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Radiation from Relativistic Particles in Nongeodesic Motion in a Strong Gravitational Field

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Abstract

The scalar and electromagnetic radiation emitted by relativistic particles moving along the stable nongeodesic trajectories in the Kerr gravitational field are described. Two particular models of the nongeodesic motion are developed involving a slightly charged rotating black hole and a rotating black hole immersed in an external magnetic field.

§ (1): *Introduction*

Considerable attention has been given lately to the theoretical description of the radiation from test bodies moving in the vicinity of black holes [1-4]. It was shown that the radiation from the relativistic particles performing an unstable circular motion in the Schwarzschild and Kerr fields called "geodesic synchrotron radiation" (GSR) [3-4] does not share the properties of usual synchrotron radiation (SR) in flat space [5]. In particular, the gravitational GSR has no maximum in the region of high frequencies. This distinction is due to the fact that the gravitational field acts both on the radiating particle and the emitted photons, while the electromagnetic field acts on the charged particle only. The transition from the SR regime to the GSR one for charged particles moving along circular trajectories in the Schwarzschild space-time in the presence of an external magnetic field was studied in [4, 6].

In the present paper a similar theory is developed for the Kerr space-time. We consider two models in which both the nongeodesic and geodesic motion of charged particles along ultrarelativistic circular trajectories can occur: (1) a rotating black hole carrying small electric charge; (2) a rotating black hole in an external magnetic field [6-8]. Though the motion of particles in the Kerr-Newman field was considered in various aspects previously [9-13], the domains of existence, stability, and binding of the circular orbits for $a \neq 0, Q \neq 0$ have not been given explicitly. Here this problem is studied both analytically and numerically (Section 2). In Section 3 an equatorial motion of a test particle in the field of a rotating black hole immersed in the uniform magnetic field is considered. The scalar radiation from the relativistic particles in nongeodesic motion in the Kerr metric is briefly discussed in Section 4; the case of electromagnetic radiation is considered in Section 5. Throughout the paper the unit system $G = c = 1$ and metric $(+ - - -)$ are used.

§ (2): *Existence and Stability Bounds for the Circular Nongeodesic Equatorial Orbits in the Kerr-Newman Field*

In the Boyer–Lindquist coordinates the space-time associated with a charged rotating black hole is described by

$$
\begin{aligned}
dS^2 = {} & \left(1 - \frac{2Mr - Q^2}{\Sigma}\right) dt^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2 \\
& - \sin^2\theta \left(r^2 + a^2 + \frac{2Mr - Q^2}{\Sigma} a^2 \sin^2\theta\right) d\varphi^2 \\
& + 2\left(\frac{2Mr - Q^2}{\Sigma} a \sin^2\theta\right) d\varphi\, dt
\end{aligned}
\tag{1}
$$

where $\Sigma = r^2 + a^2 \cos^2\theta$, $\Delta = r^2 - 2Mr + a^2 + Q^2$, M, Q, a are the mass, charge, and angular momentum of the hole, respectively. Consider the equations of motion of a particle with the mass μ and charge e

$$
\frac{d^2 X^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dX^\alpha}{ds} \frac{dX^\beta}{ds} = \frac{e}{\mu} F^\mu{}_\nu \frac{dX^\nu}{ds}, \qquad U^\mu = \frac{dX^\mu}{ds} \tag{2}
$$

where $F^\mu{}_\nu$ is the electromagnetic field tensor, the nonvanishing components of which are

$$
\begin{aligned}
F^1{}_0 &= -\frac{\Delta Q}{\Sigma^3} (\Sigma - 2r^2), \qquad F^2{}_0 = -\frac{Qr}{\Sigma^3} a^2 \sin 2\theta \\
F^1{}_3 &= -\frac{\Delta Q}{\Sigma^3} \cdot a(\Sigma - 2r^2), \qquad F^2{}_3 = \frac{Qr}{\Sigma^3} a(r^2 + a^2) \sin 2\theta
\end{aligned}
\tag{3}
$$

For the circular orbits in the equatorial plane $\theta = \pi/2$ equations (2) with $\mu = 0$, 2, 3 become trivial. The remaining equation ($\mu = 1$) leads to the following expression for the angular velocity of rotation:

$$\omega_p = (1 - a^2\omega_s^2)^{-1}\left\{\pm\left[\omega_s^2 - \frac{\omega_Q}{r_p}\left(1 - \frac{a^2\omega_Q}{4r_p}\right)\right]^{1/2} - a\left(\omega_s^2 - \frac{\omega_Q}{2r_p}\right)\right\} \quad (4)$$

