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Spontaneous Localizations of the Wave Function and Classical Behavior

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We investigate and develop further two models, the GRW model and the K model, in which the Schrödinger evolution of the wave function is spontaneously and repeatedly interrupted by random, approximate localizations, also called "self-reductions" below. In these models the center of mass of a macroscopic solid body is well localized even if one disregards the interactions with the environment. The motion of the body shows a small departure from the classical motion. We discuss the prospects and the difficulties of observing this anomaly. As far as the influence of the surroundings on submacroscopic objects (like dust particles) is concerned, we show that the estimates obtained recently in the theory of continuous measurements and in the K model are compatible. Also, we elaborate upon the relationship between the models. Firstly, borrowing a line of thought from the K model, we find the transition region between macroscopic and microscopic behaviors in the GRW model. Secondly, we refine the propagation rule of the wave function in the K model with the help of the time-evolution equation proposed in the GRW model.

1. INTRODUCTION

The Schrödinger equation leads to the unlimited coherent expansion of the center-of-mass wave function of any isolated system. However, the experimental evidence supports the existence of states with far-away coherent components only in the case of microobjects. This motivates the search for a general law of propagation of the wave function which would prevent the development of far-away superpositions in the case of macrosystems. Under such a law the freely propagating wave function should undergo self-reductions which destroy the excessive coherence created by

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the Schrödinger evolution. For more or less simple systems the interplay between the deterministic (Hamiltonian) Schrödinger evolution and the stochastic (non-Hamiltonian) self-reductions can be quantitatively studied. In particular, the behavior of the center-of-mass wave function of an isolated solid body can be described.

At this point it should be noted that according to a widespread view the investigation of the wave functions of isolated macroscopic bodies is futile, because macroobjects cannot be isolated from their surroundings. Of course, in classical physics the influence of the environment on macroobjects can often be neglected, but there are well-known reasons to believe that in quantum physics the situation is different. This view is strengthened by recent results in the theory of continuous measurements. Indeed, it has been shown there that the coherence of the density matrix of a "local system" substantially decreases due to the usual Hamiltonian interactions with the environment (air, sunlight, etc.). The loss of coherence is practically irreversible, because the correlations with the surroundings are spreading farther and farther away from the local system, to more and more microscopic degrees of freedom. On this basis it has been proposed[3,4] to consider the environment as the source of macroscopic behavior. This standpoint can certainly be upheld if one accepts that "classical properties are caused by the influence of a specific environment, and contrary to usual thought are not intrinsic to macroscopic bodies."[3]

The opposite conclusion is reached in the models under consideration: Macroscopic bodies would possess (very nearly) classical properties even if they were perfectly isolated. These properties are therefore intrinsic to the bodies. For example, in the case of a solid body of 1 g, frequent self-reductions prevent the center-of-mass wave function from expanding coherently over distances larger than 10^{-11} cm in the GRW model,[1] and 10^{-16} cm in the K model.[2] So, the center of mass is always well localized—a genuine classical property. The environment influences the behavior of the body, but it is not the principal source of this property.

As one goes toward smaller masses, the self-reductions become less frequent and the center-of-mass wave function can expand to larger domains. When the mass values of the microworld (not too large molecules, atoms, elementary particles) are reached, there are practically no more self-reductions. The center-of-mass wave functions of isolated microsystems can expand then over enormous domains where the Schrödinger equation and the superposition principle reign unrestricted. The predominantly classical behavior goes over into predominantly quantum behavior in the region of 10^{-12}–10^{-15} g.

It should be emphasized that in both models the general law of propagation formulated above is supposed to apply to measuring

apparatuses and observers, too. Therefore, not only the emergence of the classical behavior of macroscopic systems but also the measurement process should be described and interpreted on the basis of this general law. We shall indicate the salient points which show that this requirement is met, but no detailed discussion of this important issue will be given in this paper.

