

Lorenz Curves for Various Processes: A Pluralistic Approach to Equity

P. Suppes

Department of Philosophy, Stanford University, Stanford, CA 94305, USA

Received November 14, 1986 / Accepted February 3, 1988

Abstract. Theoretical Lorenz curves for a bureaucracy with n seniority levels and similar curves for a simple stochastic economy with capitalistic opportunity to move up or down are derived. In both cases it is argued that equality of distribution is not an appropriate ideal. But the main thrust is that income alone is not a sufficient criterion of equity. A more general Aristotelian viewpoint is argued for. The marked increase in the equity of education between 1940 and 1984 in the United States is taken as a good example. But even within education, it is argued, detailed policies will realistically satisfy no simple set of equity principles.

The appeal of principles that lead to an equal distribution of income is matched by the appeal of similar principles in the theory of belief. In the latter case, the result is a uniform probability distribution as the appropriate prior distribution. Several of the main principles used to defend a uniform distribution of value have also been used to defend the uniform distribution of belief, or conversely. The most salient example is the use of the principle of insufficient reason to justify a uniform distribution in either case. The Bayesian attack on a central principle of uniformity of belief has had a powerful impact. Classical or logical theories of probability that have attempted to justify the uniform distribution of belief as the only natural prior distribution are not currently well received. Of course, in real problems a uniform prior distribution has never had the role it was presumed to have in introductory discussions of the classical theory of probability. Perhaps the finest example is to be found in Laplace's classical treatise on probability. He announces that probability is to be defined in terms of the ratio of favorable cases to possible cases, but then this definition scarcely enters into any of the complex developments or applications he pursues.

The Bayesian approach to belief is, in fact, suggestive of the proper approach to matters of equity. It is a mistake to think only in terms of simple first principles that justify egalitarianism. We need to think harder and in a more subtle way about the principles that justify various inequalities that are unlikely to be eliminated, or that,

in fact, most people would not want to see eliminated. Some examples are discussed in the first section.

There is a second introductory point to be made about the viewpoint I advocate here. Too much discussion of matters of justice and equity has centered on unrealistic ideas such as those of Rawls concerning the initial distribution under the veil of ignorance, or, to take a quite different example, Nozick's idealized and equally unrealistic concept of entitlement. As in the case of Bayesian theories of belief, we are always in the middle of rebuilding the equity ship at sea. We are not going to run ourselves aground and start afresh. What actually dominates political considerations in debates about issues of equity is what we should do next, what direction we should go. It seems to me that what we need from a normative standpoint, therefore, is more analysis of long-run outcomes, rather than analysis of inevitably dim beginnings.

For this reason, I have concentrated in Sect. 2 on asymptotic Lorenz curves of distribution, but I recognize that these too should be taken with a grain of salt. We are not really interested in asymptotic results but always in much shorter-range results, as reflected in actual data in Sect. 4. The reason to discuss asymptotic results is just that it is easy to summarize the results of processes when we look at the asymptotic behavior. It may well be that the right normative consideration – right in the sense of proper modesty – is to think not at all in terms of asymptotic results but only in terms of direction of change. Such an analysis could be given for the processes discussed in Sect. 2, without much modification, and I do present in Sect. 4 some empirical results in this direction with respect to the distribution of education in the United States.

In the first section I review briefly some earlier results of mine [6] concerned with seniority and income distribution in bureaucracies. The results might be described, perhaps facetiously, as socialism with class. In any case, I want to use the resulting Lorenz curves, and the corresponding Gini coefficients, to compare with various dynamic processes that could be put in place in the part of an economy that is market-driven. (Note that I deliberately do not say “in a market economy” because it seems to me we shall be faced forever with mixed economies, and in discussion of appropriate normative questions about distribution of income we must deal both with market forces and also with bureaucratic institutions.) The Gini coefficient can certainly be criticized as an appropriate single measure, but I use it here for simple comparative purposes.

