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Diffuse Scattering Model of the Thermal Damping of a Wire Moving Through Superfluid ^{3}He-B at Very Low Temperatures

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We present a microscopic model of the scattering of quasiparticles in superfluid ^{3}He-B by a moving solid surface. This is used to calculate the thermal damping of a wire resonator in the low temperature regime. The calculated damping force is in good agreement with experimental results when the quasiparticles are assumed to be scattered diffusely by the wire.

1. INTRODUCTION

Superfluid ^{3}He-B can now be cooled to a temperature regime in which the quasiparticle excitations are so few in number that interactions between them may be neglected. In this ballistic quasiparticle regime the interaction of excitations with the superfluid ground state can be studied in great detail.

Vibrating wire resonators are often used as sensitive probes of the excitations. At wire velocities below the pair-breaking critical velocity,[1] the dominant drag force exerted by the superfluid is thought to arise from the scattering of quasiparticle excitations at the wire surface. If the effect of superflow on the excitation spectrum were neglected, there would be an equal flux of quasiparticles and quasiholes incident on a given surface element of the wire. Quasiparticles and quasiholes travelling in the same direction have almost opposite momenta and so only a very small drag force would be exerted, in disagreement with experiment.[2] However, the excitation spectrum is in fact changed by the presence of superflow. Some of the excitations moving into the superfluid back-flow around the wire are unable to continue in propagating states. These excitations change their particle/hole character and move backwards along their original trajectories, a process known as Andreev reflection. This effect gives rise to an

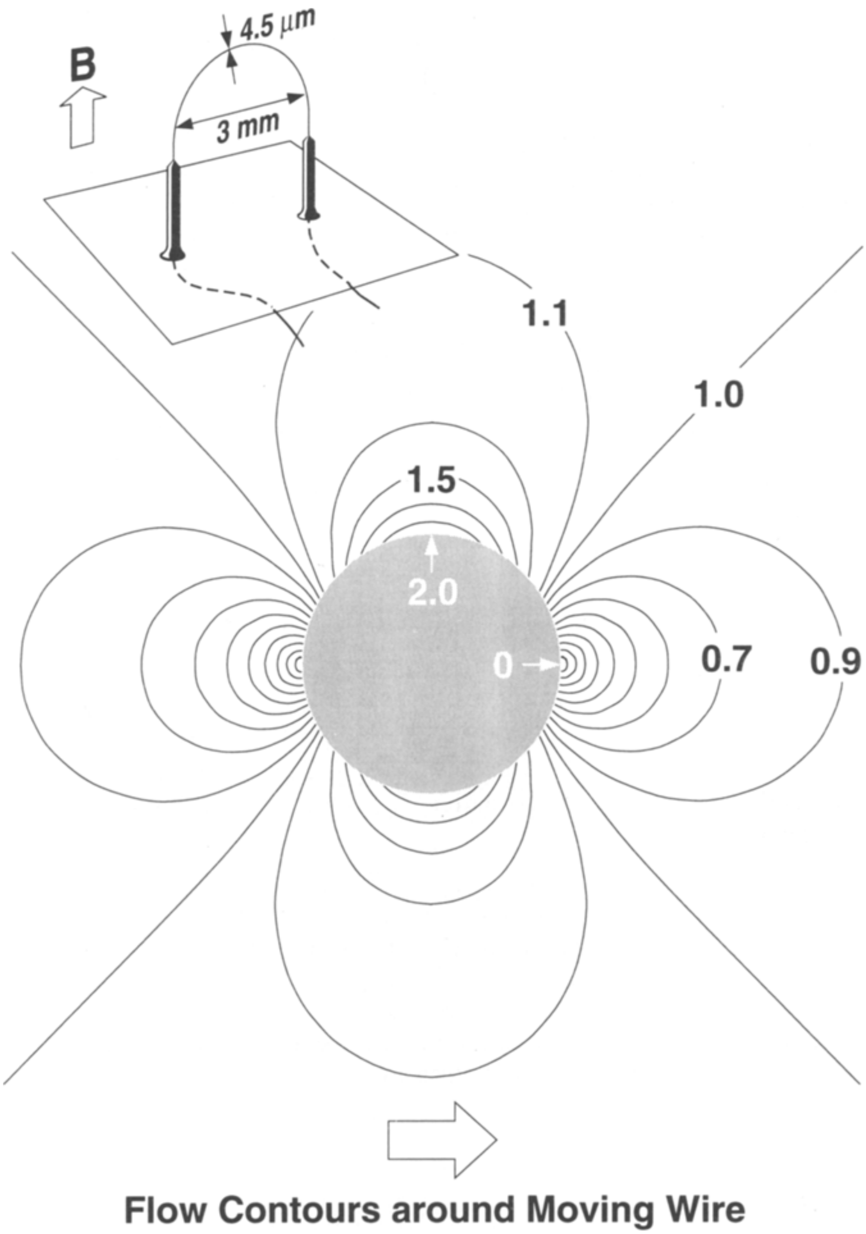

Fig. 1. Flow contours around a wire cross-section. Contours of flow speed are shown in a frame of reference in which the wire is stationary and the ^{3}He-B is moving to the right. The inset is a sketch of the experimental wire resonator geometry.