Although not the source of classical properties, the environment does play an important role in models with self-reductions, too. Its mode of action is even extended. Indeed, the environment may influence now the behavior of a local system not only through the usual Hamiltonian interactions, but also through its own ability to undergo self-reductions. As shown in Ref. 2, the latter effect leads to the "submacroscopic" decay of superpositions in the process of drop formation in a cloud chamber (see Section 7 below). The discussion of problems involving the environment will be limited to the K model, because to the author's knowledge the GRW model has not been applied yet to nonsolid systems.

The Hamiltonian influence of specific surroundings on dust particles has been estimated in Ref. 3. We show in Section 8 that these estimates are compatible with those obtained in the K model, and we comment on the relation between the theory of continuous measurements and models with self-reduction.

In Sections 2 and 3 the GRW model and the K model are discussed to the extent needed for the purposes of the present paper. In Section 4 the transition between macroscopic and microscopic behaviors is described in the framework of each model. In Section 5 it is shown that with the help of the time evolution equation proposed in the GRW model one can also describe the nonselective time evolution of the wave function in the K model, and one can even refine the propagation rule given in Ref. 2.

The repeated spontaneous localizations of the center of mass of a solid body lead to a small deviation in the propagation of the body from the purely Hamiltonian motion. The problems of the observation of the predicted anomaly are discussed in Section 6. The subject of Sections 7 and 8 has just been indicated. Section 9 contains some concluding remarks.

2. THE MODEL OF GHIRARDI, RIMINI, AND WEBER

The basic assumption underlying the GRW model is that the wave function of any isolated system spontaneously undergoes random, approximate localizations (self-reductions) which act against the expansion of the wave function due to the Schrödinger evolution. If after each self-reduction the actual result is selected, one of the possible histories of

the wave function obtains. If no selection is made, the state of the system will obviously be described by a mixture even if the initial state was a pure state. As shown in Ref. 1, the time evolution equation of the density operator $\hat{\rho}$, describing the nonselective evolution of the wave function of an isolated system, can be written in the form[2]

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}(t)] - \lambda(\hat{\rho}(t) - T[\hat{\rho}(t)]) \tag{1}$$

In the case of a system with one spatial degree of freedom the T operation

$$T[\hat{\rho}] = \left(\frac{\alpha}{\pi}\right)^{1/2} \int_{-\infty}^{+\infty} dx\, e^{-(\alpha/2)(\hat{q}-x)^2}\, \hat{\rho}\, e^{-(\alpha/2)(\hat{q}-x)^2} \tag{2}$$

in the non-Hamiltonian λ-term maps $\hat{\rho}$ into the new density operator $T[\hat{\rho}]$, resulting from approximate localizations carried out with suitable probability at any point in space. The average precision of each localization is $1/\sqrt{\alpha}$, while the average frequency of a localization is λ.

The self-reductions are viewed as a new basic element in the propagation law of isolated systems, the origin of which is not discussed in Ref. 1. They should account for the emergence of the classical behavior of macroscopic systems, without upsetting the validity of the superposition principle—more specifically, of the Schrödinger equation—for microscopic systems. Accordingly, models with self-reductions must allow the development of superpositions with far-away components in the case of microscopic systems, but not for macroscopic ones.

These desiderata are fulfilled in Ref. 1 for microsystems and for macroscopic solid bodies. First, it is noted that due to the λ-term Eq. (1) transforms pure states into mixtures, and that for a given system far-away components in a superposition lose their coherence faster than close ones. Next, it is proven that if Eq. (1) holds for individual microsystems with

$$\frac{1}{\sqrt{\alpha}} \approx 10^{-5}\ \text{cm}, \qquad \lambda = \lambda_{\text{micro}} \approx 10^{-16}/\text{sec} \tag{3}$$

then the density operator of the center of mass of a solid body also satisfies this equation with the same value of α and with[3]

$$\lambda = \mathcal{N}\lambda_{\text{micro}} \tag{4}$$

[2] If not stated otherwise, we follow the notations of Ref. 1 when discussing the GRW model. We denote the density operator by $\hat{\rho}$ in order to distinguish it from the density ρ of a body.

