RELATIONS AMONG STABILITY PARAMETERS IN THE SURFACE LAYER

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Abstract. Observations at 5 sites have been analyzed to determine relations among variables used to assess the effect of stability on dispersion. The variables compared were Monin-Obukhov length, Richardson number, bulk Richardson number and Pasquill and Turner categories. All the relationships differ with different terrain roughness.

1. **Introduction**

Pasquill's method for diffusion was developed to estimate vertical and lateral plume widths from a continuous source at various distances. These widths depend primarily on σ_{ϕ} and σ_{θ} , the standard deviations of vertical and horizontal wind direction fluctuations, respectively. According to Monin-Obukhov similarity theory, assuming horizontal homogeneity and steady state conditions, these quantities are functions of *z/zo* and z/L where z is the height, z_0 the roughness length and L the Monin-Obukhov length defined by:

$$
L = -\frac{u^3 \varphi \rho T}{kgH}.
$$
\n⁽¹⁾

Here c_p is the specific heat of air at constant pressure, ϱ is the air density, k is von Karman's constant, *T* is the absolute air temperature, g is the acceleration of gravity, *H* is the vertical heat flux and u_* is the friction velocity. The relations can be shown to have the form (see e.g., Panofsky and Prasad, 1965):

$$
\sigma_{\phi} \cong \frac{\sigma_w}{u} = \frac{k\phi_3(z/L)}{\left[\ln z/z_0 - \psi(z/L)\right]}
$$
\n(2)

and
\n
$$
\sigma_{\theta} \cong \frac{\sigma_{v}}{u} = \frac{k\phi_{2}(z/L)}{\left[\ln z/z_{0} - \psi(z/L)\right]}
$$
\n(3)

where σ_w and σ_v are the standard deviations of vertical and horizontal wind speed fluctuations, respectively, *u* is the mean wind speed and ϕ_2 , ϕ_3 and ψ are universal functions. Since u_* and *H* are taken as independent of height in the surface boundary layer, *L* can also be assumed constant with height in this layer.

Similarity theory also states that *z/L* is a function of the Richardson number only, which is defined by:

$$
Ri = \frac{g}{T} \frac{\partial \theta / \partial z}{(\partial u / \partial z)^2}
$$
 (4)

Boundary-Layer Meteorology 3 (1972) 47-58. *All Rights Reserved Copyright ©* 1972 *by D. Reidel Publishing Company, Dordrecht-Holland* where θ is the potential temperature and other quantities have been previously defined. The Richardson number can be measured more easily than *z/L,* but its vertical distribution is not known *apriori.*

However, even the Richardson number often is not available since it requires accurate wind measurement at two levels. Therefore, simpler substitutes for *z/L* have been developed which are related to *Ri* and therefore to *L.* For example, the bulk Richardson number is a nondimensional stability ratio that uses temperature at two levels but requires wind speed at only one level, and therefore can be computed more accurately from less sophisticated measurements. The bulk Richardson number (Lettau and Davidson, 1957) is defined by:

$$
B = \frac{g}{T} \frac{\partial \theta / \partial z}{u^2} \bar{z}^2.
$$
 (5)

Here \bar{z} is usually taken as the geometric mean height between top and bottom of the layer considered.

Where temperature profile measurements are not available, Pasquill (1962) has suggested a set of six stability classes specified in terms of wind speed and insolation only. Turner (1964) has modified Pasquill's scheme and has defined seven stability categories. In this method, net radiation is estimated from a knowledge of solar altitude and existing conditions of cloud cover and ceiling height. Both the Pasquill and Turner classes are independent of height and surface roughness. Therefore, roughness must be taken into account when relating these categories to *L,* which depends on wind, stability and roughness length.

In this paper, relations will be studied between *z/L* and the more easily determined quantities used in estimating atmospheric dispersion.

