point 2 in Fig. 4b), and the strain range ε_X^P reaches 1.4%. This is followed by cyclic elastic deformation (points 2-5).

Thus, the results show that the proposed method can be used efficiently for solving the problems of the SSS of actual structures with an allowance made for technological special features of their manufacture in dynamic loading.

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FEM SOLUTION OF A DYNAMIC ELASTOPLASTIC PROBLEM OF FRACTURE MECHANICS.

2. SUPERCRITICAL CRACK PROPAGATION

V. I. Kostylev and V. Z. Margolin

UDC 539.3

A method of algorithm of numerical analysis of supercritical crack growth are proposed. They make it possible to determine the release of elastic energy and the rate and direction of crack growth with an allowance made for residual stresses. Crack growth was simulated by means of finite elements with special properties. The results of calculations carried out using this algorithm were compared with analytical dependences and experimental data on crack propagation with constant and variable rate.

In solving the dynamic problem of fracture mechanics it is necessary to examine a number of main aspects, such as the start of crack propagation, crack propagation kinetics, i.e., determination of the trajectory and rate of crack growth.

1. The condition for the start of crack propagation can be analyzed on the basis of the force (the value at which the dynamic stress intensity factor reaches fracture toughness K_c) and energy (when the rate of release of electric energy reaches the critical value G_c) criteria. It should be mentioned that in contrast to the static analogs, the quantity K_c and G_c are characteristic of the material which greatly depend on the loading rate [1]. To develop this criteria for short-term pulse loading, the authors of [1] proposed criteria of

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Translated from Problemy Prochnosti, No. 7, pp. 12-19, July, 1990. Original article submitted August 29, 1989.

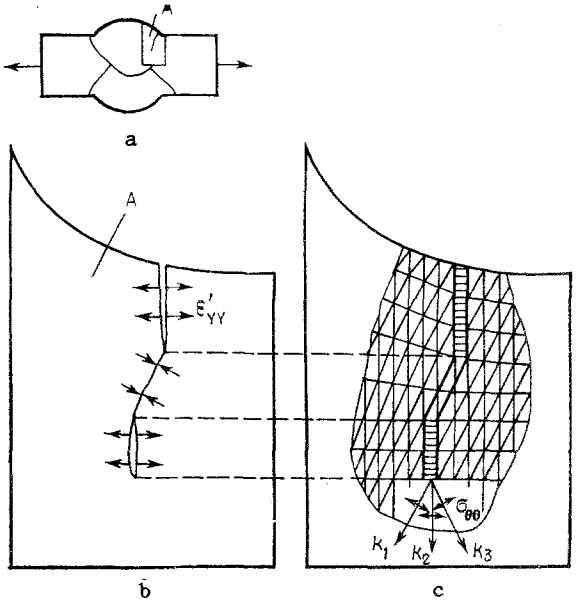

Fig. 1. Sketch of a butt welded joint (a), a fragment with a crack (b), and simulation of its movement (c).

the minimum time and minimum action which take into account the limited duration of action of the load pulse.

When using the numerical methods, the stress intensity factors (SIF) can be determined directly on the basis of stress fields at the crack tip (direct method) or using special singular elements which ensure the singularity of the stress distribution at the crack tip of the type zero ($r^{-1/2}$). In the first, to increase the accuracy of the solution, the density of the mesh of regular finite elements at the crack tip is increased. It is preferred to determine the SIF by the direct method using singular elements for a stationary crack than regular elements because it is sufficient to use a measure with a relatively low sensity. However, in measuring a crack moving along a curvilinear trajectory, there are difficulties associated with the arrangement of the mesh and movement of the singular element. Alternative methods of determining the SIF which make it possible to avoid this problem and which use only regular elements, are the energy methods based on computing either the crack closure integrals and the J-integral proposed by Cherepanov and Rice, or the rate of release of elastic energy (the compliance method, the method of displacement of a node at the crack tip) [2].

The condition of the start of crack propagation based on energy criterion in relation to the degree of plastic deformation can be determined by two methods. In localized plastic yielding at the crack tip dissipation of the energy of plastic deformation can be added directly to the energy required for the formation of a new crack surface. This is equivalent to the transition of examining an elastic solid for this the condition of crack propagation is determined from the equation $G = \gamma_{ef}$ [3]. For a stationary crack in dynamic loading, the value G can be determined conveniently by the compliance method reducing the dynamic problem to a static one. For this purpose, it is necessary to find the increment of the potential energy ΔP caused by the variation of crack length by ΔL at fixed external loads which include the inertia forces

$$G^S = -\frac{d\,\mathrm{P}}{dL} \approx -\frac{\Delta \mathrm{P}}{\Delta L} = \frac{\mathrm{P}(t-\Delta t) - \mathrm{P}(t)}{\Delta L}. \qquad (1)$$

The expression for the potential energy can be written in the following form

$$\mathrm{P} = \frac{1}{2}\int_V \{\sigma\}^T (\{\varepsilon\} - \{\varepsilon_0\})\, dV - \{u\}^T (\{F_p\} + \{F_\rho\}),$$

where $\{\sigma\}$, $\{\varepsilon\}$, $\{u\}$, $\{\varepsilon_0\}$, $\{F_p\}$, $\{F_\rho\}$ are, respectively, the vectors of the stresses, strains, displacements, initial strains, and external and inertia forces.