[3] In Ref. 1 the center-of-mass coordinate is denoted by Q and an index Q is attached to quantities referring to it. Also, λ is called λ_{macro} when $\mathcal{N} \approx 10^{23}$. We shall not introduce these distinctive notations. In our paper, as in Ref. 1, the model is restricted to one spatial dimension, but it is not difficult to verify that it works in three dimensions, too.

$\mathcal{N}$ being the number of the microscopic constituents (atoms or molecules) of the body. For a mass of 1 g at normal density $\rho \approx 1\ \mathrm{g/cm^3}$, that is, for $\mathcal{N} \approx 10^{23}$, one finds

$$\lambda \approx 10^{7}/\mathrm{sec} \tag{5}$$

So, the wave function of an isolated microscopic system would collapse only once in 10^8 years, whereas the center-of-mass wave function of an isolated macroscopic solid body would undergo typically 10^7 reductions per second. This is a considerable difference in the frequency of the self-reductions. On the other hand, since the value of α does not vary, one might think that both for microsystems and macrosystems the self-reductions occur when the components in a superposition are getting farther than 10^{-5} cm from each other. However, this is not so. As shown in Ref. 1, an initial Gaussian pure state is transformed by Eq. (1) into a mixture which at any given moment consists of Gaussian wave packets having different mean positions q and different mean momenta p, but, in good approximation, the same coherent position spread (in other words the same width) $\sqrt{Q}$, and the same coherent momentum spread $\sqrt{P}$. These spreads are calculable functions of time, and the time evolution of Q, P has a stable fixed point Q_0, P_0, where

$$Q_0 = \frac{1}{\sqrt{2}}\left(\frac{\hbar}{\alpha\lambda m}\right)^{1/2}\left[1 + \frac{\varepsilon}{\sqrt{2}} + O(\varepsilon^2)\right] \tag{6a}$$

$$P_0 = \frac{\hbar^2}{\sqrt{2}}\left(\frac{\alpha\lambda m}{\hbar}\right)^{1/2}[1 + O(\varepsilon^2)] \tag{6b}$$

The dimensionless parameter

$$\varepsilon = \left(\frac{\alpha\hbar}{\lambda m}\right)^{1/2} \tag{7}$$

appearing in (6) is very small for macroscopic bodies. For example, for $m \approx 1$ g, one finds $\varepsilon \approx 10^{-12}$.

Once the system gets to the fixed point—and Ghirardi, Rimini, and Weber claim that the domain of attraction is large—the time evolution consists in the repetition of reduction–expansion cycles. The cycles are first described in the approximation when the T operation acts at equally spaced moments with frequency λ. In this case, during each cycle, Q drops from a maximal value to a minimal one due to a self-reduction, and then goes back to the maximal value due to the Schrödinger evolution. The maximal value turns out to be Q_0, while in the case of macroscopic solid bodies the minimal value Q_{00} equals $Q_0(1 - \sqrt{2}\,\varepsilon)$. Because of the small-

ness of ε this "breathing" of the wave function is negligible. Then it has been shown that for $\varepsilon \ll 1$ the cyclic behavior is stable also if the self-reductions occur randomly with average frequency λ.

From (6) one finds for $m \approx 1$ g

$$\sqrt{Q_0} \approx 10^{-11} \text{ cm}, \qquad \sqrt{P_0} \approx 10^{-16} \text{ g cm/sec} \tag{8}$$

We see that the density operator of the center of mass of an isolated solid body of 1 g is a mixture composed of wave packets of 10^{-11} cm. In other words, the frequent self-reductions prevent the center-of-mass wave function from developing superpositions with components farther away than 10^{-11} cm, rather than 10^{-5} cm. On the other hand, since the atoms (or molecules) in a solid body are well localized with respect to the center of mass, there are practically no self-reductions related to the degrees of freedom describing the relative motion of the microscopic constituents of the body.