2. Theoretical Relations Between Stability Parameters

In unstable air, the Pandolfo-Businger hypothesis is a good approximation to the relation between *Ri* and *z/L.* It states (see, e.g., Paulson, 1970):

$$
1/L = Ri/z. \tag{6}
$$

For stable air, an empirical relation found by McVehil (1964) has been confirmed by others:

$$
1/L = \frac{Ri}{z(1 - \beta Ri)}.
$$
\n(7)

Recent analysis by Webb (1970) and Businger *et al.* (1971) have given β a value near 5, while Cermak and Arya (1969) determined that $\beta = 10$ in the wind tunnel. McVehil suggests that β may vary according to site location and assigns β a value of 7, which was also used in this investigation.

The bulk Richardson number is related to *Ri* through:

$$
Ri = \frac{Bu^2}{\left[\partial u/\partial (\ln z)\right]^2}.
$$
 (8)

In order to eliminate *u* from Equation (8), we introduce the nondimensional wind shear (kz/u_{\star}) ($\partial u/\partial z$) which is a function of *Ri* only:

$$
\phi(Ri) = \frac{kz}{u_{*}} \left(\frac{\partial u}{\partial z}\right).
$$
\n(9)

Integration of Equation (9) gives (see, e.g., Panofsky, 1963):

$$
u = \frac{u_*}{k} \left[\ln z / z_0 - \psi \left(Ri \right) \right],\tag{10}
$$

where $\psi(Ri)$ is another 'universal function'. Then, upon substitution of Equations (9) and (10), Equation (8) becomes:

$$
Ri = B \left[\frac{\ln z/z_0 - \psi(Ri)}{\phi(Ri)} \right]^2.
$$
 (11)

Under unstable conditions values of ψ and ϕ are quite well known. Paulson (1970) suggests:

$$
\phi(Ri) = (1 - 16 Ri)^{-1/4} \tag{12}
$$

and derives:

$$
\psi = 2 \ln \left[(1 + X)/2 \right] + \ln \left[(1 + X^2)/2 \right] - 2 \tan^{-1} X + \pi/2 \tag{13}
$$

where

$$
X = (1 - 16 Ri)^{1/4}.
$$
 (14)

Under stable conditions, Equation (7) leads to:

$$
\psi(Ri) = -\frac{\beta Ri}{(1 - \beta Ri)}
$$
\n(15)

and

$$
\phi(Ri) = \frac{1}{(1 - \beta Ri)}.
$$
\n(16)

We have implied here that K_h/K_m , the ratio of the exchange coefficients for heat and momentum, is near unity in stable air, an assumption in good agreement with recent measurements (Webb, 1970).

With values of ψ and ϕ , a nomogram for *Ri* as a function of *B* and ln z/z_0 can be constructed. Such a nomogram, first devised by Prasad (1967), has been extended along the abscissa to include larger values of *Ri,* while the ordinate has been increased to include values of In *z/z ^o* between 1.0 and 10.0. Richardson numbers determined from *B* and $\ln z/z_0$ by the use of the nomogram will be called calculated Richardson numbers, Ri_c . In these computations, $\psi(Ri)$ for unstable air was taken from a tabulation by Hansen (1967) which differs slightly from Paulson's suggestion, being based on the paper by Panofsky (1963). Figures 1 and 2 give the results.

Both Pasquill and Turner use estimates of insolation and wind, in order to arrive at diffusion parameters, which theoretically depend also on z/z_0 . However, *L* depends

Fig. 1. Unstable *Ri* as a function of *B* and $\ln(z/z_0)$. (Isopleths of *B* have been multiplied by -100.)

on heat flux, wind, and roughness length, so that any relationship between the Turner or Pasquill categories and *L* must differ with different roughnesses. Further, no exact equivalence is likely, because the heat flux, required for the estimation of *L,* does not depend on insolation alone.

Whereas it has been possible to connect *z/L, Ri* and *B* analytically, the relationship between *L* and the Pasquill and Turner categories will be treated empirically. There is no guarantee that such empirical relations will be universal, for the relation between *H* and insolation varies with time and place, a fact which has not been taken into account.

3. Observations and Data Handling

The data used in this investigation were obtained from five sites: Kerang, Australia; South Dartmouth, Massachusetts; O'Neill, Nebraska; Hanford, Washington; and Cape Kennedy, Florida. In all cases, the mean wind and temperature at several levels were available, as well as insolation. Also, the roughness length had been estimated previously.

