frac{\Phi_T'(T_p^0-T_0)}{2\Phi(T_p^0)}=q\frac{d\ln m^0}{dT_p^0}. \tag{19}$$

The temperature coefficient of the combustion wave k, which characterizes the nonlinear properties of the combustion wave, is one of the main parameters determining the stability of the stationary propagation of the reaction front. For real sources, the value of k can be determined according to the experimental or theoretical dependence $m^0(T^0_p)$. In the simplest case of a zeroth-order reaction ($w = k_0e^{-E/RT}$), k and m^0 have the form

*In order to avoid misunderstandings, related to the apparent uncertainty in $\partial T/\partial x$ in the presence of a δ source, we point out that this discrepancy is easily removed if we take into account the fact that the δ source can be interpreted as a bounded function, localized in a finite, arbitrarily small region, with continuous distribution of the temperature gradient. We note in this connection that the result (18) is easy to obtain by approximating the δ source by a step function, solving Eq. (17) and then passing to the limit of an infinitely narrow reaction zone.

†The thermal diffusion stability of a laminar flame taking into account the boundary condition (18) is studied in [10].

$$m^0 = \text{const}\sqrt{e^{-E/RT_p^0}R\,(T_p^0)^2/E},\quad k \simeq \frac{E\,(T_p^0 - T_0)}{R\,(T_p^0)^2}\left(1 + 2\frac{RT_p^0}{E}\right).$$

Relations (12), (15), (16), and (19) form the system of equations for calculating the unknown coefficients A_1, A_2, ε, ε_1', and m'. The equation closing this system is determined by the propagation regime of the combustion wave.

KINETIC REGIME FOR PROPAGATION OF A FILTRATIONAL COMBUSTION WAVE

In the kinetic combustion regime, the starting condensed matter is completely used up in the reaction zone ($\eta_p = \eta(x > \xi) \equiv 1$), while the oxidizer occurs on both sides of the front and does not limit the process. For a zeroth-order reaction with respect to the oxidizer, the diffusion equation separates from the general system and does not explicitly participate in the subsequent analysis, merely limiting the region of applicability of the results by the condition for realization of the kinetic combustion regime [8]

$$G^0 a_0 > u^0 \rho_0 \mu,$$

where G^0 is the flow rate of the gas entering the reaction zone; a_0, concentration of the oxidizer far from the front.

The mass rate of combustion is related uniquely to the velocity of the reaction front u relative to the uncombusted matter:

$$m = \rho_0 u,\ u = -d\xi/dt,\ m' = -\varepsilon\omega\rho_0. \tag{20}$$

The requirement that there be a nontrivial solution to the system of homogeneous equations (12), (15), (16), (19), and (20) leads to the dispersion relation

$$z_{20} - 2k\,(1 + \Omega_1) + z_{10}\left(\Omega_1 + k\frac{1+g_2}{1+g_1}\right) - z_{10}z_{20}\frac{1+g_2}{2\,(1+g_1)} = 0,$$

$$\Omega_1 = \frac{\Omega}{1-g_1}\frac{C^0}{c_1u^0},\quad z_{10} = 1 + \sqrt{1 + 4\Omega + s^2},\quad z_{20} = 1 - \sqrt{1 + 4\Omega\,(1-\delta_g) + s^2}, \tag{21}$$

$$g_1 = g\,\frac{\delta_g\,(1 + s/2)\,\Omega}{(\Omega - s/2)},\quad g_2 = g\,\frac{\delta\Omega}{\Omega - s/2},$$

For front distortions (s > 0), the quantity g is determined by the expression

$$g = -\frac{f_1c_1u^0}{2\,(f_1 + f_2)\,C^0\Omega}\left\{s + 2\Omega\frac{C^0}{c_1u^0} + \frac{G_1^0}{\mu_g m^0}\left(1 - \frac{f_2}{f_1}\right)s\right\}.$$

In the case of one-dimensional perturbations, Eq. (12) was replaced by (13). The form of the dispersion relation (21), in this case, does not change and the quantity g depends on the scheme of the combustion process and equals one of the values calculated previously.