The nonselective evolution described above represents the multitude of the possible histories of the initial wave function. Of course, only one of these histories comes into being as the center-of-mass wave function accomplishes its reduction–expansion cycles. So, the center-of-mass wave function of an isolated macroscopic solid body has a small standard size $\sqrt{Q_0}$ (we neglect the breathing). It cannot expand coherently to larger size because of the self-reductions. On the other hand, due to these same self-reductions there will be a non-Hamiltonian contribution to the spread of the mean values q, p of the position and momentum of the center of mass. Summing up, the center-of-mass wave function is a tiny wave packet–soliton wavering around the Hamiltonian trajectory. In principle this "anomalous Brownian motion"[2] could lead to observable effects. However, with the chosen values of α and of λ_{micro} the non-Hamiltonian spreads are increasing too slowly. The position spread is only 10^{-8} cm after a year [see Eq. (53) below]. Therefore, there is no way to observe this prediction of the GRW model.

As shown in Ref. 1, the departure from the Hamiltonian behavior can be conveniently described in terms of the diffusion of q and p. Indeed, due to the smallness of Q_0 and P_0 on the macroscopic scale, one can introduce a phase-space density $\sigma(q, p, t)$ which satisfies a Fokker–Planck equation

$$\frac{\partial}{\partial t}\sigma = -\frac{p}{m}\frac{\partial}{\partial q}\sigma + \frac{\lambda}{4}\left[a^2\frac{\partial^2}{\partial q^2} + 2ab\frac{\partial^2}{\partial q\,\partial p} + b^2\frac{\partial^2}{\partial p^2}\right]\sigma \tag{9}$$

with

$$a = \left(\frac{2\hbar}{\lambda m}\right)^{1/2}[1 + O(\varepsilon)], \qquad b = \sqrt{\alpha}\,\hbar[1 + O(\varepsilon)] \tag{10}$$

The diffusion coefficients for q and p read

$$d_q = \frac{\lambda a^2}{2} = \frac{\hbar}{m} \approx 10^{-27}\ \mathrm{cm}^2/\mathrm{sec} \tag{11a}$$

$$d_p = \frac{\lambda b^2}{2} = \frac{\hbar^2}{2}\alpha\lambda \approx 10^{-37}\ \mathrm{g}^2\ \mathrm{cm}^2/\mathrm{sec}^3 \tag{11b}$$

The numerical values in (11) refer to a body of 1 g. The extreme smallness of d_p accounts for the very slow increase of the non-Hamiltonian spreads.

We see that the self-reductions described by the non-Hamiltonian term in the time evolution equation leads to the desired breakdown of the superposition principle in the case of macroscopic solid bodies and to the emergence of a trajectory practically indistinguishable from the classical one. It should be emphasized that the model gives a quantitative answer to the question what "far-away" (or "macroscopically distinct") means: Positions of a solid body of 1 g with its center of mass separated by a distance larger than 10^{-11} cm are macroscopically distinct.

Let us point out that not only the position but also the velocity of the center of mass is sufficiently well determined, so that both of them behave very nearly classically. Indeed, the coherent velocity spread $\Delta v \approx \hbar/m\sqrt{Q_0} \approx 10^{-16}$ cm/sec. So, neither the position nor the velocity have perfectly sharp values; on the other hand, neither of them can acquire a large coherent spread. These features are due to the judicious choice of the two new constants of nature of the model, α and λ_{micro}.

Finally, let us remark that the existence of the stable, nonzero width $\sqrt{Q_0}$ of the wave function in the selective evolution is not in contradiction with the well-known fact[3] that the density matrix $\rho(x, x', t)$ becomes diagonal for $t \to \infty$. It can be shown that this property of $\hat{\rho}$ is due to the fact that the incoherent momentum spread tends to infinity for $t \to \infty$.