In cases with developed plastic deformation, the stress–strain state (SSS) at the crack tip and, consequently, the crack propagation condition are controlled by the J-integral proposed by Cherepanov and Rice which can be written in the following form for the dynamic problem and the stationary crack [4]

$$J = \int_{\Gamma} [(W + T) n_1 - T_i u_{i,1}] d\Gamma, \tag{2}$$

where T is an arbitrary contour surrounding the crack tip; $W = \int \sigma_{ij} d\varepsilon_{ij}$ is the strain energy density; $T = \rho \dot{u}_i \dot{u}_i / 2$ is the kinetic energy density; $T_i = \sigma_{ij} n_j$ is the vector of forces acting on the contour Γ; n_j are the components of the vector of the normal to the contour directed from the inside of the latter. The term of the J-integral, which depends on W, is calculated as the sum of increments ΔJ^W is in the equation

$$J^W = \int_{\Gamma} W n_1 d\Gamma = \sum_n \Delta J^W = \sum_n \int_{\Gamma} \{\sigma_m^n\}^T \{\Delta\varepsilon^n\} n_1 d\Gamma,$$

where $\{\sigma_m{}^n\} = (\{\sigma^{n+1}\} + \{\sigma^n\})/2$ is the mean stress in the stage $\Delta t_n = t_{n+1} - t_n$; $\{\Delta\varepsilon^n\} = \{\varepsilon^{n+1}\} - \{\varepsilon^n\}$ is the strain increment in the stage Δt_n.

2. Simulation of crack propagation and determination of the crack trajectory is the second aspect which must be examined in numerical solution of the dynamic problem of fracture mechanics. When using FEM, the crack propagation can be simulated either by consecutive release nodes distributed along the crack trajectory [1, 2, 4] or by consecutive definition of the elasticity modulus close to zero in elements at the crack tip along its trajectory: E_T $E^* \ll E$ (E is the elasticity modulus of the parent material) [5]. The second simulation method for cracks with a curvilinear trajectory is more rational because it makes it possible to take into account efficiently different boundary conditions in the elements of the crack cavity (partial contact of the crack edges, determined by interaction between the residual and service stress fields) in relation to the sign of strain ε'_{YY} in these elements (the sign ' relates to the local system of coordinates, the X' axis passes along the tangent to the crack trajectory). In this simulation, if a crack is situated in a noninformed stress field and opens in certain areas along its length, which corresponds to the condition $\varepsilon'_{YY} > 0$, we assume the $E_T = E^*$ and, consequently, the crack does not resist applied loads in these sections (Fig. 1). In areas in which the crack edges come into contact, i.e., $\varepsilon'_{YY} \leq 0$, for elements of the crack cavity $E_T = E$ and two variants are possible: ($\varepsilon'_{XY} \neq 0$, $\sigma'_{XY} \neq 0$) of slipping of its edges, and slipping of the edges ($\varepsilon'_{XY} \neq 0$, $\sigma'_{XY} = 0$). In the first variant, from the viewpoint of transfer of the force flow the solid operates as a monolith whereas only normal stresses are transferred in the second case. To apply the second variant at arbitrary orientation of crack elements (the crack trajectory is curvilinear), it is necessary to carry out a number of transformations.

In the local coordinate system X',Y' we show right the finite-element of the equation equilibrium (19) taken from [6], for the ℓ-th element of the crack

$$\left([M'_l]\frac{1}{\Delta t} + [K'_l]\frac{\Delta t}{2} + [C'_l]\right)\{\dot{u}_l\}'_n = \left([M'_l]\frac{1}{\Delta t} - [K'_l]\frac{\Delta t}{2}\right)\{\dot{u}_l\}'_{n-1} + \{f_l^p\}'_n - \{f_l^{\varepsilon^0}\}'_{n-1}, \tag{3}$$

where the stiffness matrix $[K_\ell']$ and the vector of forces $\{f_\ell{}^{\varepsilon^0}\}_{n-1}'$, determined by initial strains, is calculated in the local coordinate system using the well-known equation [6] but with an allowance made for the condition of slipping of the crack edges, i.e., the element D_{33}' of the matrix of the relationship between the stresses and strains $[D_\ell']$ is equal to zero. Since the vectors of the nodal operates {u} and forces {f} in the local and global coordinate systems are linked by the relations of the following form