imbalance in the particle/hole flux incident on each surface element of the wire and results in a much larger drag force.

We have calculated the drag force on a wire in a previous paper.[3] In that paper it was assumed that excitations hitting the wire surface were scattered specularly. Electron micrographs of real wires show that they have rough surfaces so a diffuse scattering model is likely to be more realistic. Therefore in this paper we describe calculations of the thermal damping force in the ballistic regime, when the excitations are scattered diffusely by the wire surface. A brief report on this work is published in Ref. 4.

The calculation is done in a frame of reference in which a chosen cross-section of the wire is stationary. The superfluid velocity $\mathbf{v}_S$ round the wire is a linear combination of the uniform flow $\mathbf{V}$ at large distances and a dipolar flow centred on the wire axis as shown in Fig. 1. The inset to Fig. 1 shows the approximate geometry of the vibrating wire resonator used for the measurements described in Ref. 2. The active part of the resonator is a 4.5 μm diameter NbTi filament forming an approximately semicircular loop of radius 1.5 mm.

2. MODEL OF THE SUPERFLUID

As in Ref. 3 we model the B-phase of superfluid ^{3}He in low magnetic fields by an isotropic Fermi superfluid described by a scalar order parameter Δ. The excitation spectrum is calculated by solving the Bogoliubov equations.[5] Since $\mathbf{v}_S$ varies on a scale long compared with the coherence length, the derivative terms in the equations can be neglected. The quasiparticle energy is

$$E = \mathbf{p} \cdot \mathbf{v}_S + \sqrt{\xi^2 + \Delta^2} \tag{1}$$

where $\mathbf{p}$ is the momentum of the excitation and $\xi = v_F(|\mathbf{p}| - p_F)$. We define $\hat{\mathbf{p}}$ to be the unit vector in the direction of $\mathbf{p}$. In Eq. (1) we can replace $\mathbf{p}$ by $p_F\hat{\mathbf{p}}$ with quasiclassical accuracy because $|\mathbf{v}_S| < (\Delta/E_F)\, v_F$. The excitations can be distinguished as quasiparticles if $\xi > 0$ and quasiholes if $\xi < 0$. In either case the group velocity is given by

$$\mathbf{v}_G = \mathbf{v}_S + \frac{\xi v_F \hat{\mathbf{p}}}{\sqrt{\xi^2 + \Delta^2}} \tag{2}$$

At a point on the surface of the wire where the outward normal is $\hat{\mathbf{n}}$, $\mathbf{v}_S$ is tangential. As a result $\mathbf{v}_G \cdot \hat{\mathbf{n}}$ has the same sign as $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}}$ for quasiparticles, and the opposite sign for holes. Quasiparticles incident on the surface have $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} < 0$ and quasiholes have $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} > 0$.

3. INCOMING EXCITATIONS

Incoming excitations, which come from the bulk of the superfluid and hit the wire, are assumed to be in thermal equilibrium in the frame of reference of the container. In our frame of reference the container is moving with velocity $\mathbf{V}$. We label incoming excitations with momenta $\mathbf{p}'$ and outgoing excitations with momenta $\mathbf{p}$. At low temperature the incoming excitations have a Boltzmann-like distribution function, but their energies in the container frame of reference are $E-\mathbf{p}'\cdot\mathbf{V}$. Their distribution function is

$$f(\mathbf{p}') = \exp[(E-\mathbf{p}'\cdot\mathbf{V})/kT] \tag{3}$$

in a volume $d^3\mathbf{p}'/(2\pi\hbar)^3$ of momentum space.

The flux of incoming excitations hitting a surface element $\hat{\mathbf{n}}\,dS$ of the wire is

$$|\mathbf{j}\cdot\hat{\mathbf{n}}| = 2\int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\,|\mathbf{v}_G\cdot\hat{\mathbf{n}}|\,f(\mathbf{p}') \tag{4}$$

By using Eqs. (1) and (2) this can be written as

$$|\mathbf{j}\cdot\hat{\mathbf{n}}| = 4v_F N(0)\int_{\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}<0}\frac{d\Omega_{\hat{\mathbf{p}}'}}{4\pi}\int_{\mathbf{p}'\cdot\mathbf{v}_S+\Delta}^{\infty}|\hat{\mathbf{p}}'\cdot\hat{\mathbf{n}}|\,f(\mathbf{p}')\,dE \tag{5}$$

In this expression the two spin states and the states for quasiparticles and quasiholes have been combined.