3. THE MODEL OF KÁROLYHÁZY

In the K model a definite hypothesis has been made concerning the physical origin of the breakdown of the superposition principle. Due to this hypothesis, recalled below, the linear size a_c of the spatial domain—of the "coherence cell"—inside which the superposition principle still holds for the center-of-mass wave function of an isolated solid body can be evaluated without the introduction of free parameters. Namely, the "coherence length" a_c turns out to be the following function of the mass (more

conveniently, of the Compton wavelength $L = \hbar/mc$) and of the linear size[4] R of the body[2,5]:

$$a_c \approx \left(\frac{R}{\Lambda_p}\right)^{2/3} L \qquad \text{if} \quad R > a_c \tag{12}$$

$$a_c \approx \left(\frac{L}{\Lambda_p}\right)^{2} L \qquad \text{if} \quad R < a_c \tag{13}$$

In the expressions

$$\Lambda_p = \left(\frac{G\hbar}{c^3}\right)^{1/2} \approx 10^{-33}\,\text{cm} \tag{14}$$

is the Planck length. The appearance of the gravitational constant G is a reminder of the key role played by the theory of general relativity in the model. The line of thought starts with the remark that the sharply determined structure of space-time is incompatible with two generally accepted assertions, one formulated in quantum mechanics, the other in general relativity. The first states that the position and the velocity of an object cannot have sharp values simultaneously, and the second stipulates that the structure of space-time is determined by the positions and the velocities of the masses. It follows that quantum mechanics and general relativity, taken together, limit the possibility of implementing the Minkowskian structure of empty space-time. In particular, if one tries to delineate a time interval with the help of physical objects, there will be a quantum mechanical uncertainty $(\Delta T)_Q$ in the length of the interval due to the Heisenberg uncertainty relation, and there will also be a gravitational deviation $(\Delta T)_G$ from the value in flat space. The quantity $(\Delta T)_Q$ is decreasing with the mass, while $(\Delta T)_G$ is increasing with it. There exists therefore a minimal, tiny but unavoidable departure ΔT in the delineated value of any[5] time interval from the value T in flat space, determined by the condition $\Delta T = (\Delta T)_Q = (\Delta T)_G$. The quantity ΔT turns out to be independent of the mass of the test bodies, and it is related to T only through constants of nature:

$$(\Delta T)^3 \approx \left(\frac{\Lambda_p}{c}\right)^2 T \tag{15}$$

[4] Since we are dealing here with order-of-magnitude estimates, it is immaterial whether R stands, for example, for the height h or for the radius r of a cylinder, as long as h and r are of the same order of magnitude. For simplicity, one might think of a spherical body with radius R.

[5] Between reasonable limits; also, the motion of macroscopic bodies is restricted to non-relativistic velocities.[2,5]

This relation indicates the extent one should loosen the Minkowskian structure of the empty space-time in order to remove the incompatibility mentioned above. At the same time, an opportunity arises here of introducing into the structure of empty space-time a stochastic element which may lead to the reduction of the wave function. In Ref. 2 (and also Ref. 5) this is achieved by replacing the Minkowskian metric tensor of empty space-time by a random set of metric tensors such that Eq. (15) holds in the sense of averages. Each member of the set is a gravitational plane wave with suitably chosen amplitude. The spread of the metric tensors around the Minkowskian value $g_{00} = -g_{ii} = 1$ is of the order of 10^{-13} This tiny randomness has to be a "genuine" one, i.e., it should not be the manifestation of any causal process acting behind the scene. Indeed, only a genuine randomness can induce genuinely stochastic reductions leading to a limitation of the validity of the superposition principle. Just as in Eq. (1) the introduction of the non-Hamiltonian term amounts to a basic modification of the dynamical law, the randomness associated with the metric in Ref. 2 amqounts to a small but basic modification of the structure of empty space-time.

The next step in the model building is the evaluation of the influence of the uncertain structure of the geometry on the propagation law of various systems. In order to carry out such an evaluation, the state of any isolated system propagating on the new, empty but hazy space-time is described now by a randomized wave function in configuration space. Namely, to each member of the random set of nearly Minkowskian metric tensors there corresponds a Schrödinger wave function propagating on that metric, and the physical state of the system is represented by the whole set of these Schrödinger wave functions. In particular, as we shall see presently, the degree of coherence of the state is coded in the randomized wave function. A single member of the set does not contain this information and cannot be interpreted individually. Also, no particular physical meaning should be forced upon the individual members of the classical (i.e., nonquantized) random set of metric tensors. Their role is to provide us with a model of a single but hazy empty space-time where the limitation (15) holds. In a future, deeper theory of quantum mechanics and general relativity the idea concerning the genuinely uncertain structure of space-time may well receive a different form.

