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SPONTANEOUSLY BROKEN SCALE INVARIANCE AND GRAVITATION†

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ABSTRACT

It is proposed to combine the scalar-tensor theory of gravitation with the hypothesis of 'spontaneously broken scale invariance', which has been developed in quantum field theory and seems to give a better understanding of the origin of the masses of elementary particles. The general theoretical background of this approach is reviewed. In our model theory we predict that the Newtonian gravitational potential acquires an anomalous part with a force-range typically of the order of 10^5 cm. *The experimental consequences are also discussed.*

In this paper an attempt is made to bring the physics of gravitation somehow to a contact with the physics of elementary particles in such a way that we can make certain predictions which are tested by experiment, hopefully in the near future. We try to exploit the consequences of the hypothesis of 'spontaneously broken scale invariance' [*1-3*]¶.

By a spontaneous symmetry breaking we mean the following: suppose a Lagrangian has a symmetry, or is invariant under a certain transformation. Normally the solution of the Schrödinger equation obtained from this Lagrangian has a degeneracy which is a manifestation of the symmetry. There are, however, some exceptional cases in which this is simply not true; it appears as if the symmetry is broken in spite of the fact that the original Lagrangian still has

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¶ For spontaneously broken scale invariance, see [*3*] for example.

the symmetry. Detailed analysis shows that the symmetry breaking has been smuggled into the theory through the ground state, or the vacuum. The vacuum is infinitely degenerate.

There are in fact many examples of this situation of spontaneous symmetry breaking. Perhaps the best known are the theory of ferromagnetism and the theory of superconductivity. All the phenomena called phase transitions belong to the same category. There is one thing which is common to all of these phenomena: a long-range excitation or a massless boson should occur. Such a massless boson is now called a Nambu-Goldstone boson [4].

It was proposed that scale invariance might also be broken spontaneously [5]. Scale invariance (dilatation symmetry) is obviously broken in the real world due to the fact that there are elementary particles with finite masses. One may, however, ask the question how and by what mechanism this important space-time symmetry is broken.

There are two ways to break this symmetry. One is the explicit breaking; one introduces the mass term in the Lagrangian. Another intriguing way is the spontaneous breaking. To illustrate how the latter idea works, let us consider the simplified example of a Dirac particle — the nucleon. We first assume that the nucleon is massless. We also introduce the Nambu-Goldstone boson field ϕ. In this particular case of dilatation symmetry, we may call it the 'dilaton' field, which is a neutral massless scalar field. The Lagrangian is given by

$$L = -\bar{\psi}\gamma\partial\psi - \tfrac{1}{2}\phi_{,\mu}{}^{2} - g\bar{\psi}\psi\phi, \tag{1}$$

where we have included the interaction. The coupling constant g is dimensionless. (We use the unit system in which $c = \hbar = 1$). No dimensional constant appears in the Lagrangian (1) so that scale invariance follows immediately. The energy-momentum tensor, suitably defined, is traceless.

We now assume that the ϕ field has a *non-vanishing vacuum expectation value*. This is a key recipe in all of the calculations of spontaneous symmetry breaking†. The new field $\sigma(x)$ is then defined by

$$\phi(x) = v + \sigma(x). \tag{2}$$

† One of the possible ways to bring about this non-vanishing vacuum expectation value is to introduce a quartic interaction of the ϕ field as well as its negative squared mass. Scale invariance is thus broken explicitly. It is important, however, to notice that an almost infinitesimally small amount of the explicit breaking (characterized, for example, by the mass $\sim 10^{-19} m_N$, as suggested later) can easily trigger a spontaneous breaking which results in the much larger masses of the ordinary particles.

Notice that the vacuum expectation value v has the dimension of mass. We substitute (2) into (1) and find that there appears a term which has the same form as the nucleon mass term. In this way the nucleon mass,

$$m_N = gv, \tag{3}$$

has been generated spontaneously. Equations (2,3) are the basic relations in the theory of spontaneously broken scale invariance.

Now the question is if the dilaton exists in reality. The dilaton may not be exactly massless because the whole scheme might be an approximation to some extent. Yet it must be much lighter than any other 'ordinary' particles in order that the approximation is justified. So far no such light scalar meson has ever been discovered. This, however, may simply suggest that the coupling of the dilaton, if any, is very weak; perhaps much weaker than that of the ordinary weak interaction. An attractive possibility is that *the interaction of the dilaton is as weak as the gravitational interaction.* The question is now shifted to whether one can formulate the consistent theory of gravitation to accommodate this scalar field. Here the scalar-tensor theory of gravitation gives a clue.