The analysis of inequality ordinarily centers around the Lorenz curve for income distribution. There is broad agreement on using the Lorenz curve to represent the distribution of income or wealth, but there is much less agreement over how to define a single measure of inequality to be derived from the Lorenz curve. The classical and most widely used measure is the Gini coefficient, which is used in Sects. 1 and 2.

Sections 3 and 4 are concerned with the philosophical foundations of a pluralistic approach to equity. A broadly Aristotelian viewpoint is argued for, and one distribution example from education is worked out in detail. The final section moves to the microanalysis of equity. A second example from education concerned with allocation of instructional resources is to show concretely the practical impossibility of reaching agreement about determinant principles of equity at the microlevel.

A variety of examples and arguments are considered in this paper, but the central idea is easy to state: whenever we turn to details, no egalitarian or other simple principles of equity are likely to be widely accepted. Practical decisions about particular allocations of benefits of any kind will irreducibly depend in the end on negotiation and bargaining, not the algorithmic application of categorical principles.

1. Socialism with Class

I now turn to the derivation of the Gini coefficient for a society of classes (following [6]). For the mathematical models considered here, it is sufficient to know the number n of classes and the income differential between the classes. Thus, for example, if $j=0.05$, in moving from one class to the next higher class the income benefit in the ratio model is an increase of 5 percent. A critical simplifying assumption is that the society is in equilibrium with each class occupied by the same number of members.

First, under these assumptions it is easy to show for the given model that if x is the income of a member of the lowest class, then $x(1+j)^{k-1}$ is the income of a member of the k^{th} class counting from the bottom upward. Because we are only interested in proportionality results, we may hereafter ignore the actual amount of income and thus ignore x . Second, it is then easy to show that the proportion of income y_k distributed to the first k classes is:

$$y_k = \frac{(1+j)^k - 1}{(1+j)^n - 1}. \quad (1)$$

Our next task is to compute the Gini coefficient as a function of n and j , which is done in [6]:

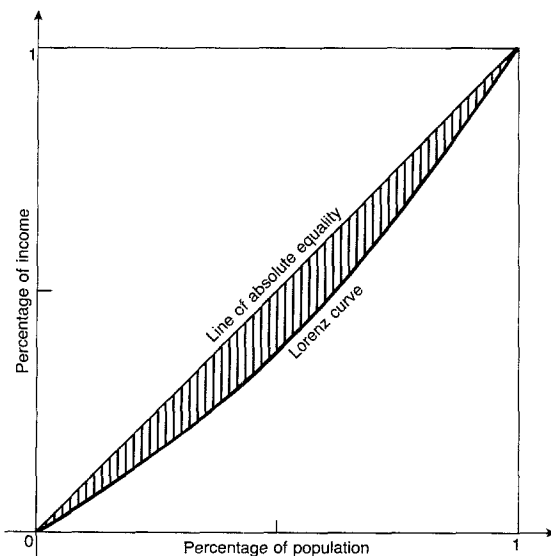


Fig. 1. Theoretical Lorenz curve for 20 classes

$$G(j,n) = 1 + \frac{2}{(1+j)^n - 1} - \frac{2+j}{jn} . \quad (2)$$

When $n=1$, it is easy to see that $G(j, 1)=0$. It is natural to restrict consideration to the cases where $j>0$ and $n \geq 1$.

Figure 1 shows the Lorenz curve for $n=20$, the number of classes and the rate of increase $j=0.05$, with the Gini coefficient $G(0.05, 20)=0.16$. If we halve the number of classes, we have $G(0.05, 10)=0.08$.

In this static class model the results are taken to be asymptotic – the distribution the bureaucracy aims to achieve.

