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Gravity's Rainbow†

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The temperature anisotropy of the cosmic microwave background may be partially the imprint left by gravitational waves. Measuring the gravitational wave component and spectral shape of the anisotropy is a critical test of inflationary cosmology and theories of large-scale structure formation.

The first instants of creation may have produced a spectrum of long-wavelength gravitational waves which have left a detectable imprint on the cosmic microwave background. The minute temperature variations observed by the COBE Differential Microwave Radiometers (DMR) [1] may, in large part, be due to gravitational waves. If so, COBE could be not only the world's first successful detector of cosmic microwave anisotropy, but also the world's first successful detector of primordial gravity waves—a fantastic notion, to be sure!

Such a detection would go well beyond verifying the existence of gravitational waves. The detection would crucially affect our understanding of how large-scale structure formed in our universe and would provide a critical test of the inflationary model of the universe [2].

The CMB anisotropy is a direct measure of the inhomogeneities in the universe just prior to the formation of large-scale structure. Two features must be extracted to test inflation and large-scale structure formation.

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First, how does the inhomogeneities' amplitude vary with wavelength? The variation is usually quantified in terms of a "power law index," n, where the amplitude is proportional to (wavelength)$^{-n}$. Second, what are the relative contributions of energy-density spatial variations and long-wavelength gravitational waves to the inhomogeneities? Both correspond to variations in the space-time metric, which induce minute red shifts or blue shifts in the cosmic gas of photons and, thereby, produce the CMB temperature anisotropy [3]. Energy density variations are scalar variations and gravity waves are tensor variations of the metric.

Measuring n and the relative contributions of energy density fluctuations and gravitational waves to the CMB anisotropy is essential input for any theory of large-scale structure formation. The power index is needed to extrapolate the amplitude from the wavelengths measured by CMB observations to the smaller scales relevant for galaxy formation. Energy density fluctuations are gravitationally unstable and can condense as seeds for large-scale structure. Gravitational waves, which propagate and red shift away as the universe expands, do not contribute to large-scale structure formation. Only by identifying and subtracting the gravitational wave component from the total anisotropy does one properly extract the much sought-after primordial spectrum of the large scale structure seeds.

Measuring n and determining the gravitational wave component also provides a powerful test of inflationary cosmology. Inflation is a proposed solution to a number of mysteries of the standard Big Bang model [4]. Why is the universe so homogeneous? Why is the universe spatially flat? Why are there no magnetic monopoles or other remnants from phase transitions that took place early in the universe? These mysteries are all explained by supposing that the expansion of the universe underwent a period of extraordinarily rapid acceleration—inflation—during the first instants (10^{-35} seconds or so) after creation. The extraordinary stretching of space would flatten and smooth the universe and dilute the density of monopoles and other remnants to negligible values.

The expansion of a homogeneous and isotropic universe is described by Einstein's equation of motion for the scale factor, R,

$$\ddot{R} = -\frac{4\pi G}{3c^2}(\rho + 3p)R, \tag{1}$$

where ρ is the energy density and p is the pressure. Hence, the expansion rate inflates ($\ddot{R} > 0$) if the equation of state linking the pressure and density, $p = \gamma\rho$ satisfies $\gamma < -1/3$. Since ρ is positive, a large *negative* pressure is required. For free particles, $\gamma = \langle v^2/c^2\rangle/3$ so that $0 \leq \gamma \leq 1/3$. Negative γ could occur, though, if microphysical interactions were to cause the universe to be in a state with large vacuum energy density. A subsequent transition to a vacuum with zero energy density would end inflation and release matter and energy.

Inflation smoothes out any initial non-uniformity while producing a new spectrum of inhomogeneities [6]. The energy density and any light fields all experience quantum fluctuations on subatomic scales which inflation stretches to cosmological dimensions. The fluctuations, somewhat analogous to the quantum fluctuations about a black hole, have an amplitude proportional to the inverse of the space-time curvature during inflation, $H/2\pi$, where $H \equiv d\ln R/dt$ is the Hubble expansion rate. If one thinks of the fluctuations as irregular waves, then the accelerating expansion will stretch the waves outside the causal horizon, beyond which no physical processes can act to change the amplitude. The earlier a fluctuation leaves the horizon, the more its wavelength is stretched. Hence, inflation acts as a prism creating a macroscopic spectrum of fluctuations whose amplitude is set on microscopic (sub-horizon) scales. If all microphysical parameters are time-independent during inflation, then all of the fluctuations are produced with the same amplitude on average and one arrives at the traditional lore that predicts a scale-invariant spectrum of energy-density perturbations, corresponding to index $n = 1$. The COBE DMR measurement of $n = 1.1 \pm 0.5$ is viewed as consistent with this lore.