$$\{\dot{u}_l\}' = [S_l]\{\dot{u}_l\},$$
$$\{f_l\}' = [S_l]\{f_l\},$$
$$\alpha = (\widehat{X', X}),$$

$$[S_l] = \begin{bmatrix} \cos\alpha & \sin\alpha & & & & \\ -\sin\alpha & \cos\alpha & & & & \\ & & \cos\alpha & \sin\alpha & & \\ & & -\sin\alpha & \cos\alpha & & \\ & & & & \cos\alpha & \sin\alpha \\ & & & & -\sin\alpha & \cos\alpha \end{bmatrix},$$

$$[S_l]^{-1} = [S_l]^T,$$

and also because of the variance of the mass matrix $[M_\ell]$ and the damping matrix $[C_\ell]$ in relation to the coordinate system, we shall write the equation (3) in the global coordinate system X, Y:

$$\left([M_l]\frac{1}{\Delta t} + [S_l]^T[K_l'][S_l]\frac{\Delta t}{2} + [C_l]\right)\{\dot{u}_l\}_n =$$

$$= \left([M_l]\frac{1}{\Delta t} - [S_l]^T[K_l'][S_l]\frac{\Delta t}{2}\right)\{\dot{u}_l\}_{n-1} + \{f_l^p\}_n - [S_l]^T\{f_l^{\varepsilon^0}\}'_{n-1}.$$

Introducing the notations $[K_\ell]_{ef} = [S_\ell]^T[K_\ell'][S_\ell]$, $\{f_\ell{}^{\varepsilon^0}\}_{ef} = [S_\ell]^T\{f_\ell{}^{\varepsilon^0}\}'$, we obtain

$$\left([M_l]\frac{1}{\Delta t} + [K_l]_{ef}\frac{\Delta t}{2} + [C_l]\right)\{\dot{u}_l\}_n =$$
$$= \left([M_l]\frac{1}{\Delta t} - [K_l]_{ef}\frac{\Delta t}{2}\right)\{\dot{u}_l\}_{n-1} + \{f_l^p\}_n - \{f_l^{\varepsilon^0}\}_{n-1_{ef}} \qquad (4)$$

It can easily be seen that the form of Eq. (4) is identical with Eq. (19) in [6], with $[K_\ell]$, $\{f_\ell{}^{\varepsilon^0}\}$ replaced by $[K_\ell]_{ef}$, $\{f_\ell{}^{\varepsilon^0}\}_{ef}$. Thus, in solving the problem taking into account slipping, it is necessary to form the resolving system finite element equations using the algorithm described in [6], assuming that in the crack elements we used the effective stiffness matrix $[K_\ell]_{ef}$ and the vector of forces $\{f_\ell{}^{\varepsilon^0}\}_{ef}$, determined by the initial strains. Subsequently, solving the system of Eqs. (4) and determining the values of the rate vectors $\{\dot{u}_\ell\}$, and the displacement $\{\Delta u_\ell\}$ and strain $\{\Delta\varepsilon_\ell\}$ increments, we can calculate the stress vector $\{\sigma_\ell\}$ taking slipping into account. For this purpose, the equations lining the vectors of the strain increments $\{\Delta\varepsilon_\ell\}$ and stress increments $\{\sigma_\ell\}$ in the local and global coordinate systems will have the form

$$\{\Delta\varepsilon_l\}' = [A_l]\{\Delta\varepsilon_l\}, \quad [A_l] =$$
$$\{\sigma_l\}' = [A_l]\{\sigma_l\},$$

$$= \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & 2\sin\alpha\cos\alpha & 0 \\ \sin^2\alpha & \cos^2\alpha & -2\sin\alpha\cos\alpha & 0 \\ -\sin\alpha\cos\alpha & \sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$[A_l]^{-1} =$$

$$= \begin{bmatrix} \cos^2\alpha & \sin^2\alpha & -2\sin\alpha\cos\alpha & 0 \\ \sin^2\alpha & \cos^2\alpha & 2\sin\alpha\cos\alpha & 0 \\ \sin\alpha\cos\alpha & -\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Consequently, the equation linking the vectors of the stresses and the strain increments in the global coordinate system, with an allowance made for fulfilling the slipping conditions, can be written in the form

$$\{\sigma_l\} = [A_l]^{-1}\{\sigma_l\}' = [A_l]^{-1}[D_l'](\{\Delta\varepsilon_l\}' - \{\varepsilon_l^0\}') =$$
$$= [A_l]^{-1}[D_l'][A_l](\{\Delta\varepsilon_l\} - \{\varepsilon_l^0\}),$$

where the matrix of relationship between the stresses and strains $[D_\ell']$ ensures the transfer of normal stresses and absence of shear stresses (element of the matrix $D_{33}' = 0$). Consequently, simulating the crack by this method, the boundary conditions on its edges are constantly fulfilled.