Stability of a Plane Combustion Wave (s = 0). Let us analyze (21), limiting ourselves to small values of the parameter $\delta_g = \mu_g c_g/c_c$, i.e., weak absorption (liberation) of gas in the reaction computed per gram of initial substance ($|\mu_g| \ll 1$).

In the linear approximation with respect to δ_g, Eq. (21) for the frequency of the perturbation Ω takes the form (s = 0)

$$\begin{gathered}\Omega^2 + b_1\Omega + b_2 = 0,\\ b_1 = -k_1^2 + k_1\,[(4\alpha - 1) - \alpha\delta_g] + \alpha\,(1-\alpha) + \alpha^2\delta_g,\\ b_2 = \alpha\left[k_1^2(\alpha - 1) + k_1\alpha\right],\quad k_1 = k - 1,\\ \alpha = u_0\,(c_1 + c_g\mu_g\rho_0 g_{0i})/\left(u^0c_1 + c_gG_1^0\right).\end{gathered} \tag{22}$$

Together with the quantity g_{0i}, the value of α is determined not only by the characteristics of the system, but also by the percolation regime. For natural percolation, regulated by the gas consumption in the reaction zone, $\alpha = 1$, both for accompanying ($g_{03} = 0$, $G_1^0 = 0$) and for counter ($g_{04} = -1$, $G_1^0 = -\mu_g m^0$) oxidizer fluxes.

For forced percolation (G_0 is the absolute value of the given gas flux) with an accompanying flow, we have

$$g_{01} = -1,\quad G_2^0 = -G_0,\quad G_1^0 = G_2^0 - \mu_g\rho_0u^0,\quad \alpha = \alpha_1 = \frac{c_2u^0}{c_2u^0 - c_gG_0},$$

and with a counter flow $G_1^0 = G_0, \quad g_{02} = 0, \quad \alpha = \alpha_2 = c_1 u^0/(c_1 u^0 + c_g G_0).$

Taking into account the relation between the oxidizer flux and the velocity of propagation of the front in the kinetic regime, as well as the restriction on the parameter α, related to stalling of combustion in a counter flow [8], we will indicate the range of variation of α in accompanying (α_1) and counter (α_2) forced percolation with given flow rate G_0

$$\alpha_1 > 1/(1-\delta_p),\ 1/(1+\delta_0) > \alpha_2 > k_1/(1+k_1),$$
$$\delta_p = c_g\mu/a_0 c_p \mu_p,\ \delta_0 = c_g\mu/c_c a_0.$$

Under conditions of fixed pressure differential,

$$g_0 = g_{05} = -\frac{r_2}{r_1+r_2}, \quad \alpha = \alpha_5 = \frac{1 - \dfrac{c_g\mu_g\rho_0 r_2}{c_1(r_1+r_2)}}{1 + c_g G_1^0/u_0 c_1}.$$

Expressing G_1^0 in terms of the pressures p_{10} and p_{20} on the outer boundaries of the porous medium taking into account the discontinuity of the fluxes on the combustion front

$$G_1^0 = \left(p_{10}^2 - p_{20}^2 - r_2\mu_g m^0\right)/(r_1 + r_2),$$

we obtain finally

$$\alpha = \alpha_5 = \frac{1 - \dfrac{\delta_g r_2}{r_1 + r_2}}{1 - \dfrac{\delta_g r_2}{r_1+r_2} + \dfrac{p_{10}^2 - p_{20}^2}{r_1+r_2}\cdot\dfrac{\delta_g}{m^0}}.$$

As for a given flow rate, the counter flow ($p_{10} > p_{20}$) corresponds to values $\alpha < 1$ and the accompanying flow ($p_{10} < p_{20}$) corresponds to values $\alpha > 1$.

