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Coalescing Binaries—Probe of the Universe[1]

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At present, coalescing binary systems containing neutron stars or black holes are thought to be the most likely sources of gravitational waves to be detected by long baseline interferometers being currently designed. In this essay we calculate the characteristics of the signal from a coalescing binary to the first post-Newtonian order. We show that at coalescence the eccentricity of the orbit, tidal effects, and magnetic interactions can be neglected. We also consider the effects of the expansion of the universe on the signal. We show that observations of gravitational waves from coalescing binaries by a network of detectors will provide a wealth of astrophysical information, e.g., determination of the Hubble constant, new rungs on the cosmic distance ladder, estimates of the masses of components of the binary systems, information about the mass distribution in the universe, highly accurate tests of general relativity, and constraints on neutron-star equations of state. Further development of laser interferometers may enable determination of the deceleration parameter, provide new information about evolution of the universe, and even enable observation of such effects as gravitational lensing.

1. INTRODUCTION

During the relatively long period over which gravitational radiation detectors have been developed (say from 1960 to the present), relativists have become accustomed to expecting that supernova explosions are the most important and interesting events for the detectors to see. But during the

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last five years or so a new class of gravitational wave sources has begun to occupy a central place in the thinking of the people who design the experiments: the coalescences of binary systems containing neutron stars or black holes. They were first discussed in this context by Clark and Eardley [1] but it was Kip Thorne who first understood their importance for modern detectors.

At present there are four groups around the world operating prototype Laser Interferometric Gravitational-Wave Observatories (LIGOs) at the California Institute of Technology, the Massachusetts Institute of Technology, Glasgow University, and the Max Planck Institute for Quantum Optics at Garching, near Munich. These groups, with another at Orsay, near Paris, are proposing to build a network of at least four detectors of large size (at least 1-km arm lengths) capable of making wide-band observations of gravitational waves. If these proposals are funded soon (and funding looks reasonably assured in three of the four cases), then by the end of the century they could be operating with full design sensitivity, and they could be detecting coalescing binaries over a large part of the universe.

The more one looks at what one can learn from such observations, the more astonishingly rich they become. One of us has elsewhere shown how they can be used to measure Hubble's constant more accurately and reliably than any other method [2]. Here we go further than that and show how they can lead to determinations of the masses of neutron stars, tests of general reltivity, measurements of the homogeneity of the universe, constraints on the neutron-star equation of state, and even (with a next generation of detectors) a measurement of the deceleration parameter and the mass density of the universe.

2. OBSERVABLE CHARACTERISTICS OF COALESCING BINARIES

Since LIGOs are broad-band detectors, one can observe both the strength h and frequency f of the gravitational waves at any time. Since post-Newtonian effects may well be observable, we use the calculations of Wagoner and Will [3], who studied the dynamics of a point-mass binary system in the post-Newtonian approximation. The actual response of a detector depends not only on how far away a given source is, but also on the orientations of the source and detector relative to the direction joining them. We have calculated values of the rms mean value $\langle h \rangle$, averaged over both orientations. For objects in circular orbits (an excellent approximation—see below), there is radiation at four different frequencies. The largest amplitude is the Newtonian mass-quadrupole radiation, whose frequency is twice the orbital frequency f_{orb}. There are additional contributions from current-quadrupole terms (f_{orb} and $3f_{orb}$), mass-quadrupole-mass-octupole interference ($2f_{orb} = f_N$), and mass octupole ($4f_{orb}$). Consider a system consisting of two objects with total mass $M \times M_\theta$ and reduced mass $\mu \times M_\theta$, at a distance $r \times 100$ Mpc, whose Newtonian radiation comes off at the frequency $f_N \times 100$ Hz (twice f_{orb}). We find for each frequency

$$\text{at } 0.5 f_N: \quad \langle h \rangle = 3.94 \times 10^{-25} \mu M f_N r^{-1} (\delta M/M) \tag{1a}$$

$$\text{at } f_N: \quad \langle h \rangle = 1.02 \times 10^{-23} \mu M^{2/3} f_N^{2/3} r^{-1} \times [1 - 0.034(1 - 0.514\mu/M) M^{-2/3} f_N^{2/3}] \tag{1b}$$

$$\text{at } 1.5 f_N: \quad \langle h \rangle = 9.15 \times 10^{-25} \mu M f_N r^{-1} (\delta M/M) \tag{1c}$$

$$\text{at } 2 f_N: \quad \langle h \rangle = 3.24 \times 10^{-25} \mu M^{4/3} f_N^{4/3} r^{-1} (1 - 3\mu/M) \tag{1d}$$

Here δM is the difference between the masses of the two objects.