2.1. The direction of crack propagation is usually selected from a criterion of its deviation: maximum hoop stresses, the minimum density of strain energy, the maximum rate of energy released [1, 7, 8]. The Yoffe criterion of the maximum hoop stresses $\sigma_{\theta\theta}$ is used most extensively because it results in a satisfactory agreement between the calculated and experimental data in analysis of supercritical crack growth [7]. The crack propagation and determination of its trajectory using the FEM can be simulated by different methods. Highest accuracy in numerical calculations is obtained using the approach in which the crack propagation direction is determined in each stage of crack propagation and the mesh of finite elements is correspondingly rearranged. It should be mentioned that rearrangement of the mesh greatly complicates the solution of the problem. The optimum approach as regards accuracy and use of application is the one proposed in [5] which is based on discretizing the region in which the direction of possible crack propagation is varied in each step in three directions (k_1, k_2, k_3 in Fig. 1) and the magnitude of deviation of the crack from the initial direction can periodically change from step to step together with the dimension of the elements. If necessary, the mesh of the crack elements is automatically rearranged in such a manner that the new pair of elements is oriented along the calculated trajectory. Using a relatively low-density mesh, this method makes it possible to describe a relatively complex crack trajectory.

3. Determination of the crack growth rate (CGR) in dynamic loading is a relatively complicated problem. Analytical expressions for CGR were obtained by a number of authors [1, 4, 8] for idealized formulations for infinite and semi-infinite solids. These authors used the energy balance in different forms. For structures with finite dimensions, the applicability of these expressions is limited by the time of arrival of reflected waves to the crack tip. For structures with a complex geometry, a mixed numerical-experiment has been used extensively in recent years [9]. In this method, CGR is determined by solving the nonlinear equation

$$K_I(t, v) = K_{ID}(v),$$

where v is the crack propagation rate; $K_{ID}(v)$ is the curve of dynamic fracture toughness determined by experiments. Application of this method is limited by the fact that in the presence of a mixed crack, determination of CGR becomes ambiguous. In addition, as mentioned previously, to determine the SIF, it is necessary to use a low-density mesh at the crack tip or introduce special infinite elements.

In this work, it is proposed to determine the CGR on the basis of an energy criterion for a moving crack [8]:

$$\dot{A} = \dot{U} + \dot{T} + \dot{D}, \tag{5}$$

where D is the sum of all irreversible energy components (free surface energy, energy used for plastic deformation, etc.). The expression for operation of external forces A, the energy of reversible (elastic) deformation U, and the kinetic energy T can be written in the form

$$A = \int_S \{p^S\}^T\{u\}\,dS + \int_V \{p^V\}^T\{u\}\,dV;$$
$$U = \frac{1}{2}\int_V \{\sigma\}^T(\{\varepsilon\} - \{\varepsilon_0\})\,dV; \tag{6}$$
$$T = \frac{1}{2}\int_V \rho\{\dot{u}\}^T\{\dot{u}\}\,dV,$$

where $\{\sigma\}$, $\{\varepsilon\}$, $\{u\}$, $\{\dot{u}\}$ are the vectors of the stresses, strains, displacements, and rates

obtained as a result of the FEM solutions; $\{\varepsilon_0\}$, $\{p^S\}$, $\{p^V\}$ are the vectors of the initial strains, and surface and volume forces.

The equation (5) can be written in the following modified form

$$\frac{d}{dt}(A-U-T-D)=\frac{d}{dL}(A-U-T-D)\frac{dL}{dt}=$$
$$=\frac{d}{dL}(A-U-T-D)v=0,$$

where L is the crack length. Since for the moving crack $v \neq 0$ the energy criterion can be written in the form

$$\frac{d}{dL}(A-U-T-D)=0.$$

Taking into account that the rate of release of the elastic energy in the dynamic case is $G^d = d/dL(A - U - T)$, and the intensity of the surface failure energy is $2\gamma = dD/dL$, the energy criterion of the moving crack assumes the form

$$G^d = 2\gamma. \tag{7}$$

It should be mentioned that at the start of crack propagation the structure can be subjected to high plastic deformation in which energy dissipation has a strong effect on the crack kinetics. During crack propagation, the plastic deformation is localized mainly at the tip of the moving crack. Formulation of the energy balance in the form of Eq. (7) makes it possible to analyze crack propagation in the elastic formulation because energy dissipation at the tip of the moving crack is included in 2γ. Thus, it is essential to solve the elastoplastic problem up to the start of crack propagation and it is convenient to use the solution of elastic problem in analyzing its propagation. This method of simulating the kinetics can be applied by increasing the yield limit of the material after the start of crack propagation.