Substituting $\Omega = i\psi$ into (22), we have an equation that determines the region of stability ($k < k^*$) of combustion

$$(k_1^*)^2 - k_1^*[(4\alpha - 1) - \alpha\delta_r] - [\alpha(1-\alpha) + \alpha^2\delta_g] = 0.$$

For large values of k^*, it is possible to set approximately

$$k^* = k_1^* + 1 \approx \alpha(4 - \delta_g).$$

With forced percolation, the accompanying flux ($\alpha = \alpha_1 > 1$) stabilizes, while the counter flux ($\alpha = \alpha_2 < 1$) destabilizes the combustion front. For natural percolation, the direction of the flux has no effect on the stability of combustion ($\alpha_3 = \alpha_4 = 1$). The region of stationary regimes is enlarged for reactions with gas absorption ($\delta_g < 0$) and narrows with gas liberation in the front ($\delta_g > 0$). The transition through the critical value $k = k^*$ into the unstable region ($k > k^*$) is accompanied by oscillations in temperature and combustion velocity with frequency

$$\psi = \sqrt{(k-1)^2\alpha(\alpha-1) + (k-1)\alpha^2}.$$

The exponential development of the instability ($\mathrm{Im}\,\Omega = 0$) begins at values of k exceeding k^{**} determined by the equation

$$\Delta = b_1^2(k^{**}) - 4b_2(k^{**}) = 0.$$

The pulsating character of the front propagation in this case remains, as numerical calculations of the nonstationary combustion of gas-free systems in similar situations show [11-13]. For $\alpha = 1/(k-1)$, one of the roots of the characteristic equation (22) vanishes. The stationary solutions of system (1)-(5) are lost at the same time, i.e., there is a disruption of combustion [8].

The stability of the combustion front to distortions with wavelength much greater than the thickness of the heated layer ($s \to 0$) is determined by the same equation (22) as the stability of a planar front. The difference between these two cases is in the value of the quantity α. In the case of long wavelength distortions ($s \to 0$)

$$g = g_{s0} = -f_1/(f_1 + f_2), \quad \alpha_{s0} = \frac{c_1 u^0}{C^0}\left(1 - \frac{f_1}{f_1+f_2}\delta_g\right).$$

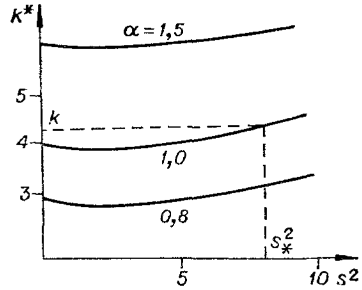

Fig. 1. The critical value k* as a function of the wave number for the kinetic combustion regime $\delta_g = 0$. For $s > s_*(k)$, the distortions are damped, and the planar front (s = 0) is unstable.

The nature of the loss in stability of stationary combustion is determined by the ratio α_{s0}/α, which depends on the parameter δ_g and the percolation regime.

For accompanying natural and counter forced percolation of gas

$$\alpha_{s0}/\alpha = 1 - f_1\delta_g(f_1 + f_2)^{-1}.$$

Since an increase in α leads to an increase in stability, in the cases being examined, destabilization of the wave begins with oscillations of the planar front with $\delta_g > 0$ (liberation of gas) and with distortion of the surface by traveling waves with $\delta_g < 0$. For accompanying forced and counter natural percolation

$$\alpha_{s0}/\alpha = [1 - f_1\delta_g(f_1 + f_2)^{-1}](1 - \delta_g)^{-1},$$

the situation is opposite to the preceding case: gas liberation in the front leads to two-dimensional instability and absorption of gas leads to one-dimensional instability. For a fixed pressure differential, the nature of the losses in stability is determined not only by the quantity δ_g, but also by the ratio of the sizes of the combusted and uncombusted regions

$$\alpha_{s0}/\alpha = [1 - f_1\delta_g(f_1 + f_2)^{-1}][1 - \delta_g f_1 l_2(f_1 l_2 + f_2 l_1)^{-1}].$$

The starting stage of combustion in an accompanying flux ($l_2 < l_1$) corresponds to two-dimensional instability with $\delta_g > 0$ and one-dimensional instability with $\delta_g < 0$. In the counter flux, the opposite inequalities hold.

