THEORY OF FILTRATIONAL COMBUSTION OF GASES

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In [1, 2], the laws of filtrational combustion of gases were considered, and it was shown that internal (interphase) and external (between the system and the surrounding medium) heat transfer plays a significant role. The presence of finite heat transfer in the combustion wave leads to the possibility of steady heat propagation on account of a high-temperature gas-phase chemical reaction in relatively narrow pore channels. This situation is characterized by low linear propagation velocities and a significantly two-temperature structure of the heat wave (LVC).

From physical considerations, it may be expected that, with increase in intensity of internal heat transfer (for example, with decrease in pore-channel diameter), the temperature difference of the phases in the combustion wave will decrease and in the limiting case the two-temperature structure of the wave degenerates to a one-temperature structure. The solid phase will play the role of a homogeneous heat-conducting inert additive here. On the other hand, with decrease in internal heat transfer, the process should pass to a new situation, in which the solid phase, actually eliminated from the sphere of thermal interaction in the flame region, will not influence the gas-phase combustion. This case of flame propagation in an inert porous medium, with high linear velocities on account of the transfer properties of the gas (HVC), was investigated experimentally in [3, 4].

Thus, with variations in the interphase heat transfer, significant change in the role of the solid phase in the filtrational combustion of the gases may be expected and, as a result, significant change in the structural and velocity characteristics of the waves. The present work is devoted to the theoretical investigation of these problems.

1. Formulation of the Problem

Steady adiabatic plane combustion waves in an infinite porous inert medium are considered, with the gas-mixture fuel supplied to the reaction zone by a filtrational flux. The equations describing the steady propagation of the flame front in a coordinate system fixed in the wave take the form

$$\begin{aligned}
&\frac{d}{d\xi}\varkappa_\eta\frac{d\eta}{d\xi}-\frac{d\eta}{d\xi}-\tau w(\eta, y)=0,\\
&\frac{d}{d\xi}\varkappa_T\frac{dy}{d\xi}-\frac{dy}{d\xi}+\tau y_b w(\eta, y)-\alpha(y-z)=0,\\
&\frac{d}{d\xi}\varkappa_\Theta\frac{dz}{d\xi}-(\omega-1)\frac{dz}{d\xi}+\alpha(y-z)=0.
\end{aligned}\tag{1.1}$$

The system of equations (1.1) is written in dimensionless variables: η is the relative mass concentration of the component present in inadequate amounts; $y = (T - T_0)/(T_r - T_0)$; $z = (\Theta - T_0)/(T_r - T_0)$; T and Θ are the temperatures of the gas and solid phase; T_0 is the temperature of the initial mixture; T_r is the minimum gas temperature in the reaction zone; $\xi = x/L$; x is an independent spatial variable; $L = [m\lambda_{Tr} + (1 - m)\lambda_{\Theta r}]/mc_T G$; m is the porosity; $\lambda_{Tr} = \lambda_T(1)$; $\lambda_{\Theta p} = \lambda_\Theta(1)$; $\lambda_T(y)$ is the thermal conductivity of the gas; $\lambda_e(y)$ is the effective longitudinal thermal conductivity of the inert layer; c_T = const is the specific heat of the gas; $G = \rho_T(v - u)$ is the flux density of gas (the mass velocity); $\rho_T(y)$ is the gas-phase density; v(y) is the flow velocity; u is the wave velocity.

From the continuity equation, it follows that

$$G = \rho_{T0}(v_0 - u) = \text{const}, \quad \rho_{T0} = \rho_T(0), \quad v_0 = v(0).$$

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Also

$$\varkappa_\eta = \rho_{T\Theta} D/GL;\ \varkappa_T = \lambda_T/c_T GL;\ \varkappa_\Theta = (1-m)\lambda_\Theta/mc_T GL;$$

D is the diffusion coefficient; $\alpha = \alpha_0 S_{sp} L/mc_T G$; α_0 is the heat-transfer coefficient; S_{sp} is the specific surface of the layer; $\omega = [v_0 - (1+\sigma)u]/v_0 - u$; $\sigma = (1-m)c_\Theta \rho_\Theta / mc_T \rho_T$; c_Θ and ρ_Θ are the specific heat and density of the solid phase; $y_b = (T_b - T_0)/(T_r - T_0)$; T_b is the adiabatic combustion temperature of the gas mixture; $\tau = \frac{L\rho_T}{G} k_0 \exp(-1/\beta)$; $\beta = RT_r/E$; E is the activation energy; R is the universal gas constant; k_0 is the preexponential factor. An irreversible first-order reaction is considered

$$w(\eta, y) = \eta \exp\left[\frac{1}{\gamma}\frac{y-1}{1+\delta(y-1)}\right],\quad \delta = \frac{T_r - T_0}{T_r},\quad \gamma = \frac{\beta}{\delta}. \tag{1.2}$$

