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Thermodynamics and Cosmology[1]

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A new type of cosmological history which includes large-scale entropy production is proposed. These cosmologies are based on a reinterpretation of the matter-energy stress tensor in Einstein's equations. This modifies the usual adiabatic energy conservation laws, thereby leading to a possible irreversible matter creation. This creation corresponds to an irreversible energy flow from the gravitational field to the created matter constituents. This new point of view results from the consideration of thermodynamics of open systems in the framework of cosmology. It appears that the usual initial singularity is structurally unstable with respect to irreversible matter creation. The corresponding cosmological history therefore starts from an instability of the vacuum rather than from a singularity. The universe evolves through an inflationary phase. This appears to be an attractor independent of the initial vacuum fluctuation.

Very few physical theories are in such a paradoxical situation as cosmology today. On the one hand, our universe is characterized by a considerable entropy content, mainly in the form of the blackbody radiation. On the other hand, classical Einstein's equations are purely adiabatic and reversible and, consequently, can hardly provide, by themselves, an explanation relating to the origin of cosmological entropy.

On the contrary, matter constituents may be produced quantum-mechanically in the framework of Einstein's equations. The energy of these produced particles is then extracted from that of the (classical) gravitational field [1]. But these semiclassical Einstein's equations are

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adiabatic and reversible as well and, consequently, also unable to provide the entropy burst accompanying the production of matter.

The aim of the present work is to overcome this problem. We propose a phenomenological macroscopical approach allowing for both particles and entropy production in the early universe. We show that thermodynamics of open systems [2], as applied to cosmology, leads very naturally to a reinterpretation in Einstein's equations of the matter stress-energy tensor [3]. This would take into account both matter and entropy creation on a macroscopical level. With this in view, we extend the concept of adiabatic transformation from closed to open systems. This applies to systems in which matter creation occurs.

This consideration leads to an extension of thermodynamics as associated with cosmology. Traditionally, in addition to the geometrical state of the universe, the two physical variables describing the cosmological fluid are the energy-density ρ and the pressure p. Einstein's equations are then solved assuming an equation of state $\rho = \rho(p)$. In our case, however, a supplementary variable, the particle density n, enters naturally into the description. This leads to an enlargement of traditional cosmology, which is presented here. An important conclusion is that, in these circumstances, creation of matter can occur only as an irreversible process, corresponding to an irreversible transfer of energy from the gravitational field to the created matter. It is quite satisfactory that this irreversible creation of matter generates cosmological entropy. This process follows the second law of thermodynamics and therefore appears to be thermodynamically possible.

Moreover, it is shown that the big-bang singularity of traditional cosmology is structurally unstable with respect to irreversible matter creation. A cosmology which includes such a phenomenon starts from an instability [4] of the Minkowski vacuum, and no longer from a singularity. We specify these properties in the framework of a simple phenomenological model of irreversible particle production. This model provides a cosmological history which evolves in three stages. First, a creation period drives the cosmological system from an initial fluctuation of the vacuum to a de Sitter space. Second, this de Sitter space exists for the decay time of its constituents. Finally, a phase transition turns this de Sitter space in a usual Robertson–Walker (RW) universe, which extends to the present. Entropy creation occurs only during the two first cosmological stages, while the RW universe evolves adiabatically on the cosmological scale.

A fundamental fact is that the de Sitter regime appears as an attractor whose parameters are independent of the characteristics of the initial fluctuation. This implies, in turn, that all the physical parameters characterizing the present RW stage are independent of this initial fluctuation.

In particular, the specific entropy per baryon S depends only on two characteristic times of the theory: the creation period time τ_c and the de Sitter decay time τ_d.

It can be noticed that this cosmological model is not part of traditional cosmology which leads to entropy conserving evolution throughout the whole cosmological history.

The traditional Einstein's equations

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{1}$$

as applied to isotropic and homogeneous universes, involve the macroscopic stress tensor $T_{\mu\nu}$ which corresponds to a perfect fluid. It is characterized by a phenomenological energy density ρ and pressure $\tilde{p}$ given by

$$\rho = T_0^0 \qquad \text{and} \qquad \tilde{p}\delta_j^i = T_j^i \tag{2}$$

In addition to Einstein's equations (1) we use the Bianchi identities

$$G^{\mu}_{\nu;\mu} = 0$$

which, for homogeneous and isotropic universes, lead to the well-known relation

$$d(\rho V) = -\tilde{p}\, dV \tag{3}$$

In traditional cosmology, this equation is used to describe an adiabatic evolution for a closed system (any arbitrary comoving volume V) and one then interprets $\tilde{p}$ as the true thermodynamical pressure.