The area around Kerang, Victoria, Australia is homogeneous, covered by an annual grass which during observation times was approximately 5 cm high. The corresponding roughness length, z_0 , was 0.2 cm (Swinbank, 1964). A small group of trees located 3 km to the south-southwest constituted the closest obstacle.

Platinum wire resistance bridges were used to measure air temperature differences over the intervals 0.5-1, 1-4, 4-8, and 4-16m. Wind speeds were determined by the use of cup anemometers mounted on 1.5-m horizontal arms supported on a 18-m tower

Fig. 2. Stable *Ri* as a function of *B* and $\ln(z/z_0)$. (Isopleths of *B* have been multiplied by 100.)

at heights of 0.5, 1, 2, 4, 8, and 16m. Continuous records of the net vertical fluxes of long- and short-wave radiation were obtained from a polyethylene-shielded radiometer mounted 2 m above the ground. For a more detailed description of the instrumentation, recorders, and the observations, see Swinbank (1964).

The South Dartmouth data were collected by a research team at the Massachusetts Institute of Technology's Round Hill Field Station. The site is located in relatively smooth nonhomogeneous terrain consisting of trimmed grass 5 to 10 cm high. The grass extends in a 100-m radius from the tower to the west and north. To the south and east, the site is bounded by Buzzards Bay. There is a tidal marsh located 300 m to the west of the tower. The roughness length at South Dartmouth is rather complex due to the nonhomogeneous terrain. A value of 0.5 cm (Panofsky and Townsend, 1964) is valid over the grass up to a height of 10 m at the most; at heights greater than 10 m, the effective roughness length increases greatly.

Measurements of wind speed were made with conventional 3-cup anemometers at heights of 0.25, 0.5, 1.0, 2.0, 4.0, 16.0, 32.0 and 40.0 m. Temperature measurements

were also made at the anemometer levels by means of resistance elements. An Eppley pyranometer measured solar radiation (Cramer and Record, 1969).

Two sites near O'Neill, Nebraska were used in two extensive field studies: The Great Plains Turbulence Field Program in 1953 (Lettau and Davidson, 1957) and Project Prairie Grass, a field program in diffusion in 1956 (Barad, 1958). The areas included about 2 km^2 of flat terrain, each covered with short mown grass.

The data used from the Great Plains Project were those observed by personnel of the Johns Hopkins University. Air temperatures were measured with copper-constantan thermocouples supported in radiation shields mounted on a tower at 0.1, 0.2, 0.4, 0.8, 1.6, 3.2, and 6.4 m above the ground. Anemometers with semi-cylindrical cups were mounted on a slender mast at the thermocouple levels with the exception of the 0.1- and 0.2-m levels. Radiation measurements were made with an Eppley pyranometer.

Two sets of z_0 's were found, one set for the lowest four anemometer levels and the other for the four upper anemometer levels, during the observational periods from 2 August to 8 September 1953. The roughness length varied from 0.6 to 1.3 cm and the appropriate value of z_0 was taken to be the mean value for the mast as a whole, 0.8 cm (Davidson and Barad, 1956).

Project Prairie Grass was conducted during the summer of 1956 under a variety of meteorological conditions. The micrometeorological data were collected by Texas A & M personnel. Anemometers at heights of 0.25, 0.5, 1.0, 2.0, 4.0, 8.0, and 16.0 m sensed the wind speeds, while radiation-shielded copper-constantan thermocouples were located at similar heights for temperature measurements. An Eppley pyranometer and a Gier and Dunkle net exchange radiometer were employed to gather radiation data. Barad (1958) determined a roughness length of 0.6 cm.

During the summer of 1959, the Green Glow Diffusion Program was conducted at the Hanford Reservation, Washington. According to Barad and Fuquay (1962), the site is located in a semi-arid basin surrounded by elevated terrain. In the immediate area of the observation site, vegetation of sagebrush 1 to 2 m high and openings covered by desert grass in relatively flat terrain combine to give a z_0 of 3.0 cm. This value of z_0 was determined during previous research at The Pennsylvania State University.