The boundary conditions for the system of equations (1.1) take the form

$$\xi = -\infty:\quad \eta = 1,\quad y = 0,\quad z = 0, \tag{1.3a}$$

$$\xi = +\infty:\quad \eta = 0,\quad \frac{dy}{d\xi} = 0,\quad \frac{dz}{d\xi} = 0. \tag{1.3b}$$

The order of system (1.1) may be reduced. Premultiplication of the first equation by y_b, adding all three equations, and integrating the result from $-\infty$ to ξ with the boundary conditions in Eq. (1.3a) gives

$$y_b \varkappa_\eta \frac{d\eta}{d\xi} + \varkappa_T \frac{dy}{d\xi} + \varkappa_\Theta \frac{dz}{d\xi} - y_b(\eta - 1) - y - (\omega - 1)z = 0. \tag{1.4}$$

It follows from the third relation in system (1.1), together with (1.3b) and (1.4), that $y = z = y_e$ when $\xi = +\infty$, where $y_e = y_b/\omega$ is the dimensionless equilibrium temperature. The condition $\omega > 0$ follows from the obvious requirement $y_e > 0$. Note that the conditions $\omega < 1$, $\omega > 1$, and $\omega = 1$ correspond to wave-velocity values $u > 0$, $u < 0$, and $u = 0$.

Introducing the variable $p(\xi)$ according to the formula

$$p = y + (\omega - 1)z - \varkappa_T \frac{dy}{d\xi} - \varkappa_\Theta \frac{dz}{d\xi}. \tag{1.5}$$

the problem of flame-front propagation with filtrational gas combustion takes the form

$$\begin{aligned} y_b \varkappa_\eta \frac{d\eta}{d\xi} &= y_b(\eta - 1) + p, \\ \frac{dp}{d\xi} &= \tau y_b w(\eta, y). \end{aligned} \tag{1.6}$$

The equation for y is Eq. (1.5), and the third relation in system (1.1) remains the equation for z

$$\xi = -\infty;\quad p = 0;\quad \xi = +\infty:\quad p = y_b. \tag{1.7}$$

The problem of finding the eigenvalue u will be solved for large activation energies or, in the notation adopted, under the condition

$$\beta \ll 1. \tag{1.8}$$

The possibility of using a Frank-Kamenetskii transformation of the reaction-rate function in the vicinity of y = 1 follows from (1.8)

$$w_0(\eta, y) = \eta \exp[(y-1)/\gamma]. \tag{1.9}$$

In view of (1.8), some modification of the approach proposed in [5] may be used; this approach is referred to below as the method of contrary extrapolation. The limiting case of filtrational combustion of a gas in conditions of infinitely intense heat transfer ($\alpha = \infty$) is considered separately below. The method of matched asymptotic expansions is used to obtain an algebraic equation for the zero approximation of the eigenvalue of the problem, which transforms, under certain formal assumptions, with an accuracy of $O(\beta)$, to the well-known formulas for the propagation velocity of a laminar flame front Le $\neq$ 1 [6, 7] and a flame in a condensed medium (Le = 0) [8, 9]. For the same limiting case, the method of contrary extrapolation is demonstrated. Further, using this method, an equation is found for the wave velocity at arbitrary α.

2. Determining the Wave Velocity When $\alpha = \infty$

Passing to the limit as $\alpha \to \infty$ in the equation for z, it is found that $y \equiv z$, and the corresponding single-temperature model takes the form

$$\begin{aligned} y_b \varkappa_\eta \frac{d\eta}{d\xi} &= y_b(\eta - 1) + p, \\ \frac{dp}{d\xi} &= \tau y_b w(\eta, y), \\ \varkappa \frac{dy}{d\xi} &= \omega y - p, \end{aligned} \tag{2.1}$$

where $\varkappa = \varkappa_T + \varkappa_e$, with the boundary conditions

$$\begin{aligned} &\xi = -\infty\colon\ \eta = 1,\ p = 0,\ y = 0, \\ &\xi = +\infty\colon\ \eta = 0,\ p = y_b,\ y = y_e. \end{aligned} \tag{2.2}$$

When $\varkappa_\eta = \varkappa_T = 0$, the given model formally coincides (in the appropriate notation) with the model of reaction-front propagation in a motionless catalyst layer [10], and when $\varkappa_\eta = 0$ (Le = 0) and $\omega = 1$, with the model of combustion-wave propagation in the condensed medium [8, 9].

In the given case, y varies monotonically from 0 to 1 when $-\infty \leqslant \xi \leqslant +\infty$, and hence $y_e = 1$, $y_b = \omega$. Following the standard procedure [11], y is assumed to be an independent variable and $p = p(y)$, $\eta = \eta(y)$. Then the problem (2.1), (2.2) is written in the form

$$\begin{aligned} \omega \frac{\varkappa_\eta}{\varkappa}(\omega y - p)\frac{d\eta}{dy} &= \omega(\eta - 1) + p, \\ \frac{1}{\varkappa}(\omega y - p)\frac{dp}{dy} &= \tau\omega w(\eta, y), \end{aligned} \tag{2.3}$$

$$\begin{aligned} \eta(0) = 1,\ \ p(0) = 0, \\ \eta(1) = 0,\ \ p(1) = \omega. \end{aligned} \tag{2.4}$$