Spectrum of Increments. For identical permeabilities $f_1 = f_2$ and negligibly small liberation of gas on the reaction front ($\mu_g = 0$), it is possible to construct the spectrum of increments of perturbations of different wavelengths. The dispersion relation (21) in this case is given by

$$\begin{gathered}\Omega^3 + B_2\Omega^2 + B_1\Omega + B_0 = 0,\\ B_2 = -k_1^2 + k_1[(4\alpha - 1)] + \alpha(1-\alpha) + s^2/4,\\ B_1 = k_1^2(\alpha - 1)\alpha + k_1\alpha(s^2 + \alpha) - \frac{\alpha s^2}{4}(2\alpha - 1),\\ B_0 = \frac{\alpha^2 s^2}{4}\left(k_1^2 + k_1 - \frac{s^2}{4}\right).\end{gathered} \tag{23}$$

Equation (23) was analyzed numerically. As in the case of gas-free systems [10, 11, 14], in the presence of distortions, the loss of stability of the filtration combustion front occurs for lower values of k than in a planar front: the function k*(s) has a weak minimum (Fig. 1).

The spectrum of increments $\Omega(s^2, k)$ for different values of α is shown in Fig. 2. The nature of the spectrum is the same for all three cases. The loss of stability begins with the excitation of long (an order to magnitude greater than the heated layer) traveling waves on the combustion surface. The increment of planar perturbations of the front increases rapidly with distance from the limit of stability. Near the boundary of exponential instability, the rate of development of autooscillations of the planar front is only insignificantly lower than the rate of growth of the distortions.

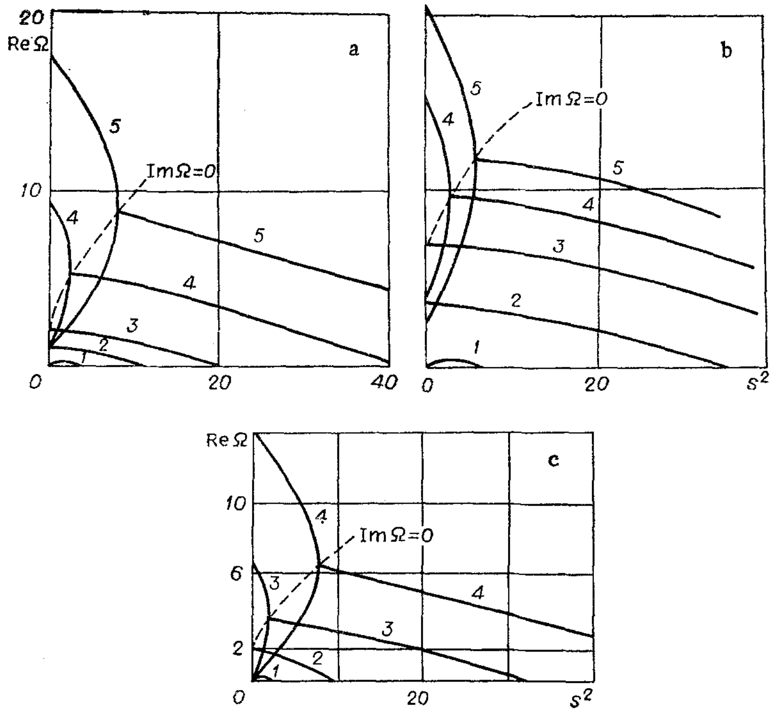

Fig. 2. The increment as a function of the wavelength of the perturbation; the kinetic regime $\delta_g = 0$. a) Natural percolation $\alpha = 1$; b) forced accompanying percolation, $\alpha = 1.5$ (k = 5.85 (1), 7 (2)-(3), 8.5 (4), and 9 (5)); c) forced counter percolation $\alpha = 0.8$ (k = 3.3 (1), 3.8 (2), 5 (3), and 6 (4)).