3.1. The rate of release of the elastic energy during formation of the new surface of a crack with length ΔL in the transition to limit at $\Delta L \to 0$ can be represented as the work of "bonding forces" at the crack edges during the time $\Delta\tau = \Delta L/v$ (the time during which the crack tip travels the distance ΔL at the speed v) whose value for a discrete model depends on the time dependence of these forces. When using finite-element models, the crack propagation act (jump) can be described as follows. The bonding forces of the crack edges, proportion to the stiffness of the elements of the cavity of the crack, are characterized by the elasticity modulus of the crack E_T, and during the period $\Delta\tau$ decrease to almost zero ($E_T = E^* = 0$) in accordance with the following law

$$E_\tau(j)=E\left(1-\sum_{i=1}^{j}\Delta t_i/\Delta\tau\right)+E^*,\ j=1,\ k;\ \sum_{i=1}^{k}\Delta t_i=\Delta\tau. \tag{8}$$

This method of simulating the crack propagation process describes most adequately the process of continuous crack propagation in a solid. In fact, a reduction of the value of E_T during time $\Delta\tau$ from the viewpoint of analysis of the rate of release of the elastic energy G can be interpreted as the process of consecutive movement of the crack tip by the value $\Delta\ell_i = v\Delta t_i$, thus reducing the effective step of crack propagation.

In this case, the rate of release of elastic energy during time $\Delta\tau$ during crack propagation by the value $\Delta L=\sum_{i=1}^{k}\Delta l_i$ is determined by the relationship

$$G^d=\sum_{i=1}^{k}g_i^d, \tag{9}$$

where g_i^d is the rate of release of elastic energy during time Δt_i. The expression for this rate in finite increments has the form

$$g_i^d \simeq \frac{1}{\Delta L}\left(\int_{t_i}^{t_i+\Delta t_i}\{F_p\}^T d\{u\} - \Delta U - \Delta T\right) = \frac{1}{\Delta L}\times \tag{10}$$

$$\times\left(\{F_p\}_m^T\{\Delta u\} - \Delta U - \Delta T\right);$$

$\Delta U = U(t_i + \Delta t_i) - U(t_i)$; $\Delta T = T(t_i + \Delta t_i) - T(t_i)$ are the increments of the strain energy and kinetic energy determined from Eq. (6); $\{F_p\}_m = (\{F_p(t_i + \Delta t_i)\} + \{F_p(t_i)\})/2$ is the vector of forces in the mean point of the interval Δt_i determined by the volume surface and concentrated forces; $\{\Delta u\}$ is the vector of increments of displacements of nodes of the entire solid during time Δt_i.

Another interpretation of the method of simulating crack propagation is also possible: use of the dependence (8) leads to a smooth reduction of the bonding forces to zero during time $\Delta\tau$ and, consequently, to the absence of noncharacteristic high-frequency oscillations. This corresponds to crack propagation in a continuous medium.

It should be mentioned that the method of simulating crack propagation, based on Eq. (8), has a number of special features. For example, in the case in which k = 1 (the most economical variant from the viewpoint of calculation time), the bonding forces decrease to E^* during period $\Delta\tau = \Delta t$. In this case, the position of the crack tip varies in jumps by the value ΔL and the crack growth v is linked unambiguously with the integration step Δt. The latter circumstance imposes a strong restriction on the selection of the integration system of finite-element equations of motion and it is therefore necessary to use unconditionally stable but less accurate integration systems (for example, variant II [6]). A more accurate system (variant I [6]) can be used because the restrictions on the integration step $\Delta t < \Delta L/2c_R$ (determined by the stability of the system) and the crack growth rate $v < c_R$ (c_R is the rate of propagation of surface Rayleigh waves) mutually exclude each other ($\Delta\tau > \Delta t$).

3.2. Equation (7) is nonlinear because, in the general case, the left and right parts are functions of the crack rate v. Opening of the nonlinearity of Eq. (7), i.e., determination of the crack growth rate at which the energy balance is satisfied, should be carried out an iteration procedure based on the approximate analytical dependence [10]

$$G^d(v) \simeq G(0)(1 - v/c_R). \tag{11}$$

We shall write Eq. (11) for two similar crack growth rates:

$$G^d(v_j)/(1 - v_j/c_R) = G^d(v_{j+1})/(1 - v_{j+1}/c_R).$$

Taking into account the fact that this equation also holds for the true CGR at which Eq. (7) is valid, the recurrent equation for the rate v can be written in the following form

$$v_{j+1} = c_R(1 - p) + pv_j,\ p = 2\gamma/G^d(v_j). \tag{12}$$

It should be mentioned that Eq. (11) holds for an infinite solid characterized by the absence of reflected waves. For real structures with limited dimensions, the form of the dependence (11) can differ. Using this dependence in the form $G^d(v) \simeq G(0)(1 - v/c_R)^M$ and applying identical assumptions, we obtain a recurrent equation similar to Eq. (12) where p is replaced by p^M. Varying the exponent M, we can control the convergence of the iteration process.