Let $\eta_0(y)$ and $p_0(y)$ denote the zero approximations of the corresponding functions in the region adjacent to $y = 0$ (the external region), and $\eta_1(y_*)$ and $p_1(y_*)$ denote the zero approximations in the region adjacent to $y = 1$ (internal region). Here $y_* = (1 - y)/\beta$ is the internal variable. In the external region $p_0(y) \equiv 0$, and the matching condition for the external and internal solutions takes the form

$$p_1(y_*) \to 0 \quad \text{as} \quad y_* \to \infty. \tag{2.5}$$

Linearizing the functions $\varkappa$, $\varkappa_\eta$, and τ in the internal region ($\varkappa(1) = 1$) and using the Frank-Kamenetskii transformation (1.9), it is found that

$$\frac{\omega \varkappa_{\eta r}}{\beta}(p_1 - \omega)\frac{d\eta}{dy_*} = \omega(\eta_1 - 1) + p_1, \tag{2.6}$$

$$\frac{1}{\beta}(p_1 - \omega)\frac{dp}{dy_*} = \tau_r \omega \eta_1 e^{-\delta y_*},$$

$$\eta_1(0) = 0, \quad p_1(0) = \omega. \tag{2.7}$$

The second relation in system (2.6) is integrated from 0 to ∞, taking account of conditions (2.5) and (2.7)

$$\frac{\omega}{2\beta} = \tau_r \int_0^\infty \eta_1 e^{-\delta y_*} dy_*. \tag{2.8}$$

Finding η_1 from the first relation in system (2.6), the result is substituted into the second relation, and the result is integrated from 0 to ∞. Taking into account that

$$\int_0^\infty \frac{d\eta_1}{dy_*} e^{-\delta y_*} dy_* = \delta \int_0^\infty \eta_1 e^{-\delta y_*} dy_*,$$

it is found that

$$\tau_r - \frac{\omega}{\gamma} = \tau_p \delta \frac{\omega}{\gamma} \varkappa_{\eta r} \int_0^\infty \eta_1 e^{-\delta y_*} dy_*. \tag{2.9}$$

Eliminating $\int_0^\infty \eta_1 e^{-\delta y_*} dy_*$ from Eqs. (2.8) and (2.9), the equation for the zero approximation of the wave velocity is found

$$\frac{2(\gamma/\omega)^2}{\varkappa_{\eta r} + 2\gamma/\omega} \tau_r = 1. \tag{2.10}$$

Here $T_r = T_0 + (T_b - T_0)/\omega$.

If there is no solid phase in the system (m = 1), then $\varkappa_{\eta r} = Le$, $\omega = 1$, and Eq. (2.10) transforms, retaining terms of order $0(\beta)$, into the well-known formula for the normal laminar-flame velocity [11]. Setting $\varkappa_{\eta r} = 0$ and $\omega = 1$, formally, Eq. (2.10) transforms to the formula for the combustion-wave velocity in a condensed medium [8, 9]. The distinctive feature of Eq. (2.10) is that it gives the wave velocity at any $\varkappa_\eta$ (including small $\varkappa_\eta \sim \beta$, the situation prevailing in the present case).

Now, the method of contrary extrapolation is demonstrated for the problem (2.1), (2.2). The first relation in system (2.1) is integrated with respect to ξ from $-\infty$ to $+\infty$. Taking account of the boundary conditions, it is found that

$$\int_{-\infty}^{\infty} \tau w(\eta, y)\, d\xi = 1. \tag{2.11}$$

In the heating zone, $p(\xi) \equiv 0$, and the functions η and y are exponential. Placing the reaction zone in the vicinity of $\xi = 0$, and linearizing the functions $\varkappa_\eta$ and $\varkappa$ at this point, the concentration and temperature values extrapolated from the heating zone to the reaction zone are found

$$\begin{aligned} \eta_-(\xi) &= 1 - \exp(\xi/\varkappa_{\eta r}), \\ y_-(\xi) &= \exp(\omega\xi) \approx 1 + \omega\xi \end{aligned} \tag{2.12}$$

(series expansion of the function η is not performed, since $\varkappa_{\eta r}$ may be as small as is desired, generally speaking). Then the concentration and temperature are extrapolated into the reaction zone from the product zone ($\xi > 0$)

$$\eta_+(\xi) = 0,\ y_+(\xi) = 1. \tag{2.13}$$

Next $\eta(\xi)$ and $y(\xi)$ are taken in the form

$$\eta_0(\xi) = \frac{1}{2}[\eta_-(\xi) + \eta_+(\xi)],\quad y_0(\xi) = \frac{1}{2}[y_-(\xi) + y_+(\xi)]. \tag{2.14}$$

The following considerations are used here. The functions $\eta(\xi)$ and $y(\xi)$ are expanded in Fourier series in the vicinity of $\xi = 0$, and the width of the reaction zone tends to zero. Then, the formal derivatives of the corresponding series tend to discontinuous functions, which take the values of the half-sums of the left-hand and right-hand limits at the point of discontinuity.