2. Capitalism with Opportunity

As a sharp contrast to the bureaucracy with ordered career marches up the ladder of classes, I turn now to a model in which, ideally, individuals are given the chance to succeed or fail. More specifically, in each time period there is a probability δ_i of moving up one income class from class i – to afford easy comparison with the bureaucratic model the mean increase from class i up to the next can be taken to be $1+j$. Second, there is a probability ε_i of moving down one class – the opportunity to fail, and finally, of course, the probability $1 - \delta_i - \varepsilon_i$ of staying in the same class, with $\delta_i, \varepsilon_i > 0$. It is implicit in the model proposed that various policies available for adoption affect the values of the parameters ε_i and δ_i . The process is assumed to be a first-order Markov chain, so that in this idealization the history of how an individual got into a given income class will not affect his probability of moving up or down. Perhaps the strongest simplifying assumption is that in a given time period an individual cannot move up or down more than one income class, but all the same for appropriate time periods it will hold for most of a population. Few individuals make or lose a fortune overnight. Let

$$q_i = \frac{\delta_i}{\varepsilon_{i+1}}$$

and

s_i = asymptotic probability of being in state i .

Then it is easy to show that the asymptotic probabilities may be expressed recursively as:

$$s_{i+1} = q_i s_i , \quad i = 1, \dots, n-1 ,$$

and explicitly

$$s_{i+1} = \prod_{j=1}^i q_j s_1 .$$

Note that if for all i , $q_i=1$, the asymptotic distribution is uniform in probability – but, of course, not in income. For this case, the proportion of the population in any one class is $1/n$, and the Gini coefficient is just the same as in (2).

The equating of the Lorenz curves and Gini coefficients is deliberate, for it should be apparent that the Lorenz curves can be made essentially identical in the two very different cases of bureaucracy and market opportunity. In fact, in a limited way there is a strong tendency to equalize the two in a mixed economy, because of the flow of employees between the government and private sectors.

3. Some Philosophical Remarks on Equity

As long as there is an economic problem of allocating scarce resources, egalitarianism in the literal sense of equal income for all seems conceptually foolish as an ideal. We may often want to make changes in income distribution that go in that direction, but adoption of such changes does not mean there is a serious intent to achieve the goal of equality.

Equity arguments against equal income distribution for all seem naturally to fall into two classes, one class associated with ideas of freedom and the other with ideas of welfare or desert. Strong intuitions support each class of concepts. For this reason alone it seems doubtful that any really satisfactory necessary and sufficient conditions for equity can be given. Perhaps we can hope to develop an ever widening circle of necessary conditions, and deepen the normative arguments for those like the two examples considered earlier – seniority and freedom of opportunity –, that many of us think are deeply embedded in the human psyche.

The battle for some necessary conditions of equity seems nearly over – the most significant example being universalizability: whatever is fair or equitable for one person must also be so for similar persons in similar circumstances. But this is a weak necessary condition. It does not help much with the more difficult issues of equity we currently face. In my view, even the apparent conflict between efficiency and equity can be regarded as a second-order equity issue. Should efficiency be sacrificed to distribute more goods to the poorest segments of society? In fact, is this not a long-run decision to decrease distribution through market mechanisms? Thus it becomes a typical problem of equity – whose ox is gored and whose ox is fed.

I am certainly not going to propose any hard and fast methodology that is not only necessary but sufficient for solving problems of equity. I do want to move in a direction that has in general not been sufficiently explored. It is easy enough to make a utilitarian move and to say that by a stringent and careful application of utilitarian principles that require, if necessary, some theory of interpersonal comparison of utility, we can resolve the more difficult cases of equity. The most notable feature of such discussions, in my judgment, is their great abstractness and generality in comparison with significant particular issues of equity we continually face in all modern societies. As far as I can see, utilitarian principles as now developed offer little hope of providing specific answers to any of the really vexing questions. No doubt the utility functions of almost all bureaucrats favor some form of seniority-based increases in salary, and the hearts of almost all young entrepreneurs are full of the desirability of freedom of market opportunities, just to refer to the two examples discussed earlier. The list goes on endlessly. No doubt the worst off are in favor of attending first to the needs of the worst off, and their utility functions clearly reflect

this. In fact, simple principles of this kind are followed in detail by no society, and are extremely unlikely to be so followed in the future.