Here

$$\omega_s^2 = \frac{Mr_p - Q^2}{r_p^4}, \qquad \omega_Q = \frac{eQ}{\mu U^0 r_p^2}$$

The upper sign holds for prograde, the lower for retrograde orbits. Using the normalization condition $g_{\mu\nu}U^\mu U^\nu = 1$ and equation (2) with $\mu = 1$, we get the relation between the particle energy and the frequencies ω_p, ω_Q

$$E = \mu\gamma = \mu\left[1 - \frac{2Mr_p - Q^2}{r_p^2}(1 - a\omega_p)\right]\left\{1 - \omega_p^2(r_p^2 + a^2) - \frac{2Mr_p - Q^2}{Mr_p - Q^2}\left[\omega_p^2 r_p^2 + \omega_Q r_p(1 - a\omega_p)\right]\right\}^{-1/2} \quad (5)$$

Introduce a dimensionless ratio of the Coulomb force to the Newtonian one $\eta = eQ/\mu M$. It is known that the ratio Q/M for the black hole of a solar mass cannot be greater than $\sim 10^{-5}$, otherwise the electron-positron pair creation takes place near the event horizon [14]. However, even in this case the parameter η can be nonsmall for particles with large charge-to-mass ratio (for the electron $e/\mu \sim 10^{21}$). When the gravitational force is dominating, $|\eta| \ll 1$, we get from (4) and (5)

$$\omega_p \simeq \omega_K\left(1 - \frac{\eta}{2\gamma_0}\left[1 - 2r_p^2\omega_s^2(1 - a\omega_K)\right]\right)$$

$$\omega_K = \frac{\pm\omega_s}{(1 \pm a\omega_s)}, \qquad \gamma_0 = \frac{1 \mp 2r_p^2\omega_s\omega_K}{\left[1 - \omega_K^2(3r_p^2 + a^2)\right]^{1/2}} \quad (6)$$

$$\gamma \simeq \gamma_0 - \frac{\eta}{2}\omega_K^2\left[1 - 2r_p^2\omega_s^2(1 - a\omega_K)\right]\left[\frac{r_p^2 + a^2 \mp 2a\omega_s r_p^2}{1 - \omega_K^2(3r_p^2 + a^2)} + \frac{2a\omega_s^2 r_p^2}{\omega_K \mp 2r_p^2\omega_K^2\omega_s}\right] \quad (7)$$

In the opposite limit $|\eta| \gg 1$ the circular orbits can exist only for $eQ < 0$. Comparing (4) and (5) one can conclude that the point $r = r_{ph}$ at which the denominator in γ_0 is zero, does not correspond to the singularity of energy (5) provided $eQ > 0$. It means that the circular orbits exist also for $r < r_{ph}$. For $\eta \neq 0$

from equation (2) with $\mu = 1$ we obtain

$$\gamma = \eta \frac{M}{r_p^3}(1 - a\omega_p) \frac{1 - \dfrac{2Mr_p - Q^2}{r_p^2}(1 - a\omega_p)}{\omega_s^2(1 - a\omega_p)^2 - \omega_p^2} \tag{8}$$

It is seen that the ultrarelativistic circular orbits with the radius r not close to r_{ph} are possible if $|\eta| \gg 1$.

Consider now small perturbations of the circular equatorial orbits described above. Substituting $\xi^\mu(s) = X^\mu(s) - Z^\mu(s)$ into equation (2), where $Z^\mu(s) = U^0 \cdot s(1, 0, 0, \omega_p)$ and separating out all terms linear in ξ^μ one gets

$$\ddot{\xi}^\mu + 2\overline{\Gamma}^\mu_{\alpha\nu}\dot{Z}^\alpha\dot{\xi}^\nu + \overline{\Gamma}^\mu_{\alpha\beta,\nu}\dot{Z}^\alpha\dot{Z}^\beta\xi^\nu - \frac{e}{\mu}(\overline{F}^\mu{}_\nu\dot{\xi}^\nu + F^\mu{}_{\nu,\lambda}\xi^\lambda\dot{Z}^\nu) = N^\mu(\xi) \tag{9}$$

where the dots mean the derivatives to the proper time: $\overline{\Gamma}$ and $\overline{F}$ are the values of the Christoffel symbols and field components at the unperturbed trajectory $Z^\mu(s)$.