In calculating the total drag force on the wire, it is convenient to separate the force into contributions from the incoming and outgoing excitations. This separation may be visualised by imagining that an incoming excitation is absorbed by the wire and then re-emitted. In the case of specular scattering the incoming and outgoing excitation momenta are related uniquely by the identity

$$2(\hat{\mathbf{p}}'\cdot\hat{\mathbf{n}})\,\hat{\mathbf{n}} = \hat{\mathbf{p}}' - \hat{\mathbf{p}} \tag{6}$$

The expression for the *total* force on the wire given in Ref. 3 can be separated into incoming and outgoing contributions by means of this identity. The incoming contribution to the force is exactly the same in the diffuse scattering model as in the specular scattering model of Ref. 3. In our present notation the incoming force is

$$F^{<} = \frac{p_F v_F a N(0)}{\pi^2}\int_{-\infty}^{+\infty} dE\,\exp\left(-\frac{E}{kT}\right)\int_0^{\pi} d\theta\, K_P(E,\theta) \tag{7}$$

where

$$K_P = -\int_{\substack{\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}<0 \\ \mathbf{p}'\cdot\mathbf{v}_S<E-\Delta \\ \mathbf{p}'\cdot\mathbf{V}<E-\Delta}} 2\pi(\hat{\mathbf{p}}'\cdot\hat{\mathbf{V}})(\hat{\mathbf{p}}\cdot\hat{\mathbf{n}})\exp\left(\frac{\mathbf{p}'\cdot\mathbf{V}}{kT}\right)d\Omega_{\hat{\mathbf{p}}'} \tag{8}$$

These expressions are obtained from the incident flux given by Eq. (4). The integrand in Eq. (4) is simply multiplied by $p_F\hat{\mathbf{p}}\cdot\hat{\mathbf{V}}$, which is the component of momentum in the $\mathbf{V}$-direction carried by the incident excitation.

4. OUTGOING EXCITATIONS

In order to have a tractable model of diffuse scattering we assume that the outgoing excitation momentum has a random direction. The only restrictions are that the excitation is moving away from the surface, so that $\mathbf{v}_G\cdot\hat{\mathbf{n}}>0$, and that it is in a propagating state. This condition means that, given the momentum direction $\hat{\mathbf{p}}$ and the energy E (which is conserved in the scattering by the wire), the dispersion relation (1) determines a real value of ξ for the outgoing excitation. This point is considered further in the next section on Andreev reflection.

Each outgoing excitation removes a momentum component $\mathbf{p}\cdot\hat{\mathbf{V}}$ in the superflow direction. The total contribution to the drag force from outgoing quasiparticles which have been scattered directly into propagating states is

$$\tilde{F}_{qp}^{>} = \frac{p_F v_F a N(0)}{\pi^2}\int_{-\infty}^{+\infty} dE\exp\left(-\frac{E}{kT}\right)\int_0^{\pi} d\theta\, I_P(E,\theta)\, J_P(E,\theta) \tag{9}$$

where

$$I_P = \int_{\substack{\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}>0 \\ \mathbf{p}\cdot\mathbf{v}_S<E-\Delta}} \hat{\mathbf{p}}\cdot\hat{\mathbf{V}}\, d\Omega_{\hat{\mathbf{p}}}$$

$$J_P = \int_{\substack{\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}<0 \\ \mathbf{p}'\cdot\mathbf{v}_S<E-\Delta \\ \mathbf{p}'\cdot\mathbf{V}<E-\Delta}} \hat{\mathbf{p}}'\cdot\hat{\mathbf{n}}\exp\left(\frac{\mathbf{p}'\cdot\mathbf{V}}{kT}\right)d\Omega_{\hat{\mathbf{p}}'} \tag{10}$$

The conditions on $\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}$ and $\hat{\mathbf{p}}'\cdot\hat{\mathbf{n}}$ ensure that the integrals I_P and J_P are taken over outgoing and incoming quasiparticles respectively. The integrand of I_P resolves the momentum carried away by the scattered excitations in the $\mathbf{V}$-direction. The restriction $\mathbf{p}\cdot\mathbf{v}_S<E-\Delta$ on the integral I_P ensures that the outgoing excitations are in propagating states.

