# LENGTH OF FREE DIFFUSION FLAMES

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Due to the wide use of diffusion flames in various technological apparatus, numerous studies [1-21] on length of laminar and turbulent flames of various combustible gases have appeared.

We will consider vertical turbulent flames. For this case we write the equation of conservation of momentum flow with consideration of lifting forces:

$$\int_0^F \rho w^2 dF = \rho_1 w_1^2 dF - \int_0^l \int_0^F (\rho - \rho_\infty) g dF dx \tag{1}$$

and the equation of conservation of some chemical element which may be present in various components of the initial mixture of reagents and in the reaction products:

$$\int_0^F (\varkappa - \varkappa_\infty) \rho w dF = \rho_1 w_1 F_1 (\varkappa_1 - \varkappa_\infty), \tag{2}$$

where $\rho_1$, $\rho_\infty$, $\rho$ are the gas density at the nozzle mouth, oxidizer density, and current density at an arbitrary point along the flow, respectively; w, $w_1$, current and mean flow rates; F, cross-sectional area of jet (the subscript 1 refers to the initial section of the jet); g, acceleration of gravity; $\varkappa_1$, $\varkappa_\infty$, $\varkappa$, mass concentration of some element present in the original gas at the nozzle mouth, in the oxidizer, and at an arbitrary point in the flame.

Introducing into Eqs. (1), (2) the values of the flow parameters on the jet axis (denoted by index m) and applying the mean-value theorem to the integral on the right side of Eq. (1), we reduce the equation to the form

$$F\rho_m w_m^2 I_1 = F_1 \rho_1 w_1^2 - (\bar{\rho} - \rho_\infty) g \bar{V}_f, \tag{3}$$

$$(\varkappa_m - \varkappa_\infty) F \rho_m w_m I_2 = \rho_1 w_1 F_1 (\varkappa_1 - \varkappa_\infty), \tag{4}$$

where $\bar{\rho}$, $\bar{V}_f$ are some mean values of flame density and volume:

$$I_1 = \int_0^1 \frac{\rho}{\rho_m} \frac{w^2}{w_m^2} \frac{dF}{F}, \quad I_2 = \int_0^1 \frac{\rho}{\rho_m} \frac{w}{w_m} \frac{\Delta\varkappa}{\Delta\varkappa_m} \frac{dF}{F}, \tag{5}$$

$$\Delta\varkappa = \varkappa - \varkappa_\infty, \; \Delta\varkappa_m = \varkappa_m - \varkappa_\infty.$$

From Eqs. (3), (4) we obtain

$$\frac{\Delta\varkappa_m}{\Delta\varkappa_1} = \frac{\varkappa_m - \varkappa_\infty}{\varkappa_1 - \varkappa_\infty} = \frac{\sqrt{I_1}}{I_2} \sqrt{\frac{\rho_1 F_1}{\rho_m F} - \frac{\Delta\bar{\rho} g \bar{V}_f}{F \rho_m w_1^2}} + \sqrt{I_1} \Delta\bar{\rho} g \bar{V}_f \Bigg/ \left( I_2 F \rho_m w_1^2 \sqrt{\frac{\rho_1 F_1}{\rho_m F} - \frac{\Delta\bar{\rho} g \bar{V}_f}{F \rho_m w_1^2}} \right). \tag{6}$$

Introducing into Eq. (6) the Froude number ($\mathrm{Fr}_1 = w_1^2/gd_1$), and considering that $F/F_1 = b^2/d_1^2$ (b, $d_1$, jet diameter at an arbitrary section and at nozzle mouth), Eq. (6) takes on the form

$$\left(\frac{\Delta\varkappa_m}{\Delta\varkappa_1}\right)^2 \frac{\rho_1}{\rho_m} + \left(\frac{\Delta\varkappa_m}{\Delta\varkappa_1}\right)^2 \frac{\Delta\bar{\rho}\bar{V}_f}{\rho_m F_1 d_1} \frac{1}{\mathrm{Fr}} = \frac{I_1}{I_2^2} \frac{\rho_1^2}{\rho_m^2} \frac{d_1^2}{b^2}. \tag{7}$$