For small ξ^μ the solution of (9) can be obtained by successive approximations. In linear approximation the equation with $\mu = 2$ decouples:

$$\frac{d^2\xi^\theta}{dt^2} + \omega_\theta^2\xi^\theta = 0, \qquad dt = ds\, dZ^0/ds \tag{10}$$

and the corresponding frequency is given by

$$\begin{aligned}\omega_\theta^2 = \omega_s^2(1 - a\omega_p)^2 \Bigg\{1 &+ \frac{a^2}{r_p^4}(r_p^2 + 4Mr_p - 2Q^2) \\ &+ \frac{2Mr_p - Q^2}{Mr_p - Q^2}\left[\frac{a^2}{r_p^2} - \frac{2a\omega_p}{(1 - a\omega_p)^2}\right]\Bigg\} \\ &- \frac{\omega_Q}{r}(1 - a\omega_p)\left[1 + \frac{a^2}{r_p^4}(3r_p^2 + 4Mr_p - 2Q^2) - \frac{2a\omega_p}{1 - a\omega_p}\right]\end{aligned} \tag{11}$$

The equations with $\mu = 0, 1, 3$ can be put into the following form:

$$\frac{d\xi^A}{dt} + \left[2(\overline{\Gamma}^A_{01} + \omega_p\overline{\Gamma}^A_{13}) - \frac{e}{\mu}(\dot{Z}^0)^{-1}F^A{}_1\right]\xi^r = 0, \qquad A \equiv t, \varphi \tag{12}$$

$$\frac{d^2\xi^r}{dt^2} + \omega_r^2\xi^r = 0 \tag{13}$$

The corresponding solution

$$\xi^r = \text{const} \cdot \sin\omega_r t, \qquad \xi^A = \text{const} \cdot \left[2(\overline{\Gamma}^A_{01} + \omega_p\overline{\Gamma}^A_{13}) - \frac{e}{\mu}(Z^0)^{-1}F^A{}_1\right]\frac{\cos\omega_r t}{\omega_r}, \tag{14}$$

describes the radial-phase oscillations with the frequency

$$\omega_r = \left\{ \omega_s^2 (1 - a\omega_p)^2 \left[\frac{\Delta_p}{r_p^2} \left(1 - \frac{Q^2}{Mr_p}\right)^{-1} + \frac{4(Q^2 - a^2 - Mr_p)}{r_p^2} + \frac{8a\omega_p}{1 - a\omega_p} \right] - \frac{\omega_Q}{r_p} [1 - a\omega_p] \left(1 + \frac{5Q^2 - 3a^2 - 6Mr_p}{r_p^2} + \frac{4a\omega_p}{1 - a\omega_p}\right) - \omega_Q^2 \right\}^{1/2} \tag{15}$$

We note that ω_r is neither equal to the axial frequency ω_θ nor to the orbital angular velocity ω_p. That accounts for the effects of periastron rotation and the Lense-Thirring precession. The stability regions of circular orbits are defined by the conditions $\omega_r^2 > 0$, $\omega_\theta^2 > 0$. The analysis of (11) in the case of neutral particles ($e = 0$) shows that ω_θ^2 is always positive, i.e., the circular motion of neutral particles in the linear approximation is axially stable. The numerical results for the existence, radial stability, and binding of the circular neutral particle trajectories in the Kerr-Newman field are given in Figure 1. Note that the existence and stability domains are enlarged with the increasing Q both for prograde and retrograde trajectories.

Taking into account the nonlinear terms on the right-hand side of (9) we observe the appearance of coupling between the orbital motion and oscillations. If the frequencies ω_r and ω_θ are in the rational relation $K_r\omega_r = K_\theta\omega_\theta$, $K = |K_r| + |K_\theta|$, where K_r and K_θ are integers, the resonances take place. The positions of low-order resonances for different charge values are shown in Figure 2.