On the contrary, in the presence of matter creation, the appropriate analysis is performed in the context of open systems [2]. In this case, the number of particles N in a given volume V is not fixed to be constant. In the case of adiabatic transformation ($dQ = 0$), the thermodynamical energy conservation law reads

$$d(\rho V) + p\, dV - \frac{h}{n}\, d(nV) = 0 \tag{4}$$

where $n = N/V$ and $h = \rho + p$ is the enthalpy per unit volume. In such a transformation, the "heat" received by the system is due entirely to the change of the number of particles. In our cosmological context, this change is due to the transfer of energy from gravitation to matter. Hence, *the creation of matter acts as a source of internal energy.*

Correspondingly, the entropy change dS, which vanishes for adiabatic transformation in a closed system as is the case in traditional cosmology, for adiabatic transformation in open systems is given as follows:

$$T\,dS = \frac{h}{n}\,d(nV) - \mu d(nV) = T\frac{s}{n}\,d(nV) \tag{5}$$

where $\mu n = h - Ts$ is the chemical potential and $s = S/V$. Therefore, according to the second law of thermodynamics, the only particle number variations admitted are such that

$$dN = d(nV) \geqslant 0 \tag{6}$$

This inequality, in the present cosmological framework of open systems, implies that space-time can produce matter, while the reverse process is thermodynamically forbidden. The relation between space-time and matter ceases to be symmetrical, since particle production, occurring at the expense of gravitational energy, appears to be an irreversible process.

The relation (4) can be written in a number of equivalent forms such as:

$$\dot{\rho} = \frac{h}{n}\,\dot{n} \tag{7}$$

$$p = \frac{n\dot{\rho} - \rho\dot{n}}{\dot{n}} \tag{8}$$

or

$$p = -\frac{\partial e}{\partial v} \tag{9}$$

with the definition

$$e = \frac{\rho}{n}, \qquad v = \frac{1}{n}$$

An overdot denotes the derivative with respect to time. It is interesting to note that energy creation $\dot{\rho}$ and particle creation $\dot{n}$ determine the pressure p. As examples, let us note that

$$\rho = mn \rightarrow p = 0 \tag{10}$$

and furthermore,

$$\rho = aT^4, \qquad n = bT^3 \rightarrow p = \frac{\rho}{3} \tag{11}$$

It is this thermodynamical analysis of open systems which provides the appropriate framework for cosmology, in the presence of matter creation. This is realized owing to a reinterpretation of the pressure in the stress-energy tensor. More precisely, creation of matter corresponds to a supplementary pressure p_c, which must be considered as part of the phenomenological pressure $\tilde{p}$, as we may write (4) in a form similar to (3), namely,

$$d(\rho V) = -(p + p_c)\, dV = -\tilde{p}\, dV$$

where p is the true thermodynamical pressure and

$$p_c = -\frac{h}{n}\frac{d(nV)}{dV} = -\frac{\rho + p}{n}\frac{d(nV)}{dV} \tag{12}$$

p_c is negative or zero depending on the presence or absence of particle production.

We now apply to cosmology these general considerations concerning open systems. In the case of an isotropic and homogeneous universe, we choose for V the value

$$V = R^3(\tau)$$

then

$$p_c = -\frac{\rho + p}{3nH}(\dot{n} + 3Hn) \tag{13}$$

where $R(\tau)$ is the RW function, and $H = \dot{R}/R$ the Hubble function.

Because of the thermodynamical inequality (6), this extension of Einstein's equations to open systems now includes the second law of thermodynamics. This implies that, in the presence of matter creation, the usual Einstein's equations

$$\begin{aligned} \kappa\rho &= 3H^2 + \frac{k}{R^2} \\ \dot{\rho} &= -3H(\rho + p) \end{aligned} \tag{14}$$

become

$$\begin{aligned} \kappa\rho &= 3H^2 + \frac{k}{R^2} \\ \dot{\rho} &= \frac{\dot{n}}{n}(\rho + p) \end{aligned} \tag{15}$$

with

$$\frac{\dot{S}}{S}=\frac{\dot{N}}{N}=\frac{\dot{n}+3Hn}{n} \tag{16}$$

The corresponding new cosmologies are more general, because they involve three functions, ρ, p, and n, rather than ρ and p only. We have, for instance, a class of de Sitter spaces with $\dot{\rho}=\dot{n}=0$, arbitrary pressure p, and $p_c=-h$.