Personnel of the Hanford Laboratories used a portable meteorological mast with modified 3-cup anemometers and copper-constantan thermocouples to measure wind speed and temperatures at 0.8, 1.5, 3.0, 6.1, 12.2, and 24.4 m. All observations were made during stable conditions, after 2100 LST.

The Merritt Island Launch Area at Cape Kennedy, Florida, serves as the Observational site for turbulence studies in the vicinity of the Apollo/Saturn V launch pads. Two towers are located approximately 5 km from the Atlantic Ocean, one 18 m, the other 150-m high. The 18-m tower has Climet wind sensors at 3, 10, and 18 m and Climet aspirated thermocouples at the 3 and 18 m levels. The larger tower has the same types of wind instruments located at heights of 18, 30, 60, 90, 120, and 150 m and the same types of temperature instruments to measure lapse rates (the 3-m level is the reference level) at the 30-, 60-, 120-, and 150-m levels.

For the purpose of this investigation, the area within 1-km radius is separated into three different roughness zones according to wind direction. The first zone consists of two subregions: 180-230 degrees includes a body of water; 230-315 deg is characterized by thick woods. The area from 315 (through 360) to 90 deg is characterized by open grassland, and the remaining sector (090-180 deg) consists of low trees and hemlock. More details relating to the site and instrumentation can be found in a report by Kaufman and Keene (1968).

According to Blackadar and Panofsky (personal communications) the roughness lengths vary with wind direction. Average roughness lengths are of the order of 40 cm.

The procedure used for determining the various Richardson numbers was similar for all sites with the exception of Cape Kennedy.

The gradient Richardson numbers, *Ri,* were computed directly from the data at each geometric mean height at every observational time according to Equation (4). Utilizing the bulk Richardson numbers and the appropriate $\ln (z/z_0)$ for a given site, a second set of Richardson numbers, *Ric,* was calculated from Figures 1 and 2. At each geometric mean height for a given observation time, values of *1/L* were computed according to Equation (6) for unstable conditions and Equation (7) for stable ones.

In the majority of unstable and neutral observations, the values of *1/L* were constant with height, yielding a fixed *1/L* for that observation time. On the stable side, slight variations of *1/L* values were more common. The values of *1/L* were then averaged to obtain a *1/L* for the given time. In some instances *1/L* could not be calculated at each level since Equation (7) is valid only for Richardson numbers between 0.0 and 0.14. Therefore, values were often estimated quite close to the ground only.

Pasquill categories of stability were determined from wind speeds at 10 m and the intensities of solar radiation. Where insolation measurements were made, values of insolation greater than 1.0 ly min⁻¹ were considered strong, 0.5 to ly min⁻¹ as moderate, and less than 0.5 ly min⁻¹ as weak (Pries, 1969). Where radiation measurements were not made (some South Dartmouth observations), insolation was estimated according to location, season, time of day, and cloud cover.

Turner stability classes were determined by using the wind speed extrapolated or interpolated to 10 m and solar altitude determined by site location, time of year, and time of day. The degree of insolation was modified, if necessary, by cloud cover and ceiling height.

The Pasquill scheme includes classes from A, strong convection, weak winds to F, stable stratification, weak winds. Another stable class was used, referred to as G by Fiedler (1968), which indicated very stable conditions (very light winds and no cloud cover at night).

Turner stability classes, ranging from 1 for extreme instability, to 4 for neutral conditions, and 7 for extreme stability, were determined for each observation time.

The Cape Kennedy data varied sufficiently from those at other sites to warrant special procedures in a number of ways. Since at all other sites the highest data level was below 25 m, only the 18- to 30-m levels at Cape Kennedy were used in order to keep the data as compatible as possible. The geometric mean height between these levels is 23 m. For computing bulk Richardson numbers according to Equation (5), the wind speed at the geometric mean was taken as:

$$
u_{23} = 1.045 u_{18},\tag{17}
$$

an interpolation formula derived from observations at 18 and 30 m.