§ (3): *Motion of the Charged Particles in the Vicinity of a Rotating Black Hole Immersed in a Homogeneous Magnetic Field*

The exact solution of the Einstein-Maxwell equations describing a slowly rotating uncharged black hole in an external asymptotically uniform magnetic field has been obtained in our paper [8] within the framework of the Ernst method [15]. If the magnetic field strength is relatively small, $B \ll B_M = 2.4 \times 10^{19}$ gs $M_\odot/M$, there is the region $Br \ll 1$ outside the black hole where the space-time is described by the Kerr metric and the nonvanishing components of the electromagnetic field tensor are

$$F_{01} = \frac{aMB}{\Sigma}\left(1 - \frac{2r^2}{\Sigma}\right)(1 + \cos^2\theta), \qquad F_{02} = -aB\frac{Mr}{\Sigma^2}(r^2 - a^2)\sin 2\theta$$

$$F_{32} = B\sin 2\theta \left\{ \frac{r^2 + a^2}{2} - \frac{2Mr}{\Sigma}a^2 \left[\cos^2\theta + \frac{a^2}{2\Sigma}\sin^2\theta\,(1 + \cos^2\theta)\right]\right\} \tag{16}$$

$$F_{31} = B\sin^2\theta \left[r - \frac{Ma^2}{\Sigma}\left(1 - \frac{2r^2}{\Sigma}\right)(1 + \cos^2\theta)\right]$$

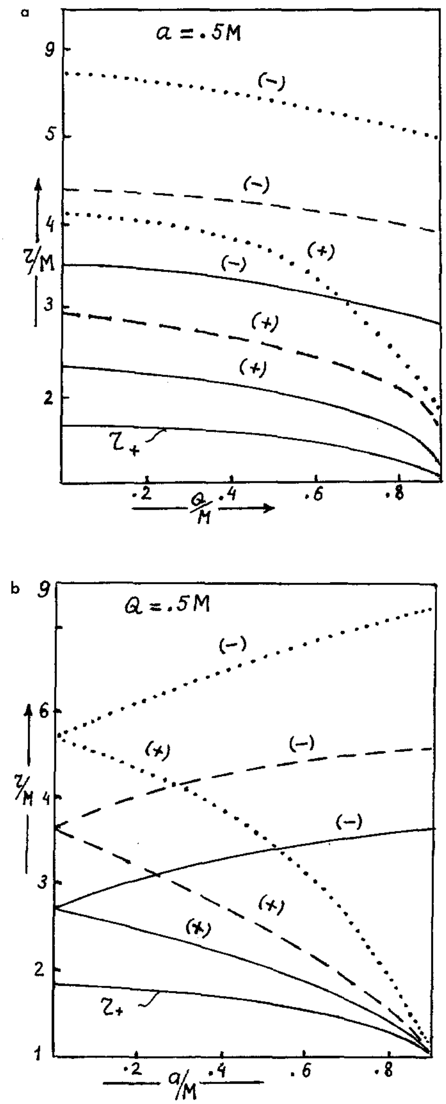

Fig. 1. The bounds of existence (——), binding (– – –) and stability (· · ·) for circular orbits of neutral particles as a function of Q/M for the given a/M (a), (c) and as a function of a/M for the given Q/M (b), (d) are shown. Notice that with the growth of the charge value Q the bounds are displaced toward the event horizon. The symbols (+), (–) correspond to prograde and retrograde orbits.

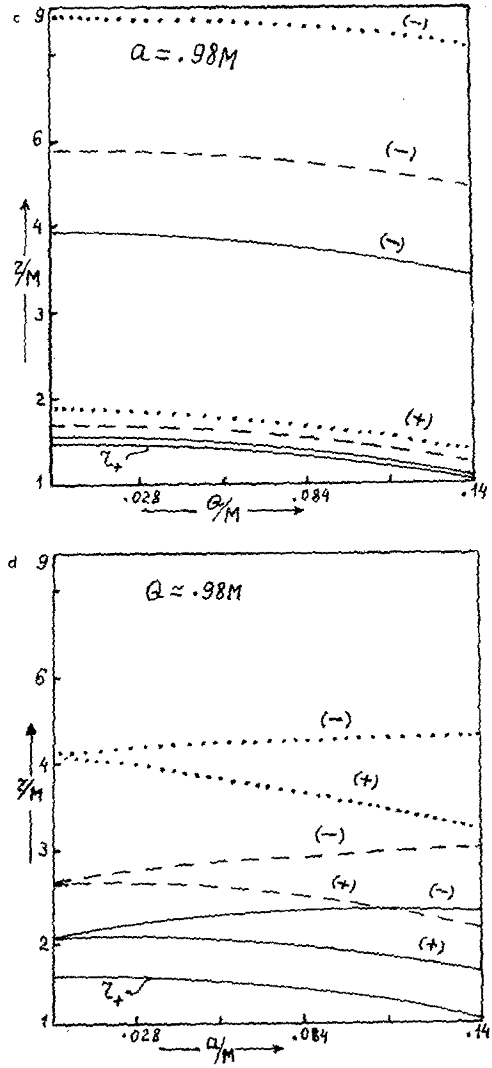

Fig. 1. Continued.