For turbulent flames at large Froude numbers, the second term on the right of Eq. (7) becomes negligibly small (self-similarity with respect to Fr sets in [17-19, 21]). In this case Eq. (7) simplifies significantly. Considering that at large distances from the source the characteristic jet width increases in proportion to the distance to the source [22, 23]:

$$b = \tilde{c} x, \tag{8}$$

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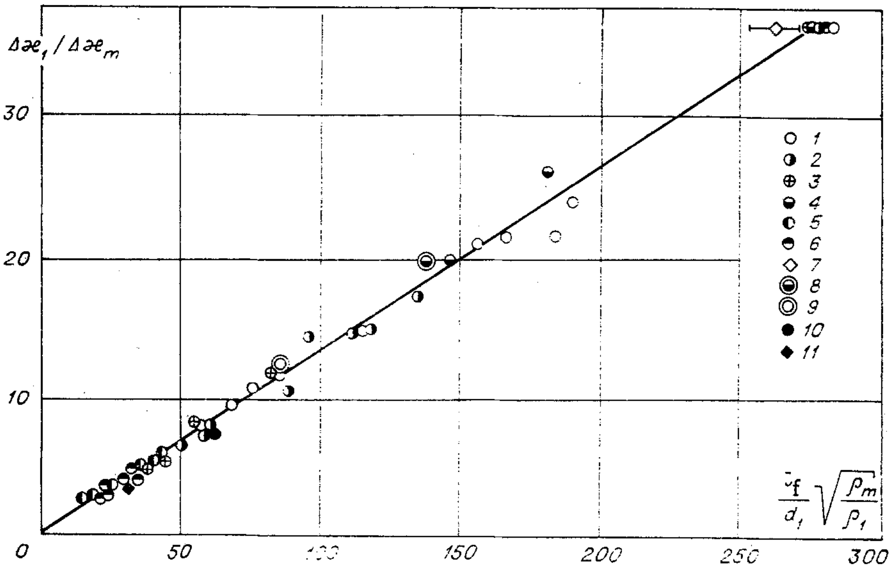

Fig. 1. Generalized experimental data for Fr independence region. $d_0$, mm ($H_2 + N_2$ mixture [20]): 1) 1.45; 2) 1.9; 3) 2.9; 4) 4.0; 5) 6.0; 6) 10.75; 7) 100% $H_2$ [17]: 8) 70% $H_2$ + 30% He [20]; 9) 49% $H_2$ + 51% He [20]; 10) city gas [6]; 11) city gas + 50% air [5].

and also considering that the quantities $I_1$ and $I_2$ at large x may be considered constant, from Eq. (7) for the region of independence from Fr we obtain

$$\Delta\varkappa_m/\Delta\varkappa_1 = c/(x/d_1\cdot\sqrt{\rho_m/\rho_1}), \tag{9}$$

where the empirical coefficient $c = \sqrt{I_1}/I_2\tilde{c}$.

Figure 1 presents experimental data of various authors, obtained for city gas, a mixture of city gas with primary air, hydrogen, and various mixtures of hydrogen with nitrogen and helium (from 0 to 0.6 volume of dilutant), for a wide range of nozzle diameters (from 1.5 to 11 mm). Study of hydrocarbon-fuel diffusion flames in the region of independence from Fr is possible only with the use of flame stabilization at the nozzle edge by a pilot flame. The data of [6], presented in Fig. 1, were obtained by such a method.

As is evident from the figure, all the presently available data on turbulent diffusion flames in the region of independence of Froude number may be described by the following equation:

$$\Delta\varkappa_m/\Delta\varkappa_1 = 7.65/(l_f/d_1\sqrt{\rho_m/\rho_1}), \tag{10}$$

where the mass concentration of the element $\varkappa_m$ is determined by the stoichiometric equation of the combustion reaction, relying on the fact that the reagent concentration in the flame front is stoichiometric [24], and the reaction-product density $\rho_m$ is taken at the calorimetric combustion temperature.

Thus, for a pure hydrogen flame

$$\Delta\varkappa_m/\Delta\varkappa_1 = \varkappa_m = 0.0279,\ \rho_m = 0.128\ \text{kg/m}^3.$$

If, in Eq. (10), instead of the reaction-product density we use the density of the surrounding medium $\rho_\infty$, then in processing the experimental results on diffusion flames we must include among the generalized variables a complex considering the thermal effect of the chemical reaction. We may choose for such a generalized variable the energy criterion $K_{en}$ [25], which in the present case represents the ratio of the enthalpy of the original fuel gas to the thermal effect of the reaction at standard temperature $Q_H^P$. For combustion of pure hydrogen $K_{en} = 1.984/(2.576\cdot 22.4) = 3.44\cdot 10^2$, and for a mixture of hydrogen with nitrogen (volume fraction of nitrogen 0.6) $K_{en} = 8.77\cdot 10^{-2}$.