The radiation of energy causes the period of the orbit to decrease and the radiation frequency to increase. The rate of change of f_N is given by

$$\tau = f_N/\dot{f}_N = 7.97 \mu^{-1} M^{-2/3} f_N^{-8/3} \times [1 - 0.030(1 + 1.24\mu/M) M^{2/3} f_N^{2/3}] \text{ sec} \tag{2}$$

again including all post-Newtonian corrections. By using these equations, one can predict the exact waveform: amplitude and frequency both increasing with time. It is this detailed prediction that enables one to dig into the noise and pick out coalescing binaries at great distance, as we describe below. It is important to remember that the radiation that will be detected is the orbital radiation emitted *before* the actual coalescence event.

Digging into the noise is done by matched filtering, digitally processing the data after it is obtained. The details of how the signal from coalescing binaries can be extracted from the noise have been studied elsewhere [4, 2, 5], so we will only sketch them in order to show how it is possible to detect such events at great distances. One can take advantage of a knowledge of the signal if the signal has many cycles in its wavetrain. In our case, (2) shows that at 100 Hz the signal may have of order 400 cycles before the orbit changes substantially. Digital filtering improves the signal-to-noise ratio by a factor of the square-root of this figure, i.e., about 20. Systems can be seen 20 times further with filtering than without. The key to this is the reliability of the predicted waveform.

The prediction is so reliable because the system is so clean that no other physical effects can significantly change h or τ. Any original orbital eccentricity rapidly disappears due to radiation reaction: the binary pulsar system PSR 1913+16 has eccentricity 0.62 today, but by the time its orbital period reduces to 0.02 s, the eccentricity will be of order 10^{-6}. For neutron stars, tidal angular momentum transfers will change the orbital period by less than 10^{-10} of the effect of gravitational radiation, and magnetic interactions are similarly weak if surface fields do not exceed 10^{12} G. The system remains clean until the neutron stars become so close that mass transfer (Roche-lobe overflow) begins. This will not occur until f_N exceeds 200 Hz if both stars exceed 0.3 M_θ [1]. A black-hole binary system is even cleaner, of course.

If the source is at a coscomological distance with redshift z, then the above formulas apply, with the interpretation that r is the *luminosity distance*, f_N is the observed frequency of the quadrupole radiation, and any mass m is replaced by $(1+z)m$.

Although one can predict the shape of the signal with confidence, there is considerable uncertainty about the rate of coalescence events. For neutron-star binaries, an early estimate based on the fact that the binary pulsar is the only known system heading for coalescence in a cosmologically short time (10^8 yr) suggests, on the basis of the known pulsar birth rate, that there will be three coalescence of Population I neutron-star binaries per year within a distance of 100 Mpc [6]. Recent improvements in the statistics of pulsars in binary systems reinforce this conclusion. But this number is very uncertain. If Population II stars formed neutron-star binaries, as seems likely, then the rate could be 10 times higher. On the other hand, if the mean number of binaries is smaller than one would estimate from the proximity of the binary pulsar to us (small number statistics), or if pulsars in binaries are not typical of pulsars as a whole, then the rate could be 100 times smaller. We adopt the figure of three per year out to 100 Mpc, bearing in mind its uncertainty. Even more uncertain is the black-hole binary coalescence rate, which may be anything between zero and the neutron-star binary rate.

3. WHAT WE CAN LEARN FROM OBSERVATIONS OF COALESCING BINARIES

In order to calculate what we can learn from observing these systems, we will here assume that the observations can determine enough information about the waves to enable one to calculate the orientation-averaged amplitude $\langle h \rangle$ for any event. We have shown elsewhere [2] that a network of four detectors can do this for events that have sufficient signal to noise ratio. Here we just assume we can use (1) and (2).

The dominant radiation is the mass-quadrupole radiation, which is the first term in (1b). Inspection of this and of the equation for τ, (2), shows that the masses μ and M enter both in the same combination, so that the product $\langle h \rangle \tau$ is independent of the objects' masses, to leading order. Since the frequency f_N is also observed, we can immediately deduce the distance to the system

$$r = 8.13 f_N^{-2} / \langle h_{23} \rangle \tau + \cdots \tag{3}$$

where h_{23} is $h \times 10^{23}$, τ is measured in seconds and r in units of 100 Mpc, and the ".." indicates that the post-Newtonian corrections have been omitted. Therefore, *gravitational wave measurements can give the exact distance to the binary system*, independent of any guesses about the masses of the objects. This is probably the cleanest known extragalactic distance measure.