For the circular orbits in the plane $\theta = \pi/2$ we obtain from equation (2) with $\mu = 1$

$$\omega_p = \beta(1 - a^2\omega_s^2)^{-1}\left\{\pm\left[1 + \frac{\omega_s^2(1 - a^2\omega_s^2)(1 + a\omega_B)}{\beta^2}\right]^{1/2} - 1\right\},$$

$$\beta = \frac{\omega_B}{2}(1 + a^2\omega_s^2) + a\omega_s^2 \quad (17)$$

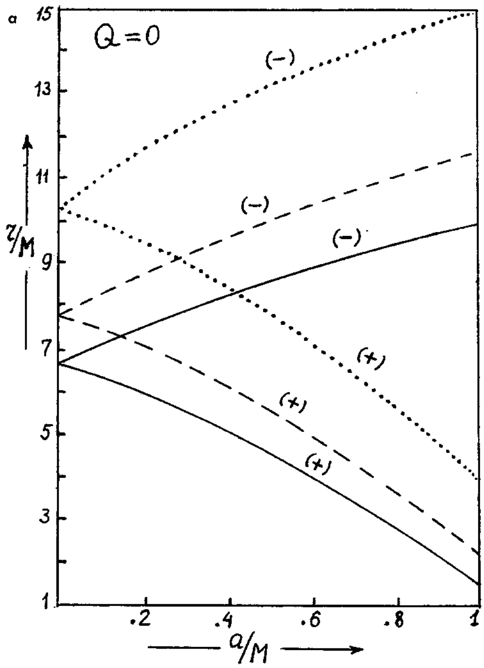

Fig. 2. The curves which characterize the position of low-order resonances for different values of Q are given. (– – –) for $k = 3$; (——) for $k = 4$; (· · ·) for $k = 5$.

Here $\omega_B = eB/\mu U^0$ is the cyclotron frequency in the gravitational field; $\omega_s = M^{1/2}/r_p^{3/2}$ is the Kepler frequency; and the plus and minus signs correspond to the prograde and retrogade trajectories, respectively. In each case the "Larmor" motion (Lorentz force is directed towards the black hole) and the "anti-Larmor" motion (Lorentz force is in the opposite direction) is possible on the condition that the expression in the square brackets in (17) is positive.

The energy of a particle can be expressed through the orbital frequency as follows:

$$E = \frac{\mu}{M}\epsilon\left[1 - \frac{2M}{r_p}(1 - a\omega_p)\right]\frac{\omega_p(1 + a^2\omega_s^2) - a\omega_s^2}{\omega_s^2(1 - a\omega_p) - \omega_p^2} \tag{18}$$

Where the dimensionless parameter $\epsilon = eB/\mu B_M$ characterize the relative influence on the particle motion of a magnetic field. For sufficiently large values of ϵ the circular motion with the ultrarelativistic velocity $\gamma \gg 1$ far from the null circular geodesic is possible. In analogy with the results of the previous section one can show that the small axial and radial-phase oscillations around the circular orbits have the frequencies