Linearizing the function τ at the point $\xi = 0$, and using Eq. (1.9), the following relation is obtained in place of Eq. (2.11)

$$\tau_r \int_{-\infty}^{0} \eta_0(\xi) \exp\{[y_0(\xi) - 1]/\gamma\}\, d\xi = 1. \tag{2.15}$$

Substituting Eqs. (2.12)-(2.14) into Eq. (2.15), a formula for the wave velocity coinciding with Eq. (2.10) is obtained. Thus, using extrapolation into the reaction zone of the solutions from the heating and product zones, in contrast to [5], the constant coefficient obtained in the formula for the wave velocity is $\sqrt{2}$ and not unity.

3. Determining the Wave Velocity at Finite α

Consider the initial problem. In the general case, the gas temperature is not a monotonic function. Suppose that its maximum, which is in the reaction zone, is reached when $\xi = 0$, i.e., $y(0) = 1$, $(dy/d\xi)(0) = 0$. The concentration is extrapolated from the heating zone into the reaction zone

$$\eta_-(\xi) = 1 + (\eta_r - 1)e^{-\xi/\varkappa_{\eta r}}. \tag{3.1}$$

Here it is taken into account that, on reaching the maximum temperature, not all the material has reacted, and the reaction continues at temperatures below the maximum. Extrapolation from the cooling zone gives

$$\eta_+(\xi) = 0. \tag{3.2}$$

Extrapolation of the finite solution of (1.6) from the vicinity of the point $\xi = 0$, $y = 1$, $dy/d\xi = 0$ to the whole of the reaction zone, it is found that

$$\begin{aligned} \eta_0(\xi) &= \eta_r e^{\nu\xi}, \\ p_0(\xi) &= y_b + (p_r - y_b)e^{\nu\xi}, \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} \nu &= \frac{1}{2\varkappa_{\eta r}}\left(1 - \sqrt{1 + 4\tau_r \varkappa_{\eta r}}\right) < 0, \\ \eta_r &= \frac{1}{1 - \varkappa_{\eta r}\nu}\left(1 - \frac{p_r}{y_b}\right). \end{aligned} \tag{3.4}$$

The latter equation is the condition of finiteness of the solution at large ξ. This requirement corresponds to the fact that the cooling zone is formed because of exhaustion of the fuel mixture. For the same reason, the result of using the method of contrary extrapolation is taken to be

$$\begin{aligned} \eta_-^0(\xi) &= \frac{1}{2}\left(\eta_-(\xi) + \eta_r e^{\nu\xi}\right), \quad \xi < 0, \\ \eta_+^0(\xi) &= \frac{1}{2}\eta_r e^{\nu\xi}, \qquad\qquad \xi > 0. \end{aligned} \tag{3.5}$$

Equation (3.2) is used here.

The temperature profiles extrapolated from the heating and cooling zones are determined from the system of equations

$$\begin{aligned} \frac{dy}{d\xi} &= \frac{1}{\varkappa_{T_r}} y + \frac{\omega - 1}{\varkappa_{T_r}} z - \frac{\varkappa_{\Theta r}}{\varkappa_{T_r}} z' + f, \\ \frac{dz}{d\xi} &= z', \\ \frac{dz'}{d\xi} &= -\frac{\alpha}{\varkappa_{\Theta r}} y + \frac{\alpha}{\varkappa_{\Theta_r}} z + \frac{\omega - 1}{\varkappa_{\Theta r}} z', \end{aligned} \tag{3.6}$$

where $f = 0$ when $\xi < 0$ and $f = -y_b/\varkappa_{Tr}$ when $\xi > 0$. The eigenvalues of the matrix of the system are determined as the roots of the characteristic equation

$$\mu^3 + \left(\frac{1 - \omega}{\varkappa_{\Theta r}} - \frac{1}{\varkappa_{T r}}\right)\mu^2 + \frac{\omega - 1 - \alpha}{\varkappa_{T_r}\varkappa_{\Theta r}}\mu + \frac{\alpha\omega}{\varkappa_{T_r}\varkappa_{\Theta_r}} = 0. \tag{3.7}$$

Analysis of Eq. (3.7) shows that, when $\varkappa_{Tr} < 1/2$, all the roots are real and $0 < \mu_1 < \mu_2$, $\mu_3 < 0$. In addition,

$$\begin{aligned} &\mu_3 < \frac{\omega - 1}{\varkappa_{\Theta r}} < \mu_1 < \frac{1}{\varkappa_{T_r}} < \mu_2 \text{ when } \varkappa_{T_r}\omega < 1, \\ &\frac{1}{\varkappa_{T_r}} < \mu_1 < \frac{\omega - 1}{\varkappa_{\Theta_r}} < \mu_2 \quad \text{when } \varkappa_{T_r}\omega > 1. \end{aligned} \tag{3.8}$$