Figure 2 shows the experimental data considered above in the form of functions of energy criterion. This functional dependence may be described with satisfactory accuracy by the following equation:

$$\Delta\varkappa_m/\Delta\varkappa_1 = [4.75(1+1/K_{en})^{0.5}]/(l_f/d_1\cdot\sqrt{\rho_\infty/\rho_1}). \tag{11}$$

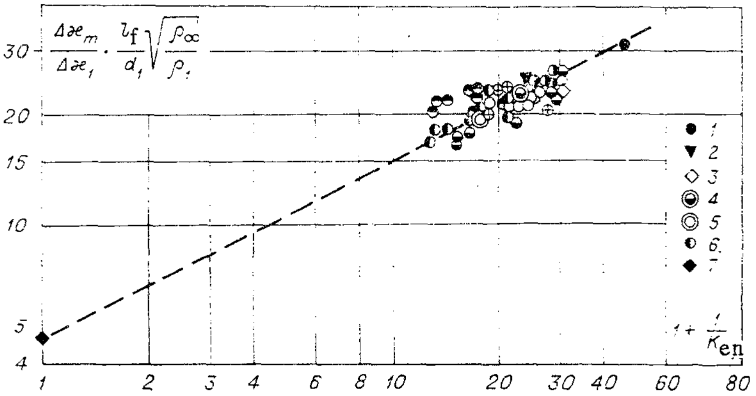

Fig. 2. Generalized experimental data on diffusion-flame length in the region of independence of Fr with consideration of energy criterion: 1) city gas [6]; 2) city gas + 50% air [5]; 3) 100% $H_2$ [17]; 4) 70% $H_2$ + 30% He [20]; 5) 49% $H_2$ + 51% He [20]; 6) 61% $N_2$ + 39% $H_2$ [20]; 7) freon, air, helium [23].

The maximum deviation of the experimental data from Eq. (11) is 25%.

Equations (10) and (11) were obtained by using the hydrogen atom concentration for the required concentration. However, as has been shown experimentally [26], in a diffusion flame there is a similarity in the ratios of mass concentrations $\Delta\varkappa_m/\Delta\varkappa_1$ of all atoms entering into the composition of the original combustible mixture. This has also been verified by processing of experimental data on diffusion flames containing hydrocarbons.

In the absence of a chemical reaction ($1/K_{en} = 0$), Eq. (11) coincides with the expression proposed in [23] for free turbulent submerged jets of various nonreacting gases (freon, air, helium, air, plasma stream). This allows the conclusion that use of the dimensionless chemical element concentration $\Delta\varkappa_m/\Delta\varkappa_1$ will permit generalization of experimental data on dispersal of different chemically reacting jets.

In the region of low Froude numbers all terms of Eq. (7) must be considered. Due to the complexity of determining $\overline{\Delta\rho}$, $\overline{V}_f$, we will use the experimental data of [11, 15, 16], obtained for turbulent diffusion flames of propane, hydrogen, and methane in the Froude number range to $\sim 3\cdot 10^4$:

$$l_f/d_1 = A\mathrm{Fr}^m. \quad (12)$$

According to the various authors, the coefficients A and m have the following values:

$$\begin{array}{ll} \text{Methane} & A=29,\ m=0.2\ [11, 16]; \\ & A=27,\ m=0.2\ [21]; \\ \text{Propane} & A=40,\ m=0.2\ [11, 16]; \\ \text{Hydrogen} & A=14,\ m=0.2\ [21]; \\ & A=16,\ m=0.2\ [11]. \end{array} \quad (13)$$

In [11], m = 0.23. However, theoretical studies [12-14, 17-27] give a value of m = 0.2. Considering this, in [16] the data of [11] for propane and methane were recalculated for m = 0.2. We have also recalculated the data of [11] for hydrogen. These A and m values are given above.