Therefore our approach "rehabilitates" (for example) the de Sitter universe, which is now compatible with the existence of matter endowed with a usual equation of state. We may even consider classes of different de Sitter universes [see Eqs. (10) and (11)], such as "incoherent" de Sitter universes ($\rho=mn$, $n=cst$, $p=0$) and or "radiative" de Sitter spaces ($T=cst$, $n=bT^3$, $\rho=aT^4$).

It has often been suggested that the expansion of the universe provides the arrow of time. A transition from an expanding universe into a contracting one would then invert the arrow of time. We do not confirm this idea, as the inequality (6) implies only that

$$\dot{n}+3Hn\geqslant 0 \tag{17}$$

which is compatible with $H\geqslant 0$, $H=0$, and $H\leqslant 0$. However, in the case of a de Sitter universe, in which $\dot{\rho}=0$, the relation (17) reduces to $H\geqslant 0$ by virtue of the relation (15).

Only an *expanding de Sitter universe is thermodynamically possible.* In our view, the arrow of time is provided by the transformation of gravitational energy into matter. In special cases, such as the de Sitter universe, this indeed prescribes an expansion of the universe.

In order to exemplify a nontraditional cosmology which includes particle creation, we present the simplest possible phenomenological model wherein the irreversible creation phenomenon is expressed in terms of the Hubble function H as follows:

$$\frac{1}{R^3}\frac{d(nR^3)}{d\tau}=\alpha H^2\geqslant 0 \qquad \text{with} \quad \alpha\geqslant 0 \tag{18}$$

To complete the model, we assume the simple relation $\rho=Mn$, hence $p=0$ according to the relation (10).

For $\alpha=0$ we recover as the solution for the spatially flat Einstein's equation

$$\kappa\rho=3H^2$$

the usual RW description with its typical big-bang singularity. However, for $\alpha \neq 0$, using (15), we obtain

$$\begin{gathered} p=0 \\ \rho=\frac{3}{\kappa} H^2 \\ \frac{1}{nR^3} \frac{d(nR^3)}{d\tau}=\frac{\alpha\kappa M}{3} \geqslant 0 \end{gathered} \tag{19}$$

This leads to

$$N(\tau)=N_0 e^{\alpha\kappa M\tau/3}$$

and

$$R(\tau)=[1+C(e^{\alpha\kappa M\tau/6}-1)]^{2/3}$$

where

$$C=\frac{9}{\kappa M\alpha}\left(\frac{\kappa M n_0}{3}\right)^{1/2}$$

The universe emerges without singularity ($R \neq 0$) at $\tau=0$, with a particle density n_0 describing the initial Minkowskian fluctuation. It therefore follows that the presence of dissipative particle creation ($\alpha \neq 0$) leads to the disappearance of the big-bang singularity. In other words, this singularity is *structurally unstable* with respect to irreversible particle creation. Hence, such a cosmology starts from an instability ($n_0 \neq 0$), and not from a singularity.

After a characteristic time

$$\tau_c=\frac{6}{\alpha\kappa M} \tag{20}$$

the universe reaches a de Sitter regime characterized by

$$\begin{gathered} R_d(\tau)=C^{2/3} e^{\alpha\kappa M\tau/9}=C^{2/3} e^{2\tau/3\tau_c} \\ H_d=\frac{\alpha\kappa M}{9}=\frac{2}{3\tau_c} \\ n_d=\frac{\kappa M}{27}\alpha^2 \end{gathered} \tag{21}$$

It is remarkable that all the de Sitter physical quantities such as H_d, n_d, and ρ_d are independent of C. This cosmological state therefore appears to be an attractor independent of the initial fluctuation. The de Sitter stage survives during the decay time τ_d of its constituents and then connects continuously (up to the first derivatives) to a usual (adiabatic) matter-radiation RW universe characterized by a matter-energy density ρ_b, and radiation energy density ρ_γ, related to the RW function by

$$\kappa\rho_b = \frac{3a}{R^3}, \qquad \kappa\rho_\gamma = \frac{3b}{R^4}, \qquad \text{and} \qquad \rho_\gamma = \frac{\pi^2}{15} T^4 \tag{22}$$

a and b are constants related to the total number N_b of baryons and photons N_γ in a volume R^3, and T is the blackbody radiation temperature. The connection at the decay time τ_d between the de Sitter and the matter-radiation regimes fixes the constants a and b:

$$\begin{aligned} a &\simeq 2H_d^2 C^2 e^{2H_d\tau_d} \\ b &\simeq H_d^2 C^{8/3} e^{4H_d\tau_d} \end{aligned} \tag{23}$$