Under conditions when the increments of two harmonics are practically equal, the quasi-steady-state combustion regime may turn out to be very sensitive to the initial perturbation of the front. It is possible that perturbations in the main mode in the form of planar deformations of the combustion wave (which usually occurs with ignition) will lead to one-dimensional pulsations of the front [12], while the predominance of the amplitude of rapidly growing distortion in the spectrum of the initial perturbation will give rise to the traveling waves on the combustion surface [15].

For a front of extent l_f less than the critical value $l_*(k) = 4\pi l s_*^{-1}(k)$, where s_* corresponds to the boundary of unstable distortions for a given value of k (see Fig. 2a), l is the thickness of the heated layer, the nonstationary nature is realized in the form of auto-oscillations that are coherent over the cross section of the front, since the distortions damp out. With an increase in size $l_f > l_*$ for corresponding initial distortion of the front, waves traveling along the combustion surface can appear. The presence of a distortion with maximum growth rate in the perturbation spectrum must lead to a successive increase in the traveling wave numbers with an increase in the extent of the front.

STABILITY OF THE PERCOLATION REGIME FOR PROPAGATION OF A COMBUSTION WAVE IN A POROUS MEDIUM

In filtration combustion regimes, the gaseous oxidizer is completely consumed in the reaction zone and the rate of burnup of the condensed substance is limited by the input of the gaseous reagent.

We obtain the equation necessary for closing the system (12), (15), (16), and (19) by balancing the oxidizer fluxes in the reaction zone. For percolation velocities higher than the diffusion velocities ($v/v_0 \sim lG_0/D\rho \gg 1$, l is the scale of the percolation zone), diffusion processes, localized in a narrow boundary layer near the reaction zone can be neglected and the reaction zone can be viewed as a surface of discontinuity in the concentration. Dropping the diffusion terms in Eq. (2) and integrating it together with (3) and (4) in the vicinity of the reaction zone, we obtain the balance relations on the nonstationary front

$$x = \xi:\ m = \mu^{-1}(G_1 a_1 - G_2 a_2), \quad m = \rho_0 u \eta_p, \qquad (24)$$
$$m = \mu_g^{-1}(G_2 - G_1).$$

The oxidizer concentrations in front of the reaction front (a_1) and behind it (a_2) are related by the condition of total consumption of the gaseous reagent $a_1a_2 = 0$. A nonzero value of the concentration corresponds to an oxidizer content a_0 in the starting mixture. Eliminating the quantity G_2 from Eqs. (24) and linearizing the relation obtained between the burnup rate m and the flux G_1, we obtain the missing equation for closing the system (12), (15), (16), and (19):

$$m' = G_1'\mu_g\frac{a_1 - a_2}{\mu_g a_2 + \mu}. \tag{25}$$

Substituting (25) into (12), we have

$$G_1' = \xi l^{-1}s\left[G_2^0 - G_1^0(f_2/f_1)\right]\Big/\left[(f_2/f_1) + \frac{\mu_g a_1 + \mu}{\mu_g a_2 + \mu}\right]. \tag{26}$$

Eq. (26) permits a transition from two-dimensional to one-dimensional (s = 0) perturbations. The same result (G'_1 (s = 0) = 0) is obtained by directly analyzing the one-dimensional stability.

