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Testing General Relativity at the Quantum Level[1]

EPHRAIM FISCHBACH[2] and BELVIN S. FREEMAN[2]

Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794

Abstract

It is shown that the effect of a gravitational field on a hydrogen atom is to admix states of opposite parity such as $2S_{1/2}$ and $2P_{1/2}$. The phase of this admixture is such as to produce circular polarization of the radiation emitted in transitions such as $2S_{1/2} \to 1S_{1/2} + \gamma$ which arises from the interference between the gravity-induced amplitude and that due to the weak neutral current. The predicted magnitude of the circular polarization, which could be sufficiently large to be detected in white dwarfs or in certain binary systems, varies from theory to theory. It is thus possible that a study of this effect could provide a feasible means of testing general relativity at the quantum level.

In a recent series of elegant experiments Colella, Overhauser, and Werner [1] (COW) have established that the quantum-mechanical behavior of thermal neutrons in a gravitational field is exactly as predicted by Newtonian gravity. Since all known theories of quantum gravity reduce to the Newtonian result in the nonrelativistic limit, their result, while important, does not allow a discrimination among competing theories at the quantum level (e.g., Einstein versus Brans–Dicke). To achieve the necessary discrimination COW would have to measure quantum effects that are characteristically smaller than the leading Newtonian contribution by a factor of $\beta^2 = (v/c)^2$, where v is the velocity of the particles being studied and c is the speed of light. Since $\beta \cong 10^{-5}$ in the COW experiment there appears to be little hope of refining this experiment to achieve the necessary sensitivity. Other proposed experiments, such as those aimed at detecting the spin precession of slow neutrons in a gravitational field,

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[2] Address after September 1, 1979: Physics Department, Purdue University, W. Lafayette, IN 47907, USA.

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are also approximately 10 orders of magnitude below the required sensitivity. It is, however, extremely important to devise tests of relativistic quantum gravity (RQG) since there is always the possibility that new gravitational effects could manifest themselves at the quantum level whose existence would not have been anticipated by an extrapolation from the macroscopic realm.

As we will see shortly, the scale of RQG effects is set by the product of β and a constant η,

$$\eta = \frac{g\hbar}{c} \tag{1}$$

where g is the local acceleration of gravity and $\hbar$ is Planck's constant. At the surface of the Earth $\eta = 2.15 \times 10^{-23}$ eV, while for a typical neutron star $\eta \approx 10^{-12}$ eV. Although there is nothing that can be done about the intrinsic weakness of the gravitational interaction, we can hope to find systems in which RQG effects are enhanced because both β and η are large, and which at the same time allow these effects to be studied by means of experimental techniques capable of great precision. What we wish to point out here for the first time is that both of these hopes may in fact be realized by the study of a new phenomenon, namely, the violation of parity selection rules for electromagnetic radiation induced by a gravitational field.

We begin with the generally covariant Dirac equation for an electron in a gravitational field, expressed in terms of a set of matrices $\gamma^\mu(x)$ which satisfy

$$\gamma^\mu(x)\gamma^\nu(x) + \gamma^\nu(x)\gamma^\mu(x) = 2g^{\mu\nu}(x) \tag{2}$$

where $g^{\mu\nu}(x)$ is the metric tensor. The matrices $\gamma^\mu(x)$ are related to the usual (constant) Dirac matrices γ^a $(a = 1, 2, 3, 0)$ by

$$\gamma^\mu(x) = e_a^\mu(x)\gamma^a \tag{3}$$

Here $e_a^\mu(x)$ are a set of tetrad fields which, for the case of a static spherically symmetric mass distribution, are given by

$$e_a^\mu(x) = \delta_{\mu a}(1 + \Phi) - 2\delta_{\mu 0}\delta_{a 0}\Phi$$

$$\Phi = \frac{-GM}{\rho c^2} \tag{4}$$

Here G is the Newtonian gravitational constant, ρ is the distance from the electron to the center of the matter distribution whose mass is M, and we assume that we are in the weak field limit $|\Phi| \ll 1$. The Dirac equation obtained from equations (2)-(4) can be expressed in terms of an equivalent Schrödinger Hamiltonian by means of a Foldy-Wouthuysen transformation. Combining the resulting expression for the electron with the analogous result for a proton we find that the Hamiltonian for a hydrogen atom in an external gravitational field is given by (neglecting various uninteresting terms)