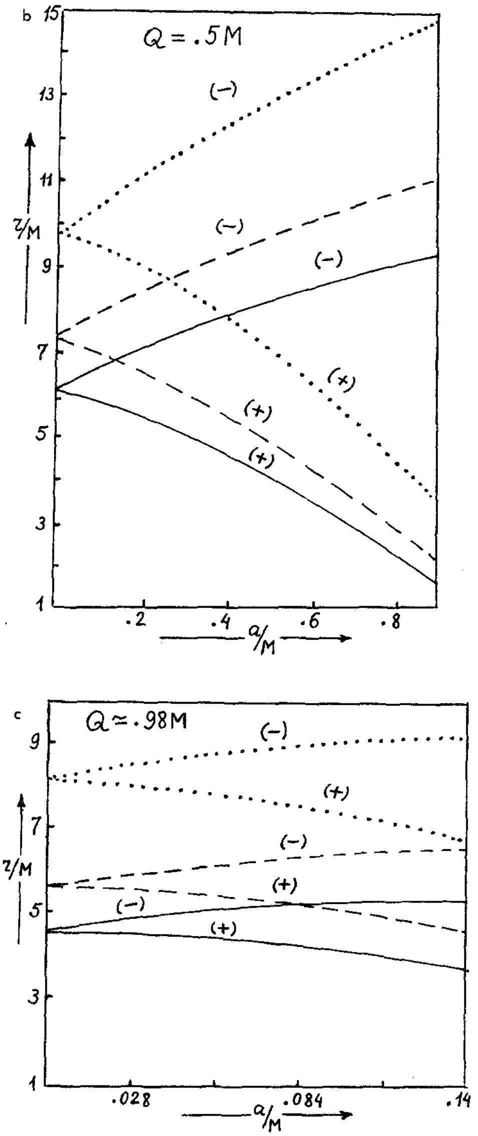

Fig. 2. Continued.

$$\omega_r^2 = \omega_p^2\left(1 - \frac{6M}{r_p} - \frac{3a^2}{r_p^2} - 8a^2\omega_s^2\right) + \omega_p\omega_B\left(1 - \frac{6M}{r_p} - \frac{3a^2}{r_p^2}\right)$$
$$+ 4a\omega_s^2(2\omega_p + \omega_B) + \omega_B^2\left(1 - \frac{2M}{r_p} - \frac{M}{r_p}a^2\omega_s^2 + 2a^2\omega_s^2\right) \qquad (19)$$

$$\omega_\theta^2 = \omega_p^2 \left(1 + \frac{3a^2}{r_p^2} + 4a^2\omega_s^2\right) + \omega_p\omega_B \left(1 + \frac{3a^2}{r_p^2}\right) - 2a\omega_s^2(2\omega_p + \omega_B) \tag{20}$$

As it is seen from (19), the radial stability regions for the anti-Larmor orbits are enhanced with the increasing magnetic field strength up to the event horizon. The analysis of the expression (20) shows that $\omega_\theta^2 > 0$ so in the linear approximation the circular motion is always axially stable.

§ (4): *Scalar Radiation in Nongeodesic Motion*

Three different radiation regimes for the circularly moving particles in the Kerr field are to be distinguished: (a) the emission of the fundamental mode in the nonrelativistic motion, (b) the emission of the harmonics of the order of γ^2 with respect to the basic frequency for the ultrarelativistic geodesic motion (GSR), (c) the generation of γ^3 harmonics in the case of relativistic motion along the essentially nongeodesic trajectory lying far from the null geodesic (SR). We consider the radiation of the scalar and electromagnetic waves for the cases $B \ll B_M$ or $Q/M \ll 1$. As is known [16], the scalar wave equation in the Kerr metric admits the solution in the separate variables form

$$\Phi(\mathbf{r}, t) = \sum_e \sum_m R_{em}(r)\, Z_e^m(\theta, \varphi)\, e^{-i\omega t}, \qquad \omega = m\omega_p \tag{21}$$

where $Z_l^m(\theta, \varphi)$ are the spheroidal harmonics.

For the nonrelativistic motion $M\omega \ll 1$ the mode $l = |m| = 1$ dominates. Using the quasistatic solutions of the homogeneous radial equation [17] we obtain the intensity of radiation at infinity and at the event horizon, respectively:

$$P_{l=1}^{\text{out}} = \frac{(f\mu)^2}{3} \cdot \frac{\omega_p^4(M^2 - a^2)}{\Delta_p} \left(1 - \frac{2M}{r_p}\right)^2 \left(r_p^2 + a^2 + \frac{2Ma^2}{r_p}\right) \cdot [(1 + 2X)^2 + 4Q^2]$$

$$P_{l=1}^{H} = \frac{3}{2}(f\mu)^2 \frac{\omega_p}{R} \cdot \frac{(1 - 2M/r_p)^2}{\Delta_p M r_+} \cdot \frac{(r_p^2 + a^2 + 2Ma^2/r_p)}{1 + 4Q^2} \cdot [(1 + 2X)\sin\varphi + 2Q\cos\varphi]^2 \tag{22}$$

$$X = \frac{r - r_+}{2(r_+ - M)}, \quad Q = -\frac{Mr_+ \cdot k}{r_+ - M}, \quad k = \omega - m\Omega_H,$$

$$\cdot\, \Omega_H = \frac{a}{2Mr_+}, \quad r_+ = M + (M^2 - a^2)^{1/2} \tag{23}$$

The expression (23) contains an oscillating factor due to the interference between the incident and scattered waves. Since in the nonrelativistic case $M\omega \ll 1$, we see that the basic part of ingoing radiation from the particles moving near the event horizon is absorbed (if $\omega_p > \Omega_H$) or is amplified (if $\omega_p < \Omega_H$).