The dispersion relation that ensures a nontrivial solution to the system of homogeneous equations (15), (16), (19), (25), and (26) has the form

$$\begin{gathered} 2k\left[1 - \beta s - z_1(1 - \beta_3)\right] = z_{10}(\beta_2 s - \beta s) + z_{20}(1 - \beta_2 s) - 0.5z_{10}z_{20}\cdot(1 - \beta_3), \\ \beta = \beta_0\left(1 + \beta_1\frac{1 + s/2}{\Omega - s/2}\right), \quad \beta_2 = \beta_0\beta_1\frac{1 + s/2}{\Omega - s/2}, \quad \beta_3 = \beta_2 s/(1 + s/2), \\ \beta_0 = \frac{1}{2}\,\frac{1 + G_1^0(1 - f_2/f_1)\left(\mu_g m^0\right)^{-1}}{1 + (1 + f_2/f_1)\dfrac{\mu_g a_2 + \mu}{\mu_g(a_1 - a_2)}}, \quad \beta_1 = \frac{c_g q}{Q}\frac{\mu_g a_2 + \mu}{a_1 - a_2}. \end{gathered} \tag{27}$$

For s = 0, there is a single root $\Omega = 0$, indicating that the planar combustion front is always stable to within the displacement.

The stability of the front relative to distortions is determined by the parameter β, which characterizes the conditions for input of oxidizer to the reaction zone and the quantity β_1, which determines the thermophysical action of the gas flux on the front distortion. We will present values of the parameters β_0 and β_1 for different combustion schemes in the case when there are no gaseous reaction products ($\mu_g = -\mu$):

1) natural counter percolation ($G_2^0 = 0$, $a_2 = 0$, $a_1 = 1$):

$$\beta_0 = -1/2,\ \beta_1 = \delta_g/(1 + \delta_g),\ \delta_g = \mu c_g \eta_p^0/c_c,$$

2) natural accompanying percolation ($G_1^0 = 0$, $a_2 = 1$, $a_1 = 0$):

$$\beta_0 = 1/2,\ \beta_1 = 0,$$

3) forced counter percolation ($a_2 = 0$, $a_1 = a_0$, $f_1 = f_2$):

$$\beta_0 = -a_0/2(2 - a_0),\ \beta_1 = \delta_g/(a_0 + \delta_g),$$

4) forced accompanying percolation ($a_1 = 0$, $a_2 = a_0$, $f_1 = f_2$):

$$\beta_0 = a_0/2(2 - a_0),\ \beta_1 = \delta_g(1 - a_0)/[\delta_g(1 - a_0) - a_0].$$

Equation (21) has real roots. Substituting the value $\Omega = 0$ into the expression for z_{10} and z_{20} gives a relation between the wave parameters on the stability boundary. For small δ_g (heat capacity of the gas is insignificant), the main parameter that determines stability is β_0.

Figure 3 shows the critical values of the temperature coefficient k* on the boundary of stability of the combustion front to distortions with scale s for different values of the parameter β_0. The instability regions ($\Omega > 0$, $k > k^*$) exist only in the case of negative values of β_0; for $\beta_0 \geqslant 0$, the combustion front is stable not only to planar deformations but also to distortions.

For $\beta_1 = 0$ ($\delta_g = 0$), the frequency of the perturbations Ω is determined by the expression

$$\Omega_{1,2} = 0.25(-b \pm 2f\sqrt{(k - \beta s)^2 - 4\beta s(k - 1)}),$$

where $b = 1 + s^2 + (2k - 1)(2\alpha s - 1) - 2f^2$, $f = 1 + \beta s - k$.

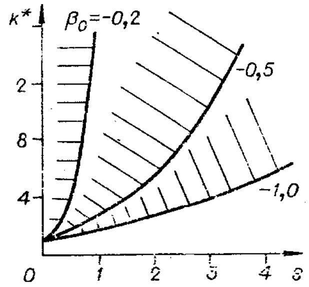

Fig. 3

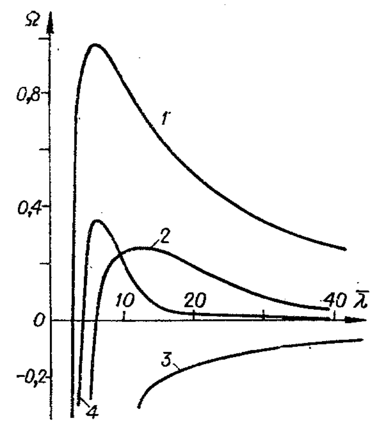

Fig. 4

Fig. 3. Boundary of stability $k^*(s)$ for different values of β_0. Percolation regime; $\delta_\Sigma = 0$. The regions of unstable combustion ($k > k^*$) are shaded.