The factors in the integrand of J_P are derived from the flux of incident quasiparticles. The limits of integration indicate that the incoming quasiparticles are moving towards the surface, and that they are in propagating states when they hit it. There is an additional restriction on J_P that the incoming quasiparticles were in propagating states when they were thermally excited in the container frame of reference, in which the superfluid velocity is $\mathbf{V}$.

5. ANDREEV REFLECTION

Physically the restriction of the excitations to propagating states is enforced by Andreev reflection. For example, a quasiparticle must satisfy the condition $\mathbf{p}' \cdot \mathbf{V} < E - \Delta$ at large distances from the wire. If it approaches a region where the superfluid velocity is such that $\mathbf{p}' \cdot \mathbf{v}_S > E - \Delta$ it will be Andreev-reflected, because its excitation energy ξ is imaginary, and its wave function is evanescent. This gives rise to a difference between the currents of quasiparticles and quasiholes hitting a particular surface element $\hat{\mathbf{n}}\, dS$ of the wire, because quasiparticles moving towards the surface have $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} < 0$ but quasiholes have $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} > 0$.

In reality, the order parameter components are depressed from their bulk values within about one coherence length from the wire surface. In this paper we neglect alterations to quasiparticle trajectories on this length scale, and explain the drag force as the effect of the variation of the superfluid velocity on a scale comparable with the diameter of the wire.

In the specular case an excitation incident on the wire surface is always scattered into a propagating state. However, in our model of diffuse scattering the excitation loses all memory of its original state and is scattered at a random angle. Some of the incident excitations will be scattered into non-propagating states. To account for this we assume that a quasiparticle scattered into a non-propagating state is Andreev reflected close to the wire surface. It returns to the wire as a quasihole where it is again scattered through a random angle. If the quasihole is scattered into another non-propagating state, it is Andreev-reflected again and returns to the wire as a quasiparticle. This continues until the excitation is finally scattered into a propagating state.

Figure 2 shows a simple picture of the condition $p_F \hat{\mathbf{p}} \cdot \mathbf{v}_S < E - \Delta$ that the excitation state labelled with $\mathbf{p}$ is a propagating one. If $E < \Delta - p_F |\mathbf{v}_S|$ there are no directions $\hat{\mathbf{p}}$ for propagating states, and if $E > \Delta + p_F |\mathbf{v}_S|$ all directions give propagating states. If $\Delta - p_F |\mathbf{v}_S| < E < \Delta + pF |\mathbf{v}_S|$ we draw a cone with semivertical angle $\psi_0 = \arccos[(E - \Delta)/p_F |\mathbf{v}_S|]$ in momentum space about $\mathbf{v}_S$ as axis. The state $\mathbf{p}$ is propagating if and only if $\mathbf{p}$ lies

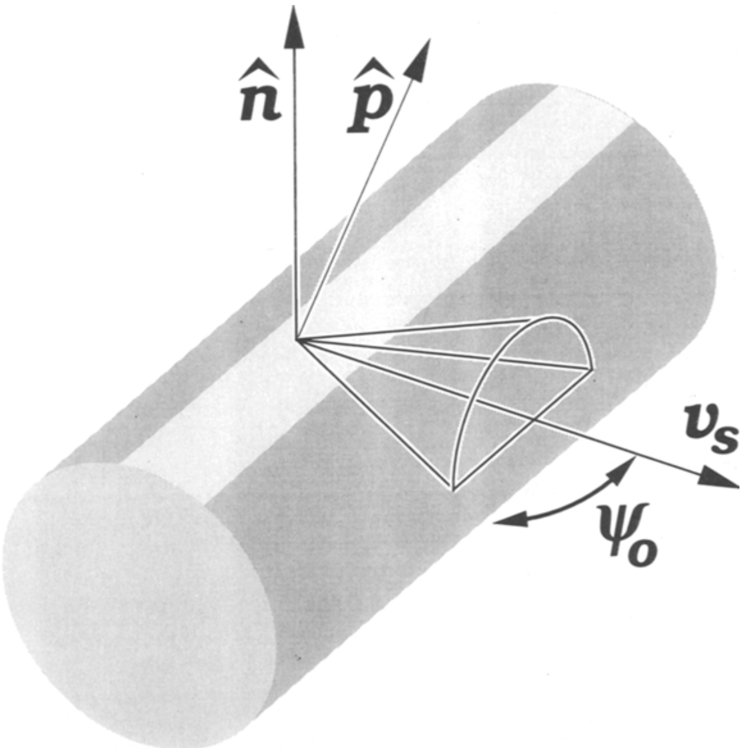

Fig. 2. Andreev reflection of scattered quasiparticles. Arrows show the normal $\hat{\mathbf{n}}$ and superfluid velocity $\mathbf{v}_S$ at the surface of the wire. A scattered quasiparticle could escape in the direction $\hat{\mathbf{p}}$ because it makes a greater angle than ψ_0 with $\mathbf{v}_S$ (see text).

outside the cone. The other condition $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} > 0$ (for an outgoing quasiparticle) slices the cone in half through its axis.