The iteration process is completed when $|G^d \times (v_{j+1})/2\gamma - 1| \leq \xi$, i.e., the crack growth rate $v_T = v_{j+1}$ at which the energy balance (7) is satisfied with the required accuracy is determined. The first approximation of the rate v_1^n in Eq. (12) in the n-th stage in respect of time is determined as follows: at the crack growth rate in the n − 1-th stage $v_T^{n-1} = 0$ the expression for v_1^n is determined from Mott's equation [8]

$$v_1^n = 0.38\sqrt{E/\rho}\Delta L/L,$$

and in the remaining case in the form

$$v_1^n = v_\tau^{n-1}(1 + \Delta t_n/\Delta t_{n-1}) - v_\tau^{n-2}\Delta t_n/\Delta t_{n-1},$$

i.e., by linear extrapolation of the crack growth rates from two preceding stages (Δt_n is the period between the n- and n − 1-th stages).

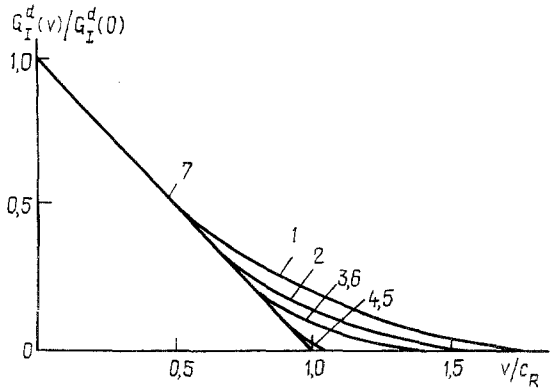

Fig. 2. Dependences of the normalized rate of release of elastic energy on the crack growth rate constructed by calculations using Eq. (7) – curve 7, and FEM – curves 1-6; 1-4, 6) for variant II of integration at the value k of 1, 4, 11, 18, 11, respectively; 5) for variant I. (1-5 – one pair of finite elements was used in the jump, 6 – three pairs of finite elements).

It should be mentioned that the rate of release of elastic energy G^d in Eq. (12) is determined from Eq. (9), i.e., the process of determination of the CGR is iteration in respect of the rate and includes several steps Δt_i in each iteration. Thus, the procedure for determining the CGR can be described as follows: from Eq. (12) we calculate the next approximation of the crack growth rate v_j and determine the value k from the condition fulfilling the quality

$$\sum_{i=1}^{k} \Delta t_i \simeq \Delta L / v_j.$$

This is followed by solving the dynamic problems [6] taking into account the deduction of the elasticity modulus of the element of the crack tip using Eq. (8), and determine the parameters of SSS of the structure and the rate of release of elastic energy $G^d(v_j)$ from Eq. (9), where the expression for g_i^d is represented in the form (10). We verify the condition of completion of the iteration process. If this condition is not satisfied, the process is continued, otherwise the crack tip is moved by the value ΔL and the procedure for determining the CGR is repeated.

4. The algorithm of solving the dynamic of fracture mechanics can be represented as follows.

I. In the current stage Δt_n the solution of the dynamic elastoplastic problem up to the start of crack propagation or of the elastic problem after the start of crack propagation. The algorithm described in [6] is used to take into account the fields of residual strains ε^0. These fields make it possible to simulate the residual stresses and take into account their effect on the crack trajectory, the crack rate, the value of the SIF, and possible contact of crack edges. It should be mentioned that in this approach the redistribution of the stress fields during crack propagation is automatically taken into account.

II. Improvement of the accuracy of the boundary conditions in the elements of the crack cavity by specifying the corresponding elasticity modulus E_T (paragraph 2) in these elements.

III. Calculation of the J-integral for a stationary crack using Eq. (2). Verification of the condition of the start of crack propagation using the criterion J-integral. If this condition is fulfilled, we transfer to paragraph V, otherwise to paragraph I.

IV. Determination of the rate of the release of elastic energy G^d using Eq. (9); verification of the energy criterion (7). If this criterion is not fulfilled, it is necessary to correct the crack growth rate in a specific direction (paragraph 3.2).