Fig. 4. The frequency Ω as a function of the wavelength of the perturbation $\bar{\lambda}$. Percolation regime: $\delta_g = 0$; $\beta_0 = -1$ (1), -0.5 (2, 4), 0 (3); k = 5 (1-3), 10 (4).

The dependence of the increment on the wavelength of the perturbation in the region of instability has a sharp maximum (Fig. 4) for particular scale of distortion $\bar{\lambda}_M(k)$, expressed in units of the heated layer of the stationary combustion wave (l). In analogy to the thermal diffusion flame with L > 1, which has a similar spectrum of increments, for unstable percolation regimes ($\beta_0 < 0$), the formation of cellular structure in the combustion wave with characteristic scale $l_M = \bar{\lambda}_M l$ should be expected [16].

The results of the analysis presented above are confirmed by available numerical solutions of the system of nonstationary equations of filtration combustion (1)-(5) in a one-dimensional formulation [17, 18]. Two-dimensional problems of filtrational combustion have not been investigated numerically.

It is interesting to compare the results of the analysis presented here with the results in [19], published after the present paper was written. The calculation presented in [19] generalizes the short-wavelength [4] and long-wavelength [20] asymptotic analysis of the stability of filtrational combustion for one of the variants of the process examined above: for counter, natural percolation of the oxidizer. In several respects, the results in [19], obtained assuming a quasistationary velocity of the combustion wave, disagree considerably with the results of the present analysis.

For the kinetic propagation regime, in both cases, the oscillatory instability of the reaction front is obtained, but the conclusion in [19] as to the destabilizing role of the gas flux (for one-dimensional and two-dimensional perturbations) contradicts both the results of the stability calculation presented above and the results of the numerical experiment in [5, 17].

Opposite conclusions are also obtained for the problem of stability to distortions of the reaction front with incomplete burnup of the solid reagent. In contrast to the conclusions in [19] concerning the stability of such a regime, in the present work, we concluded that the planar combustion wave with incomplete transformation of the porous reagent is absolutely unstable. We note in this connection the experimental data presented in [21]. For counter percolation of the oxidizer, in regimes with incomplete transformation, the burnup of the solid reagent was observed to be nonuniform over the cross section of the specimen. For accompanying percolation of the oxidizer, no nonuniformities were observed on the front. This picture agrees completely with the results of the stability analysis presented above.

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COMBUSTION OF GASEOUS SUSPENSIONS OF METAL POWDERS (THREE-ZONE MODEL)

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In recent years the ignition and combustion of gaseous suspensions of metal powders has been studied intensely. A knowledge of the principles governing these processes is important in a number of fields of the national economy. Despite the practical importance of this field, many problems are still unsolved.

As a rule, in real processes in gaseous suspensions of metal powders, ignition and combustion occur at high pressures, and the effect of pressure on combustion characteristics, as experiments reveal, produces a number of peculiarities, some of which are still unexplained in available models. Moreover, existing calculation methods do not permit a description of the ignition and combustion of gaseous suspensions within the framework of a single model, which leads to significant errors, especially in the calculation of transitional regimes, the stage of flame front formation, etc.

The present study is dedicated to an analytical description of the processes of ignition and combustion of gaseous suspensions of metal particles. We will consider the basic assumptions of the model to be used. It is assumed that the particles are of identical initial size, that the distance between particles remains constant during ignition and combustion, that the processes of heat and mass exchange are spherically symmetric, quasistationary, and occur within the limits of the reduced film.

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