If $\varkappa_{T_r}\omega = 1$

$$\mu_1 = \frac{1}{\varkappa_{T_r}}, \quad \mu_{2,3} = \frac{1}{2\varkappa_{T_r}}\left(1 \pm \sqrt{1 + 4\alpha\frac{\varkappa_{T_r}}{\varkappa_{\Theta_r}}}\right).$$

Extrapolating the solution of Eq. (3.6) from the heating and cooling zones into the reaction zones, and requiring continuity of the functions z and z' at $\xi = 0$, the finite solution of the problem may be written in the form

$$\begin{aligned} y_- &= ae^{\mu_1\xi} + be^{\mu_2\xi}, \\ z_- &= r_1 ae^{\mu_1\xi} + r_2 be^{\mu_2\xi}, \\ z'_- &= \mu_1 r_1 ae^{\mu_1\xi} + \mu_2 r_2 be^{\mu_2\xi}, \end{aligned} \tag{3.9}$$

$$\begin{aligned} y_+ &= y_e - ce^{\mu_3\xi}, \\ z_+ &= y_e - r_3 ce^{\mu_3\xi}, \\ z'_+ &= -\mu_3 r_3 ce^{\mu_3\xi}. \end{aligned} \tag{3.10}$$

where $r_i = \frac{\varkappa_{T_r}\mu_i - 1}{\omega - 1 - \varkappa_{\Theta_r}\mu_i}$. The constants a, b, and c are defined as follows:

$$a = -\frac{\mu_2\mu_3 y_e + \Delta}{r_1(\mu_2 - \mu_1)(\mu_1 - \mu_3)}, \quad b = -\frac{\mu_1\mu_3 y_e + \Delta}{r_2(\mu_2 - \mu_1)(\mu_3 - \mu_2)},$$

$$c = -\frac{\mu_1\mu_2 y_e + \Delta}{r_3(\mu_1 - \mu_3)(\mu_3 - \mu_2)}, \quad \Delta = \frac{\alpha}{\varkappa_{\Theta_r}}[y_+(0) - y_-(0)]. \tag{3.11}$$

Further, it is assumed that $y_-(\xi_-) = y_+(\xi_+) = 1$, where $|\xi_\pm| \ll \beta$. Equivalently, according to Eqs. (3.9) and (3.10)

$$\begin{aligned} a + be^{\mu_2\xi_-} &= 1, \\ ce^{\mu_3\xi_+} &= y_e - 1. \end{aligned} \tag{3.12}$$

It is taken into account here that μ_1 is a finite root when $0 \leqslant \alpha \leqslant \infty$ and hence $e^{\mu_1\xi_-} = 1$. at any α, with an accuracy of $0(\beta)$. Substituting Eqs. (3.11) into Eqs. (3.12) and eliminating Δ, y_e is found. This result satisfies the condition $0 < y_e < 1$, which follows from (3.8). Further, according to Eq. (1.5)

$$p_r = y_b - \varkappa_{T_r}\mu_3 ce^{\mu_3\xi_+}. \tag{3.13}$$

From (3.8) it follows that $0 < p_r < y_b$.

Now the method of contrary extrapolation is used:

$$\begin{aligned} y^0_-(\xi) &= \frac{1}{2}[1 + y_-(\xi)] \approx 1 + \frac{1}{2}\frac{dy_-(\xi_-)}{d\xi}(\xi - \xi_-), \\ y^0_+(\xi) &= \frac{1}{2}[1 + y_+(\xi)] \approx 1 + \frac{1}{2}\frac{dy_+(\xi_+)}{d\xi}(\xi - \xi_+). \end{aligned} \tag{3.14}$$

According to Eqs. (3.9) and (3.10),

$$\frac{dy_-(\xi_-)}{d\xi} = \mu_1 a + \mu_2 be^{\mu_2\xi_-}, \quad \frac{dy_+(\xi_+)}{d\xi} = -\mu_3 ce^{\mu_3\xi_+}. \tag{3.15}$$

The integral balance gives

$$1 = \tau_r\left\{\int_{-\infty}^{0} \eta^0_- \exp\left[\frac{1}{\gamma}(y^0_- - 1)\right]d\xi + \int_0^{\infty} \eta^0_+ \exp\left[\frac{1}{\gamma}(y^0_+ - 1)\right]d\xi\right\}. \tag{3.16}$$

Substituting Eqs. (3.5) and (3.14) into Eq. (3.16) leads, after appropriate manipulations, to the result

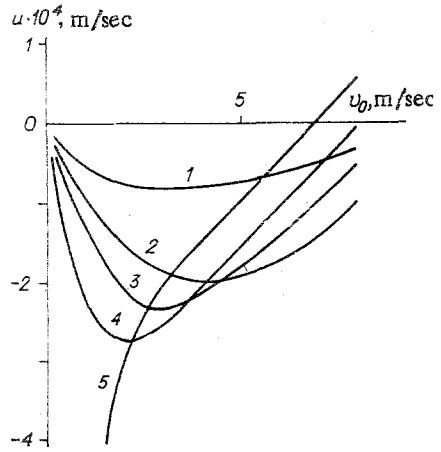

Fig. 1. Curves of $u(v_0)$ with the following values of d, m: 1) 10^{-10}, 2) $5\cdot10^{-4}$, 3) 10^{-3}, 4) $2\cdot10^{-3}$, 5) $5\cdot10^{-3}$.