$$H(e-p) = (m_e + m_p)\,\mathbf{g}\cdot\mathbf{R} + (\gamma + \tfrac{1}{2})\,\eta\alpha a_0 \hat{g}\cdot[\mathbf{r}\nabla^2 + \boldsymbol{\nabla} - \tfrac{1}{2} i \boldsymbol{\sigma}_e \times \boldsymbol{\nabla}] \tag{5}$$

Here $\mathbf{g}$ is the local acceleration due to gravity, m_e and m_p are the electron and proton masses, respectively, α is the fine structure constant, a_0 is the Bohr radius, $\mathbf{R}$ is the center-of-mass (CM) coordinate, and $\mathbf{r}$ is the relative $e-p$ coordinate. We have explicitly exhibited the dependence of $H(e-p)$ on the parametrized post Newtonian (PPN) parameter γ which distinguishes among various theories of gravity. The term proportional to $\mathbf{g}\cdot\mathbf{R}$ (which is the only one probed in the COW experiment) gives the classical Newtonian potential which corresponds to a uniform acceleration $g = |\mathbf{g}|$ of the CM. By contrast the remaining terms in $H(e-p)$ induce transitions in hydrogen similar to those produced by an external electric or magnetic field. We see immediately that the terms proportional to η mix eigenstates of opposite parity, and that the magnitude of this admixture depends directly on the specific theory of gravity through the PPN parameter γ. Thus, for example, $\gamma = 1$ for the usual Einstein theory whereas $\gamma = (\omega + 1)/(\omega + 2)$ for the Brans-Dicke theory. It follows that a measurement of the violation of parity selection rules in hydrogen (or in some other atom) would allow us to discriminate among competing theories of gravity at the quantum level.

For later purposes we note that the scale of RQG effects in hydrogen is set by $\eta\alpha$ where $\alpha = e^2/\hbar c = v/c$ for the lowest Bohr orbit. This exhibits the first advantage of the hydrogen atom for our purposes, namely, the fact that the electron is essentially relativistic. The second advantage arises from the fact that the operator proportional to $\eta\alpha$ has nonzero matrix elements between nearly degenerate states, such as the $2S_{1/2}$ and $2P_{1/2}$ states whose separation is the Lamb shift, $L = 4.38 \times 10^{-6}$ eV. The third, and perhaps most important, advantage of the H atom is that the ubiquitous presence of hydrogen in the universe allows us to study the parity-violating effects produced by $H(e-p)$ in regions where strong gravitational fields are present.

It is instructive to compare the magnitude of the matrix element of $H(e-p)$ in equation (5) with that of the weak neutral current Hamiltonian H_W which also mixes the $2S_{1/2}$ and $2P_{1/2}$ states. If we assume that the electron and proton couple via a simple V-A interaction mediated by a massive Z^0 boson, and ignore hyperfine effects, then [2]

$$\langle 2P_{1/2}\,|H_W|\,2S_{1/2}\rangle = \frac{i3^{1/2}G_F\alpha}{32\pi 2^{1/2}a_0^3} = i5.38 \times 10^{-17}\ \text{eV} \tag{6}$$

where G_F is the Fermi constant. The expression for H_W in the standard Weinberg-Salam model of H_W is more complicated [2], but the scale of the effects is the same as that given by equation (6). By comparison, using equation (5) and taking $\gamma = 1$ we find

$$\langle 2P_{1/2}\,|H(e-p)|\,2S_{1/2}\rangle = -\frac{3^{1/2}}{8}\alpha\eta = \begin{cases} -3.40 \times 10^{-26}\ \text{eV (Earth)} \\ -1.53 \times 10^{-21}\ \text{eV (40 Eri B)} \\ -5.57 \times 10^{-15}\ \text{eV (neutron star)} \end{cases} \tag{7}$$