To put the question in the frame of reference with which I began, can we develop a richer set of principles than those offered by utilitarianism or other current moral theories to derive what we think are normatively reasonable income distributions? Of course, we would ordinarily think of income distributions, it seems to me, as being a final product of principles that go much more deeply into questions of individual psychology. If we applied such principles and came out with some ghastly unequal distribution, we might use that as a basis for correction and rethinking of the principles. My point is that it is unlikely that any deep-running principles of individual psychology will strike directly at the distribution of income, but rather such distribution will only be a rather indirect byproduct. I do take it, as the two examples considered earlier are meant to reflect, that in any complex society we will not have principles that end up with a restricted egalitarianism, that is, the same income distributed to all.

The sorts of principles I have in mind in order to set equity arguments in a richer context are the kinds of principles to be found in Aristotle's conception of human activity. The human faculties we have are there to be exercised in their fullest capacities, and a person who does not so utilize his capacities is not being realized fully as a human being. Aristotle's most extensive discussion of these matters is to be found in the *Nicomachean Ethics*. In Chaps. 6 and 7 of Book X he is especially concerned with the nature of happiness. It is, first of all, not a disposition but an activity. Second, it is an activity desired in itself, not for the sake of something else. As he puts it, "those activities are desirable in themselves from which nothing is sought beyond the activity" (1176b7). A good many utilitarians might have no trouble going this far with Aristotle, but most of them would part company with what follows.

A happy life must be virtuous, and a virtuous life requires exertion; it does not consist in amusement. The bodily pleasures are certainly of an inferior sort. Moreover, happiness is not an activity of short duration. It is only fully realized "in a complete life" (1098a17). There are many virtues, and activity in accordance with any of them can contribute to happiness. But for Aristotle the most perfect activity is contemplation, because the reasoning faculty represents the highest power in man.

With his usual clarity, Aquinas – on his way to concluding that ultimate happiness consists in contemplating God – summarizes the Aristotelian argument as follows (*Summa Contra Gentiles*, Bk. III, Chap. 37):

"Accordingly, if man's ultimate happiness does not consist in external things, which are called goods of fortune; nor in goods of the body; nor in goods of the soul, as regards the sensitive part; nor as regards the intellectual part, in terms of the life of moral virtue; nor in terms of the intellectual virtues which are concerned with action, namely, art and prudence: – it remains for us to conclude that man's ultimate happiness consists in the contemplation of truth."

Of course, most of us would not now accept Aquinas' conclusion, and we would even demur from Aristotle's giving the pride of place to contemplation. What I at least find appealing about both Aristotle and Aquinas' argument is: (i) the

recognition of a natural hierarchy of faculties – just as not all pleasures are equal, so all faculties are not, and (ii) the view that happiness must consist of activities that fully use our faculties.

It is not my point to try to analyze in any faithful and detailed way what Aristotle really thought about these matters. I certainly think that from the standpoint of the late twentieth century Aristotle's psychology is seriously defective in many ways, and I would not want to begin to shore it up, even though I think his central insight about human activity is of great importance.

I do not think that the desired principles can be stated in a way to sharply resolve some critical issues, but they can be helpful. Let us consider one familiar kind of example that is something of an intellectual and political issue in almost all countries. This is the extent to which public funds should be used to support "high" culture. By its definition, high culture is meant to reach only a very select part of the population, and, in fact, different parts of high culture reach different parts of the population, even though there is a sort of myth of a small relatively homogeneous elite for whom high culture is of great importance. One can see an Aristotelian argument for such support of high culture. The artistry exemplified in the best music, the best painting, or the best literature represents the fullest exercise of many human faculties. Even those who do not appreciate or like classical music can easily understand the incredible level of technique exhibited by modern performers, just as those who do not appreciate or like pocket billiards can easily understand the incredible technique of the world's best players. One response to this line of argument might be that exclusive focus on deep cultivation of skills only benefits the very ablest in the population. However, the broader Aristotelian argument as I conceive it would be in terms of maximizing the best skills each person has. It is important to recognize the critical role of training in the development of skills. It is by no means a simple matter of inherited abilities. In fact, the strong egalitarian thesis about skill should be that each individual possesses some skill that can be developed to the highest human potential. Without adopting such a strong thesis, we can still certainly believe in the ability of everyone to develop some skill to a very high level.