Using the generalized variables proposed, all the experimental data presented, Eq. (13) and [4], for various mixtures of propane with hydrogen and primary air can be described with a maximum deviation of 25% by the function

$$\Delta\varkappa_m/\Delta\varkappa_1 = (0.61\mathrm{Fr}^{0.2})/(l_f/d_1\cdot\sqrt{\rho_m/\rho_1}). \quad (14)$$

It should be noted that Eq. (14) is valid only for turbulent diffusion flames in the Froude number range to $\sim 3\cdot 10^4$. Critical Reynolds number values, corresponding to the transition from laminar to turbulent flame, depend on the nature of the fuel gas and are presented in [3]. Carbon monoxide flames differ significantly from all other turbulent diffusion flames, having a significant laminar segment. Thus, in calculating these flames with Eq. (14) it is necessary to take only the turbulent portion of the total flame length.

For the Froude number range above ~$3 \cdot 10^4$, the region of transition to self-similarity sets in. The exponent in Eq. (12) then changes from 0.2 to 0 [7, 10, 11, 20]. In this case, for a narrow range of Fr generalized equations of the form of Eq. (14) may be obtained.

## LITERATURE CITED

1. S. P. Burke and T. E. W. Schumann, Ind. Eng. Chem., 20, 998 (1928).
2. E. W. Rembert and R. T. Haslam, Ind. Eng. Chem., 20, 1925 (1936).
3. W. Gausorn and H. Hottel, in: Questions of Combustion [Russian translation], Vol. 1, IL, Moscow (1953).
4. W. Gausorn, D. Widdel, and H. Hottel, in: Questions of Combustion [Russian translation], Vol. 1, IL, Moscow (1953).
5. K. Wohl, C. Gazley, and N. Kapp, Third Symposium on Combustion, New York (1949).
6. H. C. Hottel, Fourth Symposium (International) on Combustion, New York (1949).
7. P. G. Levchenko and B. I. Kitaev, Stal', No. 3 (1952).
8. A. S. Telegin and B. I. Kitaev, Tr. Ural. Politekh. Inst., 53 (1955).
9. O. N. Ermolaev, in: Study and Calculation of Thermal Energy and Chemical Energy Processes [in Russian], Moscow (1961).
10. O. N. Ermolaev and S. N. Shorin, Teploénergetika, No. 2 (1959).
11. K. Hess, Dissertation, Karlsruhe (1964).
12. F. R. Steward, Combust. Flame, 8, 171 (1964).
13. F. R. Steward, Combust. Sci. Technol., 2, 203 (1970).
14. P. D. Sunavala, J. Inst. Fuel, 40, 492 (1967).
15. A. A. Patnam and C. F. Speich, Ninth Symposium on Combustion, New York (1963).
16. P. G. Seeger and H. G. Werthenbach, Chem. Ind. Technol., 42, No. 5 (1970).
17. V. K. Baev, P. P. Kuznetsov, et al., Fiz. Goreniya Vzryva, 10, No. 4 (1974).
18. V. K. Baev and V. A. Yasakov, Fiz. Goreniya Vzryva, 10, No. 6 (1974).
19. V. I. Golovichev and V. A. Yasakov, Fiz. Goreniya Vzryva, 8, No. 1 (1972).
20. V. F. Komov and G. G. Shevyakov, in: Inhibition of Step-Type Gas Reactions [in Russian], Alma-Ata (1971).
21. V. F. Komov et al., in: Combustion Processes and Problems of Fire Extinguishing [in Russian], Part 1, VNIIPO (1973).
22. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Fizmatgiz, Moscow (1960).
23. G. N. Abramovich, S. Yu. Krasheninnikov, et al., Turbulent Mixing of Gas Jets [in Russian], Nauka, Moscow (1974).
24. Ya. B. Zel'dovich, Zh. Tekh. Fiz., 19, No. 10 (1949).
25. A. L. Suris and S. N. Shorin, Khim. Vys. Énerg., 5, No. 3 (1971).
26. W. Brown and G. Powell, in: Questions of Combustion [Russian translation], Metallurgiya, Moscow (1968).
27. A. G. Prudnikov, M. S. Volynskii, and V. I. Sagalovich, Mixture Formation and Combustion in Air-Reactive Engines [in Russian], Mashinostroenie, Moscow (1971).