4. Results

For each observational site, gradient Richardson numbers for the geometric mean heights were plotted against the corresponding calculated Richardson numbers *Ric* to determine if the use of Prasad's nomogram and the bulk Richardson number is a reasonable method for estimating *Ri.* Table I summarizes the various levels from which data were used, the geometric mean heights at which *Ri*, *B* and *Ri*, were computed, along with a qualitative assessment of the relationship between *Ri* and *Ric* at each site.

Although there is some slight systematic error in that Ri is greater than Ri_c under both stable and unstable conditions for the South Dartmouth and 1953 O'Neill data, the overall agreement is excellent. The systematically smaller *Ric* with these two latter sets of data could be explained if the values of z_0 were slightly too large. Also, in stable air, the omission of infrared radiation in the theory may make the nomogram (Figure 2) inaccurate.

Random differences between *Ri* and *Ric* increase as the numerical values become larger. More scatter occurred with data from Part II of the 1956 O'Neill report than with data from the other sites (except for Cape Kennedy). Presumably the sites with the least scatter had better data. The scatter is mainly due to the fact that with greater heights, the wind shear becomes smaller and slight errors in the measurement of the wind at two levels cause greater errors in the Richardson number. At Kennedy, the scatter was greatest but there was no systematic bias. It is likely that at Kennedy, *Ri,* is more accurate than *Ri* since large random errors in the wind shear produce large random errors in *Ri.*

In general, it was found that the use of Prasad's nomogram at the five sites is accurate enough to determine negative Richardson numbers in the range from 0.0 to -1.0 . On the positive side the range for satisfactory estimates was from 0.0 to about 0.10. Beyond *Ri=O.10,* the nomogram becomes inaccurate.

In determining the stability classes for each observation time at the five locations, an examination was made of the relation between the Pasquill and Turner classifications. Figure 3 shows the comparison between the two systems for all observations at all sites. The figure suggests that the best conversion rules from Pasquill to Turner classes are as follows: $A \rightarrow 1$, $B \rightarrow 2$, $C \rightarrow 3$, $D \rightarrow 4$, $E \rightarrow 6$, $F \rightarrow 7$ and $G \rightarrow 7$. In converting from a Turner to a Pasquill class, the relationship was not as simple. Observations in the Turner class 1 were almost equally divided between Pasquill categories A and B; class 2 corresponded mostly to B with a large group in C; and class 3 to C with a large group of D. Turner classes 4 and 5 went predominantly with Pasquill's D class; class 6 agreed with E with a small group in D; and class 7 corresponded to Pasquill's class F.

Data levels used, geometric mean heights, and a qualitative assessment of *Ri* versus *Ric*

The more complicated relationship is caused by a greater breakdown of the wind speeds into class-defining limits by Turner (up to 12 m s^{-1}) than by Pasquill (up to 6 m s⁻¹).

The greatest spread of Turner classes occurred with Pasquill's D class. This results from the fact that Pasquill assigned class D to neutral conditions regardless of wind speed, for overcast conditions during day or night and for any sky condition during the hour preceding or following night (night is defined as one hour before sunset to one hour after dawn). Turner assigns the neutral category 4 to cases when the total cloud cover is 10/10 and the ceiling is less than 2100 m, but places no other restrictions as Pasquill does.

At each site, the *1/L* values were grouped according to Pasquill and Turner stability classes and means and standard deviations were found for each class.

However, the means for a given class were not the same at the various sites. The variation in the neutral categories (D and 4) ranged from a relatively large negative value at Kerang to a positive value at Hanford. This is partly because the observations at Kerang were taken during daylight hours while those at Hanford were conducted

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Fig. 3. Comparison of Pasquill and Turner stability classes.

Fig. 4. 1 /L as a function of Pasquill classes and zo.

Fig. 5. $1/L$ as a function of Turner classes and z_0 .

at night. It is likely that the best average *1/L* for the neutral class under all conditions is zero.

Figures 4 and 5 show the relations between Pasquill and Turner categories on the one hand and *L* on the other. The lines on these nomograms (using the mean *1/L* values and their standard deviations for each class as a guide) were drawn subjectively to provide the best fit to the observations.

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