The introduction of the classical random set of $g_{\mu\nu}$'s describing the macroscopic empty space-time by no means contradicts the possible existence of quantum gravity. The quantized $\hat{g}_{\mu\nu}$'s are of course subject to the superposition principle and should not be confused with those proposed by Károlyházy.

As shown in Ref. 2, the uncertain structure of the geometry affects

primarily the phases in the randomized wave function. Its impact on the absolute values is negligible.[6] Since the metric couples to the mass, the more massive a system is, the stronger the phases are influenced. In the case of an isolated solid body the randomized wave function of the center of mass develops, over the members of the random set, a spread in the relative phase between any two points in space. For two points at a distance a from each other, this spread reaches a stationary value $\Delta(a)$ in a time needed for the metrical disturbance representing the uncertain structure of the geometry to traverse the body in question. For a body of size $R \approx 1$ cm this time is therefore $R/c \approx 10^{-11}$ sec.

The quantity Δ increases with the separation a and reaches the value π when $a = a_c$. As a matter of fact, Eqs. (12) and (13) were derived from the condition $\Delta(a_c) \approx \pi$. For pairs of points belonging to the same coherence cell, $\Delta \ll \pi$, except when the separation is close to a_c.

Consider now the randomized wave function $\{\psi_\beta\}$, with $\beta = 0, 1, \ldots$ labelling the individual wave functions propagating on the individual members $(g_{\mu\nu})_\beta$ of the random metric. Take all the ψ_β's equal to each other at some "initial" time. As indicated by the behavior of $\Delta(a)$ described above, the ψ_β's correspond to a single ray in Hilbert space ($\Delta \ll \pi$) as long as they are well inside a coherence cell. On the other hand, if the ψ_β's occupy several coherence cells (remember that their absolute values are the same), then very quickly—in 10^{-11} sec in our example—the spread Δ in the relative phases becomes of the order of π between points belonging to different coherence cells. Then the randomized wave function cannot be associated any more with a single ray. In this sense its coherence is lost.

Károlyházy has assumed that when such a situation develops, the coherence is re-established by a spontaneous, instantaneous, random localization of the randomized wave function to a single cell. More precisely, Károlyházy has proposed the following, somewhat bumpy but simple cyclic rule of propagation for the randomized center-of-mass wave function of an isolated solid body, in case the wave function tends to outgrow its coherence cell. The quantity $\{\psi_\beta\}$ obeys the Schrödinger equation while it expands from a single cell to a volume of linear size $2a_c$. The time needed for this expansion is

$$\tau_c \approx \frac{m a_c^2}{\hbar} \tag{16}$$

Then a spontaneous, stochastic reduction to a single cell takes place with probability proportional to the weight of the wave function in that cell just before the reduction. (The sharp edge of the wave function made by the

[6] As required,[6] the weak equivalence principle holds in the quasiclassical limit.

projection is smoothened by a Gaussian of width a_c. This procedure can be viewed as a prediscovered approximation to the selective T-operation of the GRW model. It is described in detail and its mathematical consistency is shown in Ref. 2.)

Since immediately after the reduction the wave function is again occupying a single cell, the stage is set for the next expansion–reduction cycle. The reductions are instanteneous, stochastic jumps of the wave function, and the mathematical description of the process of the reduction itself is lacking, just as in the orthodox measurement theory and in the GRW model. However, as explained above, these stochastic reductions are triggered and conditioned now by the uncertainty of the structure of the empty space-time. They occur at predictable time intervals τ_c onto domains of calculable size a_c. They are perceived as part of the general law of propagation of the wave functions of isolated systems. Since no external agent (except for the uncertainty of the structure of empty space-time) is required to trigger them, we shall call them self-reductions, similarly to the spontaneous localizations in the GRW model.

As stressed in the conclusion, there are reasons to believe that the stochastic aspect of the propagation of the wave function will remain with us. Accordingly, we think that even if a mathematical description of the reduction process is worked out in a future theory, it will not replace the reduction by a purely causal process.