With our assumption of random-angle scattering by the wire surface, the probability of a scattered quasiparticle escaping is $A = \frac{1}{2}(1 + \cos \psi_0)$. With probability $1 - A$ it will be Andreev-reflected as a quasihole if it was originally a quasiparticle. After Andreev reflection the quasihole is moving towards the surface but its value of $\mathbf{p}$ is unchanged. The surface scatters it at random into a state with a new value of $\mathbf{p}$ such that $\hat{\mathbf{p}} \cdot \hat{\mathbf{n}} < 0$. With probability A this value of $\mathbf{p}$ lies outside the cone and the quasihole escapes, and with probability $1 - A$ it is Andreev-reflected a second time into a quasiparticle and it is scattered again by the surface. The process can be indefinitely repeated. The result is that if the quasiparticle escapes into the bulk after an odd number of scatterings by the surface it is a quasiparticle, and if after an even number of scatterings it is a quasihole. The total contribution to the drag force $F^{>}_{qp}$ from outgoing excitations resulting from incoming quasiparticles can be calculated from Eq. (9) by replacing the factor I_P with the factor

$$I = \frac{I_P}{A(2-A)} - \frac{(1-A)\, I_H}{A(2-A)} \tag{11}$$

where

$$I_H = \int_{\substack{\hat{\mathbf{p}}\cdot\hat{\mathbf{n}}<0 \\ \mathbf{p}\cdot\mathbf{v}_S<E-\Delta}} \hat{\mathbf{p}}\cdot\hat{\mathbf{V}}\, d\Omega_{\hat{\mathbf{p}}} \tag{12}$$

An identical contribution results from excitations scattered from incoming holes. Thus the total force is $F = F^{<} + 2F^{>}_{qp}$ for a given cross section of the wire moving with velocity V.

6. COMPARISON WITH EXPERIMENT

To compare this with the experimental data obtained on a vibrating wire loop by Fisher *et al.*,[2] the damping force is integrated over a time period of oscillation and around the length of the loop which is assumed to be simicircular. The force is then normalised by the initial damping force

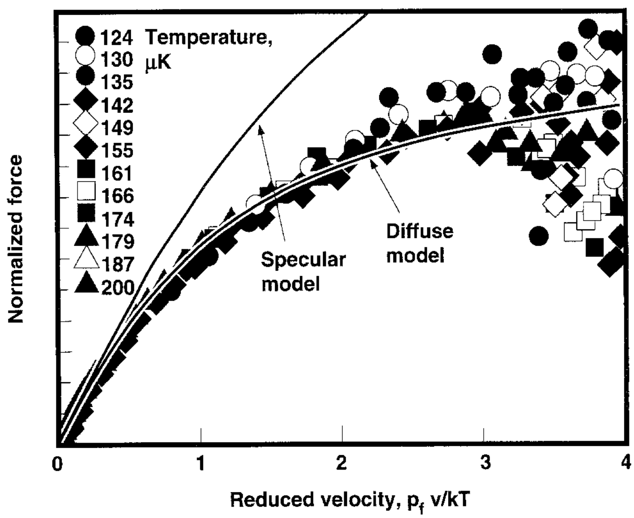

Fig. 3. The normalised thermal damping force as a function of reduced wire velocity $p_F V/kT$. The solid curves are our theoretical predictions for specular and diffuse scattering of quasiparticles. These are normalised at low velocity to the slope of the experimental points, shown by symbols.

per unit $(p_F V/kT)$ and is shown in Fig. 3 along with the experimental data. The result of the specular model is also shown for comparison. When plotted in this way the experimental data is seen to be temperature independent as predicted. Also the dependence on $(p_F V/kT)$ is very close to that obtained from the diffuse model whereas the specular model is less satisfactory. It is more difficult to comment on the absolute magnitude of the damping force owing to the exponential temperature dependence and the difficulties of thermometry at these very low temperatures. However, using a quasiparticle black body radiator, Fisher *et al.*[6] have deduced the damping of a vibrating wire resonator as a function of the quasiparticle number density. Their results are in good agreement with the calculation given above, with an estimated uncertainty of 20% arising from the non-ideal geometry of a real vibrating wire loop.

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