Now is not the occasion to pursue in detail the psychological basis for this Aristotelian thesis about human potential, but it is clear how it contrasts with straightforward utilitarian ideas. From a subjective standpoint, an individual might for a variety of reasons have no interest in developing any particular skill to any particular level of competence. He might in a more general way have no interest in an educational system that strongly encourages such ideas. A neo-Aristotelian, on the other hand, would take it not to be a subjective matter but an objective fact about human beings, well supported by a great variety of psychological evidence, and would argue that a principal problem of equity in a modern society is that of providing the opportunities for such development of human potential.

A concept of equity that emphasizes development of individual human potential as the primary equity consideration certainly runs up against ordinary concepts of distribution according to welfare or market mechanisms. It would be useful to try to attack both of these positions from the neo-Aristotelian viewpoint I am advocating. Concerning welfare, it can be said against Rawls and others that it is a clear mistake to have a simple difference principle that emphasizes improvement for those that are

worst off as the most important goal of change. The argument from the standpoint of individual human potential would be that it will be inevitable that some individuals for reasons that have played a role always and everywhere in human societies will not want to pursue a program that permits such unfettered development of their potential. Those individuals should be given some kind of support according to the view I am advocating, but they should not be the main focus of concern in considerations of equity. As to the dominance of market mechanisms, one straightforward reply is that in no current societies, nor in any foreseen in the near future, will the distribution of education be primarily a matter of the exercise of market mechanisms, but rather the public sector will continue to man the vast majority of educational institutions. To carry out the kind of program I am talking about, what is required is the infusion of capital and talent into educational institutions in order to make them much better than they now are.

Finally, in advocating such a neo-Aristotelian position I want to emphasize it is in no sense meant to be a program for the elite. The variety of skills that can be developed effectively and usefully in human beings is in no sense restricted to the ordinary narrow range of academic skills. It is, from a psychological standpoint, surely more satisfying to be a first-class mechanic than a mediocre bureaucrat. I come at the development of skills from the standpoint that is in general very sympathetic to lines of thought developed by that great democratic thinker John Dewey many years ago. Especially in *How We Think* [1], Dewey emphasizes that practical skills are as important as theoretical ones, the concrete as important as the abstract, – and education should be properly balanced in accordance with this recognition of the importance of the practical. Here and elsewhere he makes the important point that most members of a society – even those most responsible for its leadership –, are not going to be scientists or scholars. (This fact brings out the weakness of Aristotle's life of rational contemplation as a realistic goal, even for the elite of a modern society.) Dewey, like Aristotle, was not a utilitarian but held what he would have called a naturalistic view of ethical and moral phenomena.

Naturalism, Psychology, and Rights. Dewey's and Aristotle's naturalism raises a new range of questions for equity and social choice theory. Some of these questions have already been anticipated by Sen [4] in his concern to go beyond the behavioral data of revealed individual preferences to include the motivations of individuals in order to provide a richer theory of social choice and justice to solve the Sen paradox of the impossibility of a Paretian liberal [2, 3]. A thorough analysis is to be found in Wriglesworth [11].

From a naturalistic standpoint, such concerns about motivation provide the merest glimpse of the psychological iceberg. The theory of equity and social choice is remarkable for the thinness of its psychological assumptions about the drives, impulses, and capacities of human beings that play a major role in shaping their actual preferences. However, once we take into consideration such concepts, a conflict with the theory of rights is almost inevitable. The imposition of structural constraints generated by psychological theory or even by less well-defined experimental research can easily infringe on well-delineated areas of freedom defended by various theories of rights. And yet movement in the direction of introducing a

deeper psychological analysis of choice behavior seems inevitable, no matter how difficult. Aristotle and Dewey can provide important guidelines, but the really hard work of scientific development is still to be done.