3.1. Observations with Planned LIGOs

The proposed LIGOs expect to reach a sensitivity that would enable them to see a coalescing binary signal with a signal-to-noise ratio S/N of

$$S/N = 40\, r^{-1} \mu^{1/2} M^{1/3} f_N^{-7/6} \tag{4}$$

(See [4] for a full discussion.) When four detectors are operating, a four-way coincidence with a S/N of 4 in each will occur much less often than once per year, so we will adopt this as our detection criterion. Then (4) shows that the planned detectors should see neutron-star binaries out to roughly 800 Mpc, and 10 M_θ black-hole binaries out to 4 Gpc. What can these teach us?

(i). *Hubble's constant.* Events within 100 Mpc have such large signal-to-noise ratio that a determination of the distance to the system accurate to a few percent is possible, and of the direction to $\pm 3^\circ$ [2]. If a coalescence is accompanied by an optically visible supernova-like event, then by measuring the redshift of the "host" galaxy one can determine H_0 to a few percent, much better than we know it today. Even if no optical event is seen, if there are enough such events a statistical method will suffice to determine H_0 after 10–20 events are recorded [2].

(ii). *Standard mileposts.* Apart from Hubble's constant, the determination of the distance to galaxies that are hosts to optically detected coalescence events provides mileposts for the calibration of all sorts of distance measures and intrinsic luminosity measurements.

(iii). *Masses of neutron stars and black holes.* Detecting the dominant mass-quadrupole radiation allows us to measure only the combination

$\mu M^{2/3}$. The statistics of this number may convey some information, in particular about the maximum mass of neutron stars and the existence of massive black holes in binaries, provided that the event rate is high enough to generate good statistics (predicted to be four per day out to 800 Mpc). Possibly more significant is the fact that in nearby sources with high S/N it may be possible to detect the post-Newtonian terms in (1) and (2). If so, then they will give independent information that will allow a determination of the individual masses of the objects.

(iv). *Mass distribution in "local" region of the universe.* Again, given good enough statistics, events out to 800 Mpc (where the Hubble redshift should be 0.1–0.2) can be used to test for isotropy, homogeneity, superclustering, and the existence of voids. At these distances, galaxies begin to be difficult to detect, and complete surveys of them are impractical. Binary coalescences ought to be randomly distributed among galaxies, so they should be ideal survey objects. The quality of distance and direction information degrades with distance, so the surveys will yield more detailed information at 400 Mpc than at 800.

(v). *Tests of general relativity.* Any gravitational wave observation of sufficient S/N to be observed by four detectors will test Einstein's predictions regarding gravitational wave polarization: four detections overdetermine the solution for a transversely polarized quadrupole wave, so any inconsistency among them would be evidence for other polarization states [2]. If an event is accompanied by an optical detection, then a test of the speed of propagation of gravitational waves is possible: since the optical emission should brighten up within a day after the gravitational waves leave, the arrival at Earth of the two emissions, with a day or so separating them, would be evidence that the gravitational waves travel at the speed of light to an accuracy of better than one part in 10^9. Coalescence events provide other tests as well: the post-Newtonian terms predicted in (1) and (2) might be independently detectable in radiation at four different frequencies and must be consistent with one another. A black-hole coalescence event would be the strongest evidence one could imagine for the existence of black holes: not only would it identify the holes, but it would also test the predictions of general relativity regarding their radiation in the strong-field limit. (By the time observations are possible, numerical calculations should have determined these predictions to a high accuracy.)

(vi). *Neutron-star equation of state.* Once a coalescing neutron-star binary has been detected, it can be followed until the S/N drops to about 1. For moderately strong sources, this should enable one to get information about the dynamics of the actual coalescence of the two objects: when mass transfer begins, whether the coalescence is accompanied by collapse to a black hole, etc. Numerical simulations should be able to transform such observations into constraints on the neutron-star equation of state.