Consider now the scalar radiation from the ultrarelativistic particle in the circular nongeodesic motion under the condition $\omega(r - r_{ph}) \gg 1$. In this case the high harmonics of orbital frequency dominate and one can use the WKB approximation for R_{lm} that leads to

$$P^{\text{out}} = \sum_{l,m>0} \frac{8m}{3\gamma^2} \Omega_p^3 r_p \left(\frac{f\mu}{U^0}\right)^2 \frac{1 + \psi^2}{\Omega_p^2 (3r_p^2 + a^2) - 1} \cdot |Z_e^m(\pi/2, 0)|^2 K_{1/3}(z), P_H^{\text{in}} \simeq 0 \quad (24)$$

where $K_{1/3}(z)$ is the McDonald function,

$$\psi = \frac{2\gamma^2}{r_p^2 \Omega_p^2} \cdot \frac{l - |m|}{|m|} (1 - a^2 \Omega_p^2)^{1/2}, \qquad Z = \frac{2|m|}{3\gamma^3} r_p^2 \Omega_p^3 \frac{\Delta_p^{1/2} (1 + \psi^2)^{3/2}}{\Omega_p^2 (3r_p^2 + a^2) - 1}, \cdot \Omega_p = \frac{2Ma \mp \sqrt{\Delta_p} \cdot r_p}{r_p^3 + a^2 r_p + 2Ma^2} \quad (25)$$

Since the function $K_{1/3}$ falls exponentially for large values of the argument, it is clear that the main contribution comes from the harmonics

$$m \lesssim m_{\max} = \frac{\gamma^3 [\Omega_p^2 (3r_p^2 + a^2) - 1]}{2r_p^2 \Omega_p^3 \Delta_p^{1/2}} \quad (26)$$

The cutoff frequency is proportional to the cubic function of energy as in the case of flat space-time; however, when the radius of the orbit approaches that of the circular null geodesic, the dependence of the cutoff on γ becomes quadratic. In fact, for the case $|\eta| \ll 1$ one can see that for $r \to r_{ph}$ the quantity

$$\gamma [(3r_p^2 + a^2) \Omega_p^2 - 1] = \eta \frac{2M}{r_{ph}} \cdot \frac{r_{ph} - M}{(r_{ph} + 3M)^2} (2r_{ph}^2 - 3Mr_{ph} + a^2) \quad (27)$$

is finite as $\gamma \to \infty$. In the region of high harmonics $l \gg 1$, $|m| \gg 1$ the spheroidal function $Z_l^m(\pi/2, 0)$ can be approximated by the expression

$$|Z_l^m(\pi/2, 0)|^2 = \frac{1 + (-1)^{l-m}}{2} \cdot \frac{1}{\pi^2} \frac{\gamma}{\psi} \cdot \frac{1}{\Omega_p r_p} (1 - a^2 \Omega_p^2)^{1/2} \quad (28)$$

Pass from the sum over l and m in (24) to the integral over ψ and

$$y = \frac{4|m|}{3\gamma^3} r_p^2 \Omega_p^3 \frac{\Delta_p^{1/2}}{\Omega_p^2 (3r_p^2 + a^2) - 1} \quad (29)$$

we get

$$\frac{d^2 P^{\text{out}}}{dy\, d\psi} = \frac{9}{16\pi^2} \left(\frac{f\mu\gamma^2}{r_p}\right)^2 \frac{[\Omega_p^2(3r_p^2 + a^2) - 1]^2}{r_p^2 \Delta_p^{3/2} \Omega_p^5} \left[1 - \frac{2M}{r_p}(1 - a\Omega_p)\right]^2 y^2 (1 + \psi^2) K_{1/3}^2(z) \quad (30)$$

Integrating over ψ in (30) we obtain the following spectral distribution:

$$\frac{dP^{\text{out}}}{dy} = \frac{3\sqrt{3}}{16\pi} \left\{\frac{f\mu\gamma^2}{r_p} [\Omega_p^2(3r_p^2 + a^2) - 1] \left[1 - \frac{2M}{r_p}(1 - a\Omega_p)\right]\right\}^2 \frac{y \int_y^\infty K_{1/3}(x)\, dx}{r_p^2 \Delta_p^{3/2} \Omega_p^5} \quad (31)$$