Pluralism. There is one point on which I could easily be misunderstood. From what I have said above it may sound as if I am putting the entire emphasis of equity on the fullest development of individual powers and capabilities, but this I would think also mistaken. We need a more pluralistic approach. Even within education, the concern for various social values and graces – to use an old and genteel term for something that is important – must also be emphasized. Again, a philosophical and psychological analysis seems appropriate, and again the educational institutions, above all, of a society should have a clear view of goals in these matters, but of course they should not be left entirely to education and they also should be the responsibility, as they are in practice, of other institutions, ranging from the family outward to larger groups.

4. A Pluralism of Lorenz Curves

The philosophical remarks I have just been making lead to the central view I want to argue for in this paper. There is, I would claim, no single unifying idea of equity. It is a pluralistic concept to be realized in many different ways. There is no interesting and meaningful set of necessary and sufficient conditions. What we should have and what we should emphasize is something we have not yet seen much of: Lorenz curves for a variety of features of societies or social groups. The emphasis for too long has been on income distribution. Probably from a political standpoint if interpersonal comparison of utility is admitted, utilitarians should opt for Lorenz curves of utility distribution. I am certainly not against such a view but I would stress the desirability of many other kinds of curves. A surrogate for some of the things that I have been arguing for, but not a fully satisfactory one, would be a Lorenz curve for number of years of education. Providing Lorenz curves for some specific and deeper psychological properties such as the measure of individual development is a more complicated and difficult matter but not one that I think is out of reach of modern methodology. It is also important to have Lorenz curves for opportunities in societies. Educational and career opportunities are perhaps the ones most open to measurement in terms of data currently available. It would be especially interesting to examine opportunity curves for societies that have emphasized their openness. Roughly speaking, one would expect the various kinds of Lorenz curves I am describing to correlate well with the standard income distribution curves but not with anything like geometric duplication. The main point, however, of emphasizing the need for a pluralistic view of Lorenz curves for the distribution of different features of a society is to emphasize the necessary pluralism of a conceptually rich approach to equity.

As an example of such pluralism in equity analysis I show in Fig. 2 Lorenz curves for distribution of education in the United States in 1940 [9, Series H602-617] and in 1984 [10, Table 216, p 133]. To construct the curves I have treated years of education (0–16+) the same as income. The tables referred to use the same

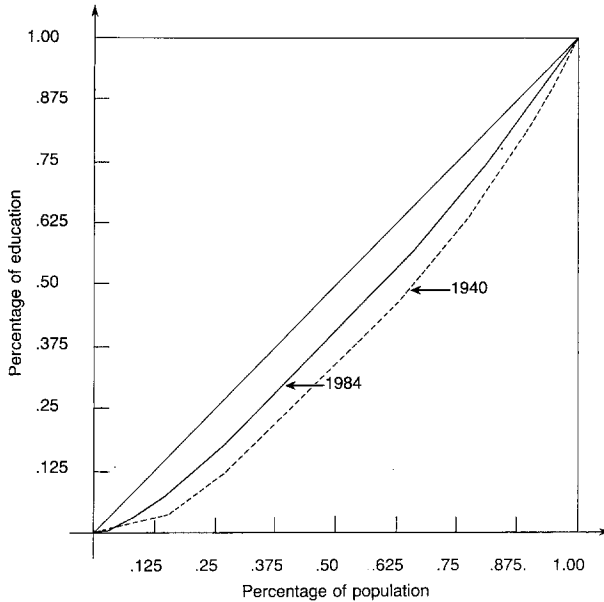


Fig. 2. Empirical Lorenz curves for distribution of education in the United States

grouping of years, with seven data points. There is one difference, however. The 1984 data are for the entire population, but the 1940 data are for the entire male population. Fortunately, the 1940 data for the female population are very similar. (The data points are for 2, 6, 8, 10, 12, 14, and 16 years of schooling, with 2 years the average for 0–4 years, etc. Undoubtedly this distribution assumption for the intervals 0–4, 5–7, 8, 9–11, 12, 13–15, and 16+ are not entirely accurate, but good enough to sustain the generalization asserted below. There are also some minor discrepancies in these intervals for the 1940 data but again not enough to be significant for the purposes at hand.)