3.2. Observations with Possible Future LIGOs

The limits on the sensitivity of planned LIGOs can in principle be circumvented to make more sensitive detectors, but not easily. A combination of higher laser power, better and heavier mirrors, longer baselines, the use of "squeezed states," and active seismic isolation, might achieve a factor of 5–10 in S/N. If the "standard quantum limit" cannot be circumvented [4], then the S/N will improve more at the higher frequencies, but the benefit of this will be limited by the frequency at which coalescence takes place, redshifted as appropriate for distant sources. In what follows we assume a factor of 10 improvement on (4). This will illustrate what the incentive can be to improve the technology. Neutron-star binaries can then be seen to a luminosity distance of 8 Gpc, which corresponds to a redshift of 1 or 2. The event rate will not be a problem with such a volume of galaxies: even on the most pessimistic assumptions, there would be thousands of events per year. Black-hole binaries would be seen at essentially any redshift, and those at redshifts of order unity would have $S/N > 40$.

The improved statistics and greater S/N of nearby events will improve the information gained in (iii)–(vi) above. In addition, new possibilities open up.

(vii). *Deceleration parameter and the mass density of the universe.* At first it might seem that observing coalescing binaries at cosmological distances would not give the deceleration parameter, because the waves contain no intrinsic measure of the redshift of the system. The radiation arises from purely gravitational effects, which have an arbitrary mass (or time) scale in them. So, as noted above, only the combination $(1+z)m$ can be determined from the waveform. But, with luck, there are two ways in which nongravitational neutron-star physics can affect the observations in a way that permits one to determine the mass scale, and hence the redshift. Once the redshift is known, it can be compared to the luminosity distance to get q_0, which in turn measures the mean mass density of the universe.

The first method involves observing the actual coalescence of the neutron stars. The details of the frequency at which mass transfer begins, and of the subsequent evolution of the system, should depend only on the masses of the two stars, since the system is so clean. But because it will depend on the neutron-star equation of state, it will not scale with redshift in a simple way. By comparing the observed distant system with a library of local ones observed by the first generation of detectors, it should prove possible to determine the true mass parameter $\mu M^{2/3}$ of the system, and hence its redshift.

The second possibility involves gathering statistics on the mass parameter observed in distant systems, which scales as $(1+z)^{5/3}$, and comparing it with the distribution determined from local observations. If its upper limit is associated with the maximum possible mass of a neutron star, then this calibrates the upper limit observed for the distant systems, and allows the deceleration parameter to be determined statistically.

Such determinations of q_0 have a great advantage over other methods: they are not sensitive to evolutionary effects, such as the changing chemical composition of the universe. This is because they involve only the neutron-star equation of state, which will not depend on the chemical composition of the progenitor star. Therefore, these methods may prove valuable even if other methods have already given a value for q_0, e.g., observations with the Hubble Space Telescope.

(viii). *Evolutionary effects.* If the chemical evolution of the universe changes the rate at which neutron stars and blck holes are formed, then these should presumably be reflected in observable changes in the coalescence event rate with distance.

(ix). *Gravitational lensing.* At quasar distances, coalescing systems are likely to be gravitationally lensed as often as quasars. Since LIGOs are essentially omnidirectional, they will automatically detect all images of the event that have sufficient brightness. But since different images traverse different paths through the lensing galaxy or cluster, the events will arrive at different times, separated perhaps by months to years. The signature of a lensed event will be the arrival of a later event at a nearly position with the same waveform (different images have the same redshift). Once lensing is detected, weaker images may be sought by matched filtering using the observed waveform. Gravitational waves can convey *more* information about the lens then optical QSO measurements can: in general, the waves are elliptically polarized, and so measuring the *sense* of the polarization will distinguish those secondary images that are parity-inverted from those that are the right way around. This is in addition to determining the relative amount of rotation each image has undergone, by comparing the orientation of the polarization ellipses. These measurements will allow better reconstruction of the mass distribution of the lens.

(x). *Redshift at which star formation began.* If black-hole binaries are formed in reasonable numbers (a few coalescences per galaxy per Hubble time), then after a few years of observation, a clear maximum on their luminosity distance should become apparent. This would pinpoint the epoch at which star formation began.

4. CONCLUSION

The list we have given here is surely not exhaustive, but it does already illustrate the importance of the effort to fund and build gravitational wave detectors. Gravitation theory blossomed in the past two decades because electromagnetic observations showed how important general relativity is for astrophysics. Now it begins to look like gravity will be able to make its own observations, and indeed to feed back information into the mainstream of astronomy. By observing coalescing binaries all over the universe, gravitational wave astronomy will become an equal partner with the other branches of observational astronomy.

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