For small y the radiation intensity grows as $y^{5/3}$, for the large y it falls exponentially. Integrating (31) over y we find the total radiation intensity

$$P^{\text{out}} = \frac{1}{12} \left\{\frac{f\mu\gamma^2}{r_p^2} \cdot \frac{\Omega_p^2(3r_p^2 + a^2) - 1}{\Delta_p^{3/4} \Omega_p^{5/2}} \left[1 - \frac{2M}{r_p}(1 - a\Omega_p)\right]\right\}^2 \quad (32)$$

Note that (32) is γ^2 times greater than the intensity of scalar GSR [2, 3].

§(5): *Electromagnetic Radiation*

In the Teukolsky formalism [16] the electromagnetic field is determined by the Newman-Penrose scalar ϕ_2 which has a separate variables form.

For the case of nonrelativistic motion one can use the quasistatic solutions [17] that lead to the following expressions for the wave flux at the horizon, for $r_p \gg M$:

$$P^H \simeq \tfrac{4}{3} e^2 \omega_p (\omega_p - \Omega_H) M r_+ \quad (33)$$

The flux at infinity in this approximation is given by the flat-space formula $P^{\text{out}} \simeq \frac{2}{3} e^2 \omega_p^4 r_p^2$. For the case of ultrarelativistic motion, for the two independent polarization states we get

$$P_\varphi^{\text{out}} = \sum_{l,m>0} \frac{8}{3} \frac{e^2}{\gamma^4} m \Omega_p^5 \Delta_p r_p \frac{(1 + \psi^2)^2}{\Omega_p^2(3r_p^2 + a^2) - 1} |Z_l^m(\pi/2, 0)|^2 K_{2/3}^2(z)$$

$$P_\theta^{\text{out}} = \sum_{l,m>0} \frac{8}{3} \frac{e^2}{\gamma^2} \frac{\Omega_p^3 \Delta_p}{m r_p} \cdot \frac{1 + \psi^2}{\Omega_p^2(3r_p^2 + a^2) - 1} \left|\frac{dZ_l^m(\pi/2, 0)}{d\theta}\right|^2 K_{1/3}^2(z) \quad (34)$$

Using the asymptotic formula

$$\left| \frac{dZ_l^m(\pi/2, 0)}{d\theta} \right|^2 = \frac{1 - (-1)^{l-m}}{2} \cdot \frac{1}{\pi^2} \frac{\psi}{\gamma} r_p \Omega_p m^2 (1 - a^2 \Omega_p^2)^{1/2} \tag{35}$$

in analogy with the results of the previous section, we obtain

$$\frac{dP_{\varphi,\theta}^{\text{out}}}{dy\, d\psi} = \frac{9e^2}{32\pi^2} \gamma^4 y^2 \frac{[\Omega_p^2(3r_p^2 + a^2) - 1]^2}{\Delta_p^{1/2} \Omega_p^3 r_p^4} (1 + \psi^2) \cdot \begin{cases} (1 + \psi^2) K_{2/3}^2(z) \\ \psi^2 K_{1/3}^2(z) \end{cases} \tag{36}$$

After the integration over ψ in (36) the spectral distribution takes the form

$$\frac{dP_{\varphi,\theta}^{\text{out}}}{dy} = \frac{3\sqrt{3}}{32\pi} e^2 \gamma^4 \frac{[\Omega_p^2(3r_p^2 + a^2) - 1]^2}{\Delta_p^{1/2} \Omega_p^3 r_p^4} \cdot y \left[\int_y^\infty K_{5/3}(x)\, dx \pm K_{2/3}(y) \right] \tag{37}$$

These expressions generalize the well-known formulas for the synchrotron radiation in flat space-time [5] to the case of the Kerr metric. Integrating (37) over y we find the total radiation intensity

$$P_{\text{tot}} = \frac{e^2 \gamma^4}{6} \cdot \frac{[\Omega_p^2(3r_p^2 + a^2) - 1]^2}{\Delta_p^{1/2} \Omega_p^3 r_p^4} \tag{38}$$

The total electromagnetic radiation power in the SR regime is γ^2 times greater than that in the GSR regime [1-3]. Note that the degree of polarization

$$\Pi = \frac{P_\varphi - P_\theta}{P_{\text{tot}}} = \frac{3}{4}$$

does not depend on the gravitational field parameters.

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