Now it is a well-known and familiar fact that the number of years of education has certainly increased in the United States on the average between 1940 and 1984. For the populations indicated above, the median number of years of education in 1940 was 8.3, and in 1984 it was 12.6. (For females in 1940 it was 8.5.) But as in the case of income an increase in the median does not necessarily imply an increase in equity. On the other hand, it is clear from the two Lorenz curves of Fig. 2 that there has been a marked decrease in education inequality between 1940 and 1984, and consequently an increase in equity on this dimension. I take this result to be a surrogate measure for the improvement in equity of the quality of life in the United States from 1940 to 1984 – at least from an Aristotelian standpoint.

In so far as we can accept that more education on average prepares individuals to realize better their capacities and powers, we might even argue from the standpoint advocated in this article that Lorenz curves for the distribution of education provide more significant data about equity in a society than do those for income distribution. It is not my objective to push this point, but only, as I have already said, to stress the desirability of a pluralistic approach to equity. It would be desirable to construct – to the extent the data make it possible – Lorenz curves for health and housing, as well as education, for a variety of times and countries. Are

there countries, for example, in which the median level of education has increased significantly over the past forty years but equity in education has decreased? Is it possible to construct Lorenz curves for the actual distribution of human rights in various countries? The pluralism about equity I advocate calls for extended conceptual and empirical analyses of the distribution of many different features of modern societies.

5. Microanalysis of Equity

The changes in the distribution of education described in the previous section are very much of a global character. The pluralism of viewpoint advocated here occurs, however, even within education once a microanalysis of allocation is attempted.

To illustrate this general idea in a concrete way, I focus on a familiar conceptual problem in American elementary schools, i. e., schools educating children in the age range 5–12 years. The problem is the allocation of resources to students' instruction in the basic skills of reading and elementary mathematics. The traditional American view of public elementary education in this century has been that the allocation should be strictly egalitarian, even if this has often not been realized in practice. There have, however, been at least three kinds of students whose needs have seemed to demand breaking the egalitarian rule. The three groups of students most often discussed are the socially and economically disadvantaged, the handicapped, and, at the other end of the scale, the gifted.

Various arguments are advanced for providing additional or special instructional resources for each of these groups. It is not feasible to review the details of the many viewpoints and responses supporting them that have been put forth. The positive arguments have in a certain definite sense carried the day. For more than two decades there have been categorical federal funds distributed to school districts to provide additional instructional resources for the disadvantaged and handicapped. The amounts have been significant but not overwhelming – something on the order of three percent of the total local and state budgets for public education.

Now suppose we have instructional resources to allocate – they may be teacher time, instructional computer time, etc. The egalitarian solution is to allocate them equally to all students. Let us further suppose that we are primarily focused on achievement gain in reading and elementary mathematics. This is by no means the only goal of importance, but, for example, in allocating instructional computer time it could easily be the most important. But even with this restricted goal a lot of vagueness remains. In allocating resources what should we try to maximize if we think, as I do, that the strict egalitarian solution is too simple. I am not really trying to argue here that the egalitarian solution is not the right one, but, rather, am concentrating on the question of what are the other possibilities that there are reasons to advocate. Qualitatively described, here are some taken from [5] formulated for a given school or classroom within that school:

1. Maximize mean achievement level;
2. Minimize variance of achievement level;
3. Maximize mean achievement level subject to the constraint of not increasing the variance;
4. Maximize the number of students at a given achievement level.

Notice that (1) leads to a strategy of giving the most time to fast learners. Of course, contrary to much popular talk, the fast learners in a classroom or school do not constitute a fixed group of students, but, *ceteris paribus*, will depend on where each student is in his nonlinear learning curve. And so the group is properly dynamically defined. Goal (2), in contrast, leads to giving the most time to students who are farthest behind, and is therefore close to Rawls' difference principle.