V. Calculation of hoop stresses $\sigma_{\theta\theta}$ at the crack tip for three directions k_1, k_2, k_3 (Fig. 1) and determination of the direction of crack propagation (paragraph 2.1) in which its tip moves by the value ΔL (paragraph 3.1).

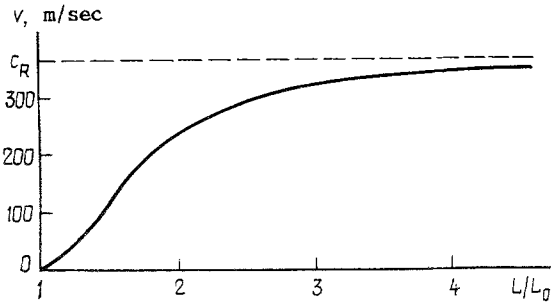

Fig. 3

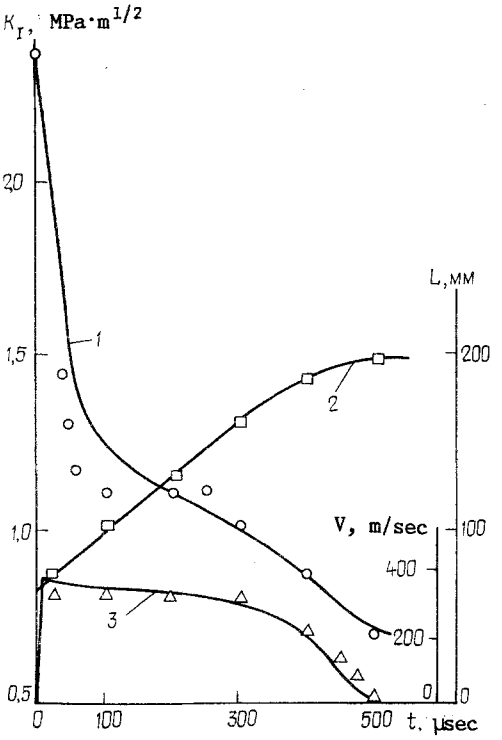

Fig. 4

Fig. 3. Variation of the crack growth rate in relation to the relative crack length.

Fig. 4. Calculated (lines) and experimental [11] (points) dependences of the dynamic stress intensity factor (1), crack length (2), and the crack propagation rate (3) on time. (K_{Iq} = 2.32 MPa·m$^{1/2}$).

VI. Transition to paragraph I.

Numerical Calculations

To verify the efficiency and determine the boundary of applicability of the proposed methods, calculations were carried out for several simulation problems of crack propagation which have approximate analytical solutions. Figure 2 shows the dependences of the normalized rate of release of elastic energy on the crack growth rate for the problem of movement of an infinite crack at a constant rate in a homogeneous field of tensile stresses [4]. Since in the examined problem the SSS in a moving coordinate system, connected with the crack tip, is stationary, the adequate numerical FEM solution can be obtained for a solid of limited dimension. The plate and crack length should be selected in such a manner as to ensure that the reflective stress waves have no effect during the examined time period. It is also necessary to take into account the fact that the stationary regime is reached during three of four crack jumps after start of its propagation.

It is evident that the accuracy of the results is influenced by various factors, such as the integration method, the size of the integration step Δt, the number of finite elements in a jump, the number of time subintervals k into which the interval $\Delta\tau$ is divided. Figure 2 shows that, using variant II of integration [6] at k = 1, 4, 11, 18, the difference of the calculated data from those obtained using the approximate analytical dependence (11) was equal to, respectively, 0.19; 0.14; 0.08; 0.01 G(0) ($v = c_R$).

Thus, at $k < 10$, the error of the calculated system is large. In the problem of determining the CGR this results in an unjustified increase of the crack growth rate, especially in the range of high values of this rate ($v \lesssim c_R$). It should be mentioned that the integration step Δt corresponding to k = 11 and $v = c_R$ is equal to the duration of passage of an expansion wave through the smallest finite element at the crack tip. The attempts to describe adequately the dependence $G^d(v)$ using more accurate simulation of crack opening by increasing the number of finite elements in a jump did not improve this dependence greatly (curve 6 in

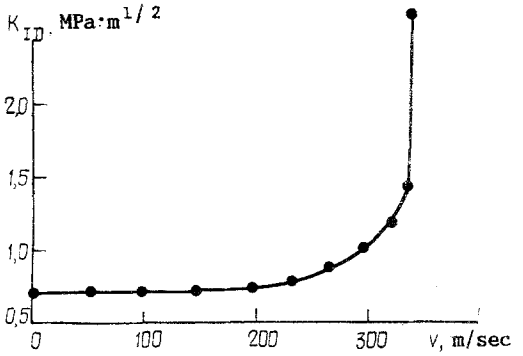

Fig. 5. Dependence of K_{ID} on crack growth rate v: (points are experimental data from [11], the curves indicate approximation of the experimental data used in the calculations).