In the past year, though, it has become clear that the traditional assumptions about inflation are flawed. First, microphysical parameters must necessarily change as inflation ends: γ must increase above $-1/3$ from its original value near -1 so that the expansion of the universe will decelerate ($\ddot{R} < 0$) down to its present expansion rate. A slowing expansion rate means that fluctuations created closer to the end of inflation will have a lower amplitude ($\propto H$). This corresponds to a power index

$$n \approx 1 - 3(1+\gamma) \tag{2}$$

where $\gamma < -1/3$ is the equation of state when the fluctuations are produced during the inflationary phase. The most plausible models predict n between roughly 0.5 and 0.98 for the wavelengths ranging from galactic to horizon size [7]. Secondly, inflation generates gravitational waves which can produce significant CMB anisotropy [2,8]. The gravitational waves are created by quantum fluctuations of massless gravitons [9,10] They have nearly the same index n as energy-density perturbations, but their amplitude depends differently on γ. The ratio of gravitational wave (T) to energy-density perturbations (S) contributions to the CMB quadrupole anisotropy is predicted to be

$$\frac{T}{S} \approx 21(1+\gamma). \tag{3}$$

At first, it is a disappointing conclusion that inflation does not have a simple, unique prediction for n or for T/S. However, since both n and T/S are found to be simple functions of the equation of state, γ, a tight, model-independent relation emerges [2]:

$$n \approx 1 - \frac{1}{7}\frac{T}{S}. \tag{4}$$

This relation constitutes a new, critical test for inflationary cosmology. Confirming it would not only support inflation, but would provide direct information about the evolution of the universe just a few instants after the Big Bang.

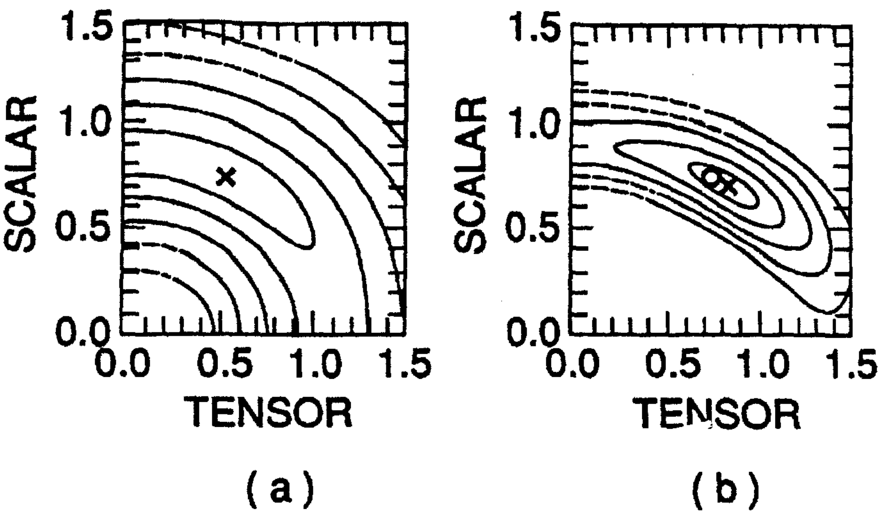

Figure 1. The cosmic fingerprint (from Ref. 11). (a) A likelihood contour plot for scalar versus tensor amplitudes assuming an $n = 0.85$ standard cold dark matter model obtained by fitting COBE DMR (Ref. 1) at large-angular scales plus the 1° South Pole [12] and Owens Valley (OVRO) [13]. For these experiments, maximum likelihood corresponds to $T/S \approx 1$, consistent with the inflationary prediction, but with a low confidence level. (b) Simulation of likelihood contour plot with $n = 0.85$ and $T/S = 1$ (circle) as input of future experiments with the improved experimental sensitivities possible within the next few years (see Ref. 11 for full details). The maximum likelihood (marked 'x') is close to the input signal and the gravitational wave detection is improved to the 95% confidence level.

Can CMB measurements extract the key parameters, n and T/S, needed to test inflation and construct models of large-scale structure? For the wavelengths outside the horizon when the CMB was emitted, energy-density fluctuation and gravitational waves are both static metric perturbations with nearly indistinguishable imprint on the CMB. Large-angular scale measurements, such as from COBE DMR, only provide information about the sum of the two contributions. On the other hand, CMB anisotropy at $\lesssim 1^\circ$ is dominated by wavelengths smaller than the horizon at decoupling where dynamical effects differentiate the two. Energy-density fluctuations grow due to gravitational instability, whereas the gravitational waves propagate and red shift away. The obvious tactic, therefore, is to combine large- and small-angular measurements.

To do so requires calculating the evolution of energy-density fluctuations and gravitational waves and incorporating these into a numerical code to predict their imprint on the CMB anisotropy [11]. Such a code has been written and successfully executed, and the results have been introduced into a statistical analysis code that compares the theoretical predictions to data. The initial results (see Figure 1a) are quite tantalizing. The combination of COBE DMR and small-angular scale anisotropy measurements made at the South Pole and Owens Valley suggest a large component of gravitational waves and power index $n = 0.85$. At this point, the confidence level is quite low, and the conclusions should be regarded as quite tentative. However, simulations using this code show that future, planned experiments will greatly refine the test (Figure 1b).

These calculations demonstrate that there is real reason to hope that, perhaps in just a few years, we will be able to quantitatively test inflationary cosmology and properly measure the seeds for large-scale structure. And, we may be able to determine whether COBE DMR has glimpsed the imprint of gravity's rainbow as dispersed through inflation's prism.

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