To reduce these qualitative formulations to specific quantitative ones, a number of additional assumptions must be made. At least in the case of computer-assisted instruction, both theoretical and empirical arguments have been given for using a power function [7, 8]:

$$x_i = a_i t^{k_i} + b_i, \quad (3)$$

where x_i is achievement level after time t , and a_i , b_i , and k_i are parameters estimated for student i , i. e., individually for each student. Using (3) and individual estimates of parameters, Lorenz curves of achievement can be derived for various allocations as described above. A quantitative analysis short of deriving Lorenz curves is to be found in [5].

One point to emphasize is that the egalitarian solution of equal time for each student will in general be far from the allocation that produces the most egalitarian distribution of achievement. It is my view that in fact either of these goals is much too simple to be taken seriously as the only principle of allocation.

A familiar *general* slogan is this. Nondiscrimination requires that all educable children be taught in school to an adequate level of achievement, at least in the basic skills of elementary mathematics, reading and writing. This general goal is laudable and can be effectively used to prevent certain gross forms of discrimination, but it does not offer much help at the microlevel of allocation of instructional resources to individuals.

I see no reason to believe that there are any principles of equity sufficiently specific and noncontroversial to settle in a definite way the kind of problems of allocation just discussed. How will the allocations be made? As they have in the past, by "political" argument and compromise within some broad framework of general rather than specific principles. One specific solution is used in one place, and another in the next municipality, county or state. There is really no objective way to decide that one detailed solution is more equitable than another.

Finally, I emphasize that at the microlevel I have given a detailed analysis of one rather special case of educational resource allocation. A list of detailed allocation problems is easily drafted just by examining the current allocation of funds in developed countries by public or private (corporate) sources to: medical services, dental services, mental health services, control of alcoholism, control of illegal drug distribution, reduction of smoking, food programs for children, reduction of illiteracy, reduction of school dropouts, reduction of crime rate, reduction of suicides, improved correctional institutions, reduction of pollution, control of pesticides, prevention of flood and storm damage, provision of parks, support of public museums, public assistance to the poor, vocational rehabilitation programs, provision of special benefits for the handicapped, etc.

Who is foolish enough to believe that a small number of categorical principles of equity can be found to adjudicate between and within these competing claims?

Pluralism of equity principles is a necessity, for there are not now, and in all likelihood there never will be, convincing forcing arguments to reduce dramatically the number of viable principles.

References

1. Dewey J (1933) *How we think*, 2nd edn Heath, Boston
2. Sen A (1970a) *Collective choice and social welfare*. Holden-Day, San Francisco; Oliver and Boyd, Edinburgh. Distribution taken over by North-Holland
3. Sen A (1970b) The impossibility of a Paretian liberal. *J Polit Econ* 78:152–157
4. Sen A (1976) Liberty, unanimity and rights. *Economica* 43:217–245
5. Malone TW, Macken E, Suppes P (1979) Toward optimal allocation of instructional resources: Dividing computer-assisted instruction time among students. *Instruct Sci* 8:107–120
6. Suppes P (1977) The distributive justice of income inequality. *Erkenntnis* 11:233–250
7. Suppes P, Fletcher JD, Zanotti M (1976) Models of individual trajectories in computer-assisted instruction for deaf students. *J Educat Psychol* 68:117–127
8. Suppes P, Macken E, Zanotti M (1978) The role of global psychological models in instructional technology. In: Glaser R (ed) *Advances in instructional psychology*, vol 1. Erlbaum, Hillsdale, NJ, pp 229–259
9. U.S. Bureau of the Census: *Historical Statistics of the United States, Colonial Times to 1970, Bicentennial Edition, Part 2*, Washington, DC 1975
10. U.S. Bureau of the Census: *Statistical abstract of the United States: 1986* (106th edn), Washington, DC 1985
11. Wriglesworth JL (1985) *Libertarian conflicts in social choice*. Cambridge University Press, Cambridge