$$\tau_r\left(\frac{2\gamma_-^2}{\varkappa_{\eta_r}+2\gamma_-}+\eta_r\varphi\right)=1, \tag{3.17}$$

where

$$\varphi=2\gamma_-\left(\frac{\gamma_-+\varkappa_{\eta r}}{2\gamma_-+\varkappa_{\eta_r}}-\nu\gamma_-\right)-\frac{\gamma_+}{1+2\nu\gamma_+},\quad \gamma_\pm=\frac{\gamma}{\frac{dy_\pm(\xi_\pm)}{d\xi}}.$$

The quantities ν, η_r, y_b, $\frac{dy_\pm(\xi_\pm)}{d\xi}$ are determined from (3.4), (3.12), (3.13), and (3.15), respectively. Here $T_r = T_0 + (T_b - T_0)/y_b$. Detailed analysis of Eq. (3.17) shows that, when $\alpha \to 0$, it is transformed to give

$$\frac{2\gamma^2}{Le_b+2\gamma}\varkappa_{Tb}\tau_b=1, \tag{3.18}$$

which, in the appropriate notation, corresponds up to terms of order $0(\beta)$ with the formula for the normal laminar-flame velocity. As $\alpha \to \infty$, Eq. (3.17) transforms to Eq. (2.10). In addition, in the given limiting cases, $\eta_r = 0$, which is completely to be expected (the gas temperature is monotonic and there is no precombustion zone).

4. Discussion of Results

Taking account of the phase interaction with no limitation on its intensity in the region of a thermal wave allows the role of various characteristics of a porous medium in filtrational gas combustion to be more completely analyzed over a broad range of α, including the limiting cases $\alpha = 0$ and $\alpha = \infty$. Below, the results of this analysis are shown, with the following choice of empirical formulas for determining the parameters of the problem.

The heat-transfer coefficient is determined from the formula [12]

$$\mathrm{Nu}_E = 0{,}725\mathrm{Re}_E^{0.47},\quad \mathrm{Re}_E \leqslant 30,$$
$$\mathrm{Nu}_E = 0{,}395\mathrm{Re}_E^{0{,}64},\quad \mathrm{Re}_E \geqslant 30,$$

where

$$\mathrm{Nu}_E=\frac{\alpha_0 d_E}{\lambda_T},\quad \mathrm{Re}_E=\frac{2}{3}\frac{m d v_0 \rho_{T_0} c_T}{(1-m)\lambda_{T_0}}=\frac{v d_E}{\nu}.$$

It is taken into account here that, for LVC, $\rho_T v \approx \rho_{T0} v_0$ and Pr = 1. If $\mathrm{Nu}_E < 2$, then $\mathrm{Nu}_E = 2$ is assumed. The effective thermal conductivity of the porous medium is found from the formula [12]

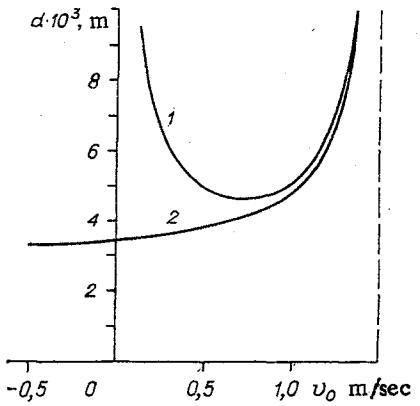

Fig. 2

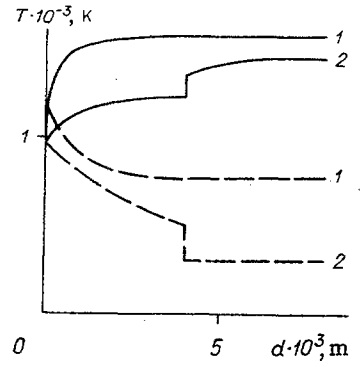

Fig. 3

Fig. 2. Regions of existence of LVC and HVC.

Fig. 3. Dependence of T_r (continuous curves) and T_e (dashed curves) on d when v_o= 2 (1) and 0.5 m/sec (2).

$$\lambda_\Theta = \lambda_T(1+\zeta\,\mathrm{Re})\Phi + \alpha_r d,$$

where $\zeta = 0.1;\ \Phi = 0.1;\ \alpha_r = 0.227\frac{p}{2-p}\left(\frac{T_{0p}}{100}\right)^3$ is the radiant heat-transfer coefficient in the porous medium; p = 1; T_{0_p} = 1000°K, $\lambda_T \ll \lambda_c$ is the thermal conductivity of the material of the porous medium; $\mathrm{Re}_E = [2m/3(1-m)]\mathrm{Re}$.