In the version of Brans and Dicke [6] the gravitational part of the Lagrangian is given by

$$\mathcal{L} = \sqrt{-g}\phi^2 R.$$

The field $\phi(x)$ is assumed to be constant to a very good approximation. The constant is obviously related to the Einstein or the Newtonian gravitational constant G,

$$\phi(x) = \frac{1}{\sqrt{G}} + \sigma(x), \tag{4}$$

where we have ignored some unimportant numerical coefficient. In the present unit system we have

$$G{m_N}^2 \sim 10^{-38}. \tag{5}$$

Equation (4) is to be compared with equation (2). It seems rather surprising that this remarkable similarity between the two equations has never been fully appreciated. Schwinger noticed this point [7], but never tried to exploit the consequences.

Perhaps the simplest assumption is to identify the dilaton field with the scalar gravitational field. We immediately obtain from (2,4)

$$v \sim G^{-\frac{1}{2}} \sim 10^{19} m_N, \tag{6}$$

where use has been made of (5). Combining this with (3) yields the result

$$g \sim 10^{-19}, \tag{7}$$

which is indeed extremely small.

The theory is not entirely the same as Brans and Dicke's theory. In particular we obtain the equation

$$\Box\phi = 0. \tag{8}$$

In Brans and Dicke's theory the right hand side is the trace of the energy-momentum tensor which now vanishes in the scale invariant theory. Although the D'Alembertian in (8) is a covariant one, equation (8) reduces to a free equation for ϕ in the limit of weak fields. Only in this limit we can make definite predictions subject to the experimental tests. For this reason this simplest theory has virtually no testable consequences.

As the next simplest theory we consider a *two-scalar model* in which another scalar field ψ is introduced. The basic Lagrangian is given by

$$\mathcal{L} = \sqrt{-g}L,$$

$$L = \frac{1}{2} f^{-2}\phi^2 R - \frac{1}{12}\Phi^2 R + L_M,$$

$$L_M = -\frac{1}{2}\phi_{,\mu}\phi^{,\mu} - \frac{1}{2}\psi_{,\mu}\psi^{,\mu} - \frac{1}{2}\Phi_{,\mu}\Phi^{,\mu} + L',$$

where f is a dimensionless constant which corresponds to Brans and Dicke's ω; Φ is a 'spinless nucleon'. The term L' describes the interaction among ϕ, ψ and Φ, with the dimensionless coupling constants. The Langrangian does not contain any dimensional constant so that scale invariance still holds. As a generalization of equation (2) we assume

$$\phi(x) = v_1 + \sigma_1(x), \qquad \psi(x) = v_2 + \sigma_2(x),$$

with two vacuum expectation values v_1 and v_2. We can apply the standard technique to analyze the consequences. The result is summarized as follows:

(i) General covariance and the weak principle of equivalence are maintained. The nucleon mass is generated spontaneously;

(ii) The static potential is calculated to be

$$V(r) = -G_\infty \frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}), \tag{9}$$

where $G_\infty \sim v^{-2}$, just as expected. The coefficient a depends on some details of the model, but is expected to be of the order of unity. The anomalous part comes from the scalar field. Roughly speaking, one of the two scalar fields acquires a non-vanishing mass μ; the range λ is given by $\lambda = \mu^{-1}$. The usual long-range part comes from the tensor field which remains massless. The same type of potential was also considered by Pechlaner and Sexl [*8*], on a different ground. The potential (9) is the same as that of O'Hanlon [*9*], Acharya and Hogan [*10*];

(iii) The mass μ of the scalar field is also generated spontaneously. An order of magnitude estimate results in

$$\mu^2 \sim Gm_N{}^4 \sim 10^{-38}\, m_N{}^2,$$

$$\lambda \sim 10^{19}\, m_N{}^{-1} \sim 10^{19} \times 10^{-14}\ \text{cm} = 10^5\ \text{cm} = 1\ \text{km}.$$

Although this estimate is extremely crude, it seems reasonable to expect that the force-range is of a macroscopic distance;

(iv) A careful study of the accuracy of the Cavendish experiment [*11*] and some of the geological and astronomical measurements gives two allowed regions of the value of the force-range† λ:

$$\lambda \lesssim 1\ \text{cm},$$

$$\text{several}\ m \lesssim \lambda \ll 1\ \text{km};$$

(v) In the famous tests of general relativity, the relevant distances are much larger than the expected value of λ. The scalar part does not affect these tests unlike in Brans and Dicke's theory.

As a final remark we emphasize that the proposed theory provides us with a better understanding of the origin of the masses of elementary particles from the point of view of Mach's principle: the scalar field which embodies this principle through equation (4) is

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† See references [*1*] for details. The theoretical basis of references [*2*] which is presented here is somewhat different from that of references [*1*] although they share the same phenomenological predictions. The most important difference is that the vacuum expectation value v is very large in references [*2*] (as in equation (6) in the present paper), while it was assumed to be of a hadronic size in reference [*1*]. There is now no reason why the present theory cannot be applied to the leptons as well. Also the Yukawa form for the anomalous term in the static potential is exact in references [*2*] while it is only approximate in references [*1*].

responsible at the same time for creating the particle masses in accordance with equations (2,3).

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