In equation (7) 40 Eri B is a white dwarf with mass $0.372M_{\odot}$ and radius $0.0152R_{\odot}$. The mass and density of the neutron star have been taken to be $0.4M_{\odot}$ and 1×10^{15} g cm^{-3}, respectively. We observe from equations (6) and (7) that the phases of $\langle H_W\rangle$ and $\langle H(e-p)\rangle$ are such that their interference will produce a circular polarization P_γ in the transition $2S_{1/2} \rightarrow 1S_{1/2} + \gamma$. On Earth $|P_\gamma| \sim |\langle H(e-p)\rangle/\langle H_W\rangle| \simeq 6 \times 10^{-10}$, whereas on 40 Eri B, $|P_\gamma| \simeq 3 \times 10^{-5}$. Since circular polarization has already been detected [3] in white dwarfs at the level of $\sim 10^{-3}$, it is conceivable that such observations could be refined to the point where a gravity-induced contribution to P_γ could be detected. From equations (6) and (7) we also note that at the surface of a typical neutron star the gravity-induced $2S_{1/2}$-$2P_{1/2}$ admixture is actually much larger than that arising from the weak interactions, and they become comparable only at a distance of ≈ 10 radii from the star. If we assume that a circular polarization at the level of 10^{-4} could be detected in the foreseeable future, then accreted hydrogen as far away as $\approx 10^3$ radii from the center of the star could contribute to the effect that we are looking for.

To distinguish between gravitational and nongravitational sources of circular polarization we first note that the gravity-induced contribution to P_γ depends on $\vec{g} \times \vec{k}$ where **g** is the local acceleration at the point of emission and **k** is the photon momentum. It follows that radiation of a given frequency reaching the Earth from different regions of a source (such as the Sun) will be characterized by different values of $\mathbf{g} \times \mathbf{k}$ and hence different P_γ (see Figure 1). For radiation from distant objects which appear as point sources use can be made of the fact that some of these sources are members of eclipsing binary systems. An example of this is the system Hercules X-1/HZ Herculis [4], which is believed to contain a neutron star and accreted hydrogen orbiting a visible companion. During an eclipse (which occurs ~ every 1.7 days and lasts for several hours) $\mathbf{g} \times \mathbf{k}$ varies in a well-defined way for light reaching the Earth, and this variation can be used to discriminate between the gravitational and nongravitational contributions to P_γ. Other observations can also be used to effect such a discrimination including the variation of P_γ with $|\mathbf{g}|$, as inferred by comparing different systems, and the characteristic dependence of P_γ on the nuclear charge Z for ionized hydrogenlike atoms.

Although additional theoretical work will be needed to elaborate the details of the calculations that we have outlined here, it appears from our present

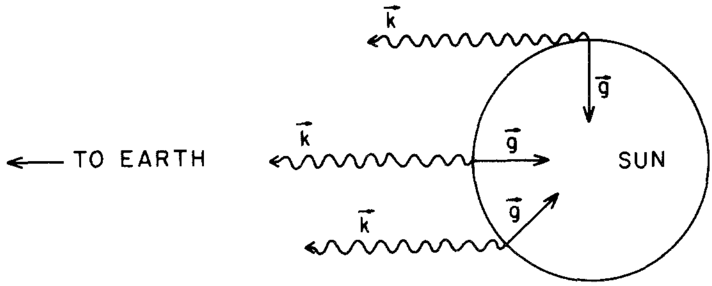

Fig. 1. Variation of P_γ, which is a function of $\vec{g} \times \vec{k}$, over the surface of the Sun.

vantage point that such a program may offer the best hope for testing general relativity at the quantum level.

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References

1. Colella, R. C., Overhauser, A. W., and Werner, S. A. (1975). *Phys. Rev. Lett.*, **34**, 1472.
2. Dunford, R. W., Lewis, R. R., and Williams, W. L. (1978). *Phys. Rev. A*, **18**, 2421.
3. Angel, J. R. P., and Landstreet, J. D. (1971). In *White Dwarfs*, Luyten, W. J., ed. (D. Reidel, Dordrecht-Holland), p. 79.
4. Gerend, D., and Boynton, P. E. (1976). *Astrophys. J.*, **209**, 562.