In determining S_{sp}, it is assumed that the porous medium consists of elements of surface equal to a sphere of diameter d

$$S_{sp} = \frac{4m}{d_e} = \frac{6(1-m)}{d}.$$

The results below (Figs. 1-5) are obtained from theoretical analysis with the following numerical parameter values: $k_o = 5\cdot10^{10}$ sec^{-1}; E = 126 kJ/mole; m = 0.5; ρ_{T_0} = 0.5 kg/m^3; $\rho_\Theta = 5\cdot10^3$ kg/m^3; $c_T = c_\Theta$ = 1 kJ/kg·°K; λ_T = 0.1 W/m·°K; T_b = 1500°K.

The Case Where $\Pi \to \infty$. In this case Eq. (3.17) transforms to Eq. (2.10). In Fig. 1, the combustion wave velocity u is shown as a function of the flow velocity of the fuel mixture v_o. It is evident that the behavior of the curve of $u(v_o)$ is typical for LVC: it is U-shaped.

The result obtained may be given the following physical interpretation. In a coordinate system fixed in the flame front, the fuel gas enters the combustion zone at a rate $v_o - u$, and enters the porous medium at a rate $|u|$, i.e., each value of $|u|$ when $u < 0$ corresponds to the combustion of a gas mixture with some fixed homogeneous inert additive. As $|u|$ increases, the gas is "diluted" with inert additive, and the flame temperature $T_e = T_o + Q/\omega c_T$ falls; ω increases. In the "standing-wave" state ($\omega = 1$), there is no dilution by inert material, and the flame temperature, as in HVC, is equal to the adiabatic temperature. However, the flame velocity with respect to the fuel gas in this case is higher than in HVC (Fig. 1). This is because the inert additive not only reduces the flame temperature when $u < 0$ but also increases the flame velocity on account of more effective heat conduction. In fact, if reduction in flame temperature is eliminated while retaining the same immobile additive in the flame zone (the case when $u = 0$ or $\omega = 1$), it follows from Eqs. (2.10) and (3.18) that

$$\frac{v_{0,\alpha\to\infty}}{v_{0,\alpha\to0}} = \sqrt{\frac{\lambda_{ef}}{\lambda_T}\frac{\mathrm{Le}+2\gamma}{\varkappa_\eta+2\gamma}} \approx 6.3,$$

where $\lambda_{ef} = \lambda_T + [(1-m)/m]\lambda_\Theta$; $\gamma = RT^2/E(T_b - T_o)$. This estimate shows the capacity of the heat-conducting additive for increasing the velocity of gas-flame propagation.

The Case Where $\alpha \to 0$. In this case, Eq. (3.17) transforms to Eq. (3.18) for the normal velocity of a laminar flame [11]. Thus, the case of combustion when $\alpha \to 0$ lies in the HVC region, and corresponds to the usual process of laminar flame propagation; it is characterized by the normal velocity S_u and occurs on account of the molecular transfer properties of the gas phase. As noted in [11], the connections in the system (1.1) break down here: The

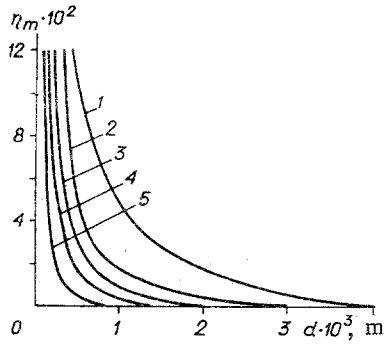

Fig. 4

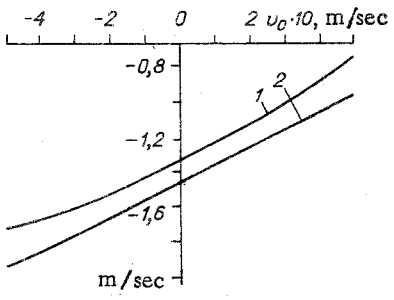

Fig. 5

Fig. 4. Curves of $\eta_r(d)$ for the following values of v_o, m/sec: 1) 0.5, 2) 1, 3) 1.5, 4) 2, 5) 5.

Fig. 5. Curves of $u(v_o)$ in HVC with $d = 5 \cdot 10^{-3}$ (1) and 10^{-2} m (2).

weak heat flow from the gas, taken into account by the equation for the porous medium, has practically no influence on the flame propagation against the flow. This means that the cycle of heat recuperation in the system combustion products—porous medium—fresh gas—combustion products is broken.

The model does not give a solution for HVC with motion of the wave downstream in the direction of the combustion products, and this corresponds to reality: In a sufficiently long tube, the flame cannot propagate downstream indefinitely at large linear velocities, since at the small but finite real value of α the temperature of the porous medium rises over time to large values determined by the value of T_e, and HVC gives way to LVC. Hence, the natural constraint for HVC is the condition $v < S_u$.