Fig. 2). When using the integration variant I [6], the dependence $G_{(v)}^{d}$ differs from the analytical dependence (11) byless than 1% (curve 5). At the same time, it should be mentioned that as a result of a restriction placed on the integration step determined by the stability of the given variant, it is not efficient to use this step at $v < c_R$ because the number of steps Δt_i rapidly increases (at $v = c_R$ $k = 18$, at $v = c_R/2$ $k = 36$, etc.).

Figure 3 shows the dependence of the CGR on the relative crack length for the start of propagation in the field of a constant tensile stress. It can be seen that the crack growth rate reaches an asymptote equal to c_R when its tip travels the distance equal to approximately four initial crack lengths. The presence of the asymptote c_R (for a normal separation crack), predicted theoretically in [7] is caused by the fact that the crack cannot propagate at rates higher than c_R since the effective surface energy cannot be negative.

Figure 4 shows the calculated FEM and experimental [11] data obtained in loading a DCB specimen (321 × 127 × 10 mm; initial crack length 66 mm) with a wedge with an opening angle of 20°. The material have the following characteristics: Young's modulus E = 3380 MPa, Poisson's coefficient $\mu = 0.33$ [11]. The crack was initiated in blunt slits at different values of K_{Iq} (1.08-2.32 MPa·m$^{1/2}$). The iteration process of determining the CGR was carried out using the proposed procedure. The dependence of the effective surface energy on crack growth rate $\gamma = \gamma(v)$ in the expression (7) is determined on the basis of experimental data on the dynamic fracture toughness: $K_{ID} = K_{ID}(v)$ [11] (Fig. 5).

Figure 4 shows that the experimental and calculated result do not differ by more than 10% in respect of rate v and 5% in respect of crack length L. The maximum difference is recorded in the initial stage of crack propagation in which the experimental data are not completely accurate because of procedure problems. The difference between the dynamic SIF, obtained using FEM, and the corresponding experimental values (Caustic method) does not exceed 15% and decreases to 2% during crack propagation up to its arrest. After crack arrest the SIF oscillates with a decrease in amplitude around the certain value. The frequency of these oscillations is close to the frequency of inherent oscillations of the rod whose length is equal to the dimension of the specimen less the length of the arrested crack.

Thus, the described method of calculating the parameters of the dynamic fracture mechanics (SIF, G, v) with suitably integration step Δt makes it possible to apply the given procedure with high reliability and accuracy, with an allowance made for wave phenomena and redistribution of stress fields during crack propagation.

CONCLUSIONS

1. A method was developed of calculating the kinetics of supercritical crack growth which can be used to determine the rate of crack propagation and crack trajectory taking into account the fields of residual stresses and also possible contact of the crack edges.

2. The numerical experiments to find the ranges of applicability of the described methods of integration of the equations of motion. These equations can be used for calculations with minimum computing time and a relatively small error.

3. Comparison of the calculated results obtained by the proposed method, with experimental data on the crack kinetics in a DCB (double cantilever) specimen made of Araldite-V epoxy resin indicates that they are in complete satisfactory agreement.

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FORMAL ANALYSIS OF SUBCRITICAL CRACK GROWTH UNDER MONOTONIC LOADING

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UDC 539.4

The article analyzes the equation of energy balance in integral form which made it possible to formulate a number of considerations concerning the conditions of applicability of criterial characteristics used in fracture mechanics, and to suggest a new criterion which, in the present author's opinion, characterizes more rigorously the conditions of failure of brittle materials.

We state the problem of analyzing the process of crack growth under monotonic loading, examining the phenomenon solely from the positions of mechanics, and touching upon the aspect of materials science to a minimal extent only. We assume that there is a crack 2ℓ long in a plate with unbounded dimensions, made of isotropic material, having linear elasticity up to some stress level above which plastic deformations arise.

To obtain relations characterizing the process of crack growth upon monotonic increase of the load, we use the equation of energy balance which takes into account the energy liberated upon an increase of crack length (or, conversely, expended on additional deformation in dependence on the loading conditions), and the energy expenditures connected with the increase of crack length. For that we adopt the following conditions:

1. The intensity of the liberated energy with increasing crack length is determined by the stress intensity factor (SIF):

$$\frac{\partial W}{\partial l} = G \sim K^2/E. \qquad (1)$$

Kiev. Translated from Problemy Prochnosti, No. 7, pp. 19-23, July, 1990. Original article submitted September 26, 1988.

0039-2316/90/2207-0953$12.50 © 1991 Plenum Publishing Corporation