This implies that the (constant) specific entropy S per proton is

$$S = \frac{n_\gamma}{n_b} = \frac{\zeta(3)}{3\pi^2}\left(\frac{45}{\pi^2}\right)^{3/4} \kappa^{1/4} m_b \left(\frac{3\tau_d}{2}\right)^{1/2} e^{2\tau_d/3\tau_c} \tag{24}$$

where m_b stands for the proton mass. Similarly, the value of the adiabatic invariant $\rho_\gamma/T\rho_b$ is:

$$\frac{\rho_\gamma}{T\rho_b} = \frac{b^{3/4}}{a}\left(\frac{\pi^2\kappa}{45}\right)^{1/4} \tag{25}$$

Hence these values [(24) and (25)] are fixed entirely by the knowledge of the two characteristic times τ_c and τ_d [see (21)]. In our previous work [5] the subject was treated quantum-mechanically. In this context, both quantities τ_c and τ_d were expressed in terms of one single parameter, namely, the mass M of the produced particles. These values are

$$\tau_d = \frac{640}{81\pi}\kappa^2 M^3 = \frac{640}{81\pi}\frac{M^3}{M_p^4} \simeq 2.5\left(\frac{M}{M_p}\right)^3 \tau_p$$

and

$$\tau_c = \frac{2}{3H_d} = \left(\frac{20}{\pi^2}\right)^{1/2}\frac{M^2}{M_p^3} \simeq 1.42\left(\frac{M}{M_p}\right)^2 \tau_p$$

where M_p and τ_p are the Planck mass and the Planck time, respectively.

Although highly sensitive (exponentially) to the value of the mass M, it is quite remarkable that correct observed values for S, namely, $10^8 \leqslant S \leqslant 10^{10}$, are then obtained for values of the mass M very close to the quantum-mechanically produced one (53.3 M_p) [6], for example,[4]

$$M/M_p = 40 \to S = 8.46\ 10^2$$
$$M/M_p = 50 \to S = 1.38\ 10^8$$
$$M/M_p = 53.3 \to S = 7.17\ 10^9$$
$$M/M_p = 60 \to S = 2.16\ 10^{13}$$

This fact provides an unexpected link between the microscopical and the macroscopical approaches.

Moreover, the present blackbody temperature is deduced from the continuity requirements to be

$$T_p = \left(\frac{45}{\pi^2}\kappa^{-1}\right)^{1/4} \frac{b^{1/4}}{a^{1/3}} H_p^{2/3} \tag{26}$$

$$T_p(^\circ\mathrm{K}) \simeq 2.82\ 10^{-9} \left(\frac{H_p}{75\ \mathrm{km/s/Mpc}}\right)^{2/3} \left(\frac{M}{M_p}\right)^{1/3} e^{0.3926 M/M_p} \tag{27}$$

where H_p is the presently observed value for the Hubble function: $50 \leqslant H_p(\mathrm{km/s/Mpc}) \leqslant 100$.

Values well in the range of the observed blackbody radiation temperature (2.7°K) are also obtained with the same values of the mass M:

$$M/M_p = 50 \to T_p = 3.49 \left(\frac{H_p}{75\ \mathrm{km/s/Mpc}}\right)^{2/3} {}^\circ\mathrm{K}$$

In conclusion, a main feature of our model is that it takes into account the second law of thermodynamics at the cosmological scale, from the very beginning. Indeed, the energy transfer from space-time curvature to matter appears to be an irreversible process leading to a burst of entropy associated with the creation of matter. It follows therefore that the distinction between space-time and matter is provided by entropy creation. The question of the cosmological arrow of time is far from being a simple one, as both expansion and contraction can occur reversibly, as in

[4] When one takes into account the N helicity states associated with the massless particles present in the cosmological medium, the critical mass M is increased by a factor of $O(\sqrt{N})$. Moreover, this critical mass can be showed to originate in the Minkowskian vacuum in a process very similar to the inverse Hawking black-hole evaporation process. These two points will be reported in detail in a forthcoming publication.

traditional cosmology. However, the conditions for the creation of matter from space-time involve the sign of the Hubble function. As mentioned above, in the case of the de Sitter universe, expansion only is thermodynamically possible. As, according to the nontraditional cosmology presented here, the universe always develops through a de Sitter stage, there is indeed a direct relation between the existence of cosmological entropy and the expansion of the universe.

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