Note that the given model of the process takes no account of the aerodynamic interaction of the gas with the porous medium. The gas-velocity field in the transverse directions is taken to be homogeneous. Therefore, the limiting case $\alpha \to 0$ corresponds to a process of laminar flame propagation at the normal velocity $S_u = v - u$. In fact, as shown experimentally [3, 4], not laminar but turbulent combustion is observed in HVC, as a rule. The velocity of combustion-wave propagation relative to fresh gas here is $S_{pu} > S_u$. Taking this into account entails a modification of (1.1), using, in particular, the turbulent characteristics of heat and mass transfer in the gas phase, which goes beyond the scope of the present investigation.

The Case Where $0 < \alpha < \infty$. At finite α, LVC may be realized with adjacent singularity conditions [1, 2]. The dependence $u(v_o)$ has the characteristic U shape with a minimum velocity u_{min} at some value v_m (Fig. 1). When $\omega > 1$, $v_o > 0$, the heat wave moves against the flow, and when $\omega < 1$, $v_o > 0$ it moves with the flow; the condition $\omega = 1$ corresponds to a standing-wave state. The velocity characteristics of the wave depend on the internal heat transfer, in particular, on the pore-channel size d and the flow velocity v_o.

It is evident from Fig. 1 that as α decreases (as d increases), the curve of $u(v_o)$ is shifted toward negative values of u. At some values $v_{o.cr}$, d_{cr}, transition to HVC occurs. From a mathematical viewpoint, the following picture is seen. At a fixed $v_o < S_u$ and sufficiently small d, Eq. (3.17) with the condition $\omega > 0$ has a single solution corresponding to LVC. With increase in d to d'_{cr}, two more solutions appear. One corresponds to HVC and the other (unstable) solution is shifted toward the root corresponding to LVC with further increase in d, coalescing with it when $d = d''_{cr}$. With further increase in d, there remains a single solution, corresponding to HVC. In Fig. 2, bifurcational curves are shown in the plane of the parameters v_o, d. Below curve 2, only LVC is possible; above curve 1, only HVC is possible; in the intervening region, both LVC and HVC are possible, depending on the initial state of the system (the region of nonuniqueness of the solution). The question of the attainment of a particular outcome in this case is not investigated specially in the present work. It is evident from Fig. 2 that as $v_{ocr} \to S_u$ (dashed curve), $d'_{cr} \to \infty$ and $d''_{cr} \to \infty$. When $v_o > S_u$, HVC is impossible.

The distinctive feature of LVC is the peak profile of the gas temperature in the chemical reaction zone. In the course of the reaction, the gas temperature "breaks away" from the

temperature of the porous medium, passes through a maximum T_r, and then relaxes to the final temperature of the system T_e. Curves of T_r and T_e as a function of the pore-channel diameter at different flow velocities are shown in Fig. 3. It is evident that as $d \to 0$, the difference between T_r and T_e decreases, and the phase temperatures coincide at the limit. When $d > d_{cr}$, the combustion process transforms to HVC, and T_r and T_e instantaneously take new values (the steps on curve 2). With increase in v_o, the two temperatures increase. Because of the large values of $|u|$ in HVC, T_e is very low. Thus, HVC is fundamentally a two-temperature situation. However, on account of the slow relaxation of temperature in the gas, the existence of the two temperatures is not a significant factor influencing the flame propagation.

A new result of the present analysis is the incomplete burnup η_r at T_r. Curves of $\eta_r(d)$ with different flow velocities are shown in Fig. 4. It is evident that, other conditions being equal, the greatest incompleteness of burnup corresponds to small d and v_o.

The given model of filtrational gas combustion admits of the existence of combustion waves in the region $v_o \leqslant 0$. As in the case where $v_o > 0$, two cases of combustion are possible in principle: LVC at large α and HVC at small α (Fig. 1). In fact, this means that combustion is maintained only by the intrapore heat content of the gas. Since interphase heat transfer is small in HVC, the porous medium does not hinder flame propagation and the practical realization of the given situation in HVC provokes no doubts. Curves for this case are shown in Fig. 5. As regards LVC, the very low values of T_r and T_e make this situation very improbable in normal conditions. However, at high pressure, with high porosity of the medium and large thermal effects, this situation may evidently be realized experimentally in LVC. The heat losses to the surrounding space will be significant here.

CONCLUSIONS

1. A two-temperature adiabatic model of filtrational gas combustion has been investigated; the model takes account of molecular transfer in the gas phase and the finite width of the chemical reaction zone, thereby permitting a general analysis of the role of thermal interphase interaction, including the limiting cases. Using a modified extrapolational model, approximate relations have been obtained for determining the basic characteristics of the process.

2. The possibility that two cases of combustion — LVC and HVC — may be realized has been demonstrated. The parameter ranges corresponding to these two cases have been determined. The nonuniqueness region has been identified.

3. The possibility and conditions of realization of three types of flame propagation have been shown: combustion-wave motion in the direction of the combustion products, in the direction of the fresh gas, and in the direction opposite to that of the fresh gas, in particular, with zero filtration velocity.

4. The possibility of slowing of the reaction on account of internal thermal interactions leading to incomplete burnup in the region of maximum gas temperatures has been shown.

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