Let us now look at numerical values of a_c and τ_c. For a solid body of linear size $R \approx 1$ cm at normal density, from (12) and (16) one finds

$$a_c \approx 10^{-16} \text{ cm}, \qquad \tau_c \approx 10^{-4} \text{ sec} \tag{17}$$

So, in the K model the localization of the center of mass is much tighter than in the GRW model. Since the center-of-mass wave functions undergoing the cyclic behavior are nearly minimal wave packets in both models, this means also that the coherent momentum spread is much larger in the K model than in the GRW model [by a factor 10^5 for a body of 1 g, as can be seen from the comparison of the value of a_c—pedantically of $2a_c$—in (17) with the value of $\sqrt{Q_0}$ in (8)]. If follows that the departure in the motion of the center of mass from the classical trajectory is also appreciably larger in the K model. In Refs. 2 and 5 this departure has been expressed in terms of an "anomalous Brownian motion" of the center of mass. Like in the GRW model, this motion is due to a small, stochastic velocity jump of the mean velocity of the center-of-mass wave function at each self-reduction. For a body of 1 g, the value of a jump is of the order of $a_c/\tau_c \approx 10^{-12}$ cm/sec [see Eq. (22) below]. The prospects of the verification of the existence of the anomaly will be discussed in Section 6, where an estimate of the energy produced by the self-reductions is also given.

For our mass of 1 g, $R \gg a_c$. Let us take now a solid grain with $R \approx 10^{-6}$ cm, $\rho \approx 1$ g/cm^3. Then $m \approx 10^{-18}$ g, and Eqs. (13), (16) give $a_c \approx 1$ km, $\tau_c \approx 10^{21}$ sec. We see that the grain belongs to the domain $R \ll a_c$. If such a grain could be isolated, its center-of-mass wave function would reach coherently the size of 1 km and would undergo one self-reduction per 10^{13} years.

For an elementary particle, Károlyházy finds

$$a_c \approx \left(\frac{L}{\Lambda_p}\right)^2 L \tag{18}$$

This is the same formula as that in (13). The condition $R \ll a_c$ is now trivially satisfied, since elementary particles are pointlike in quantum mechanics. Instead o the center of mass coordinate we have now simply the position coordinate of the particle, and a_c is the coherence length of the wave function of the particle.

From (18) and (16) one finds for an electron

$$a_c \approx 10^{35} \text{ cm}, \qquad \tau_c \approx 10^{70} \text{ sec} \tag{19}$$

These values exceed astronomical scales. Huge values are obtained also for other elementary particles, as well as for the wave functions of composite microsystems like atoms and not too large molecules. So, these wave functions in reality occupy only a tiny part of their coherence cells. Consequently, the superposition principle holds for them practically unrestricted. The reason is that the mass of a microobject is so small that its wave function hardly feels the tiny uncertainty of the space-time structure. However, the interaction of a microparticle with a macroscopic system often results in a superposition in which wave functions of the particle are entangled with those of the macrosystem. The relative phase of the randomized wave function of the composite system may then developed spreads of the order of π leading to self-reductions of the total wave function. Thereby the wave function of the microparticle is also reduced, independently of the presence or the absence of an observer.[2,5]

4. THE TRANSITION BETWEEN MACROBEHAVIOR AND MICROBEHAVIOR

4.1. The Transition Region in the K Model

We have been in the preceding section that in the region where $R \gg a_c$ the center of mass of an isolated solid body is well localized and its

propagation differs from the classical one only by a small anomaly. On the other hand, when $R \ll a_c$ the center-of-mass wave function, obeying the Schrödinger equation, can expand over large spatial domains, and its expansion is only very rarely interrupted by self-reductions. Consequently, the region $R \gg a_c$ ($R \ll a_c$) has been identified in Ref. 2 with the region where classical (quantum mechanical) behavior dominates. In the gradual transition between the two behaviors the mass region where $R \approx a_c$ plays a prominent role. Indeed, from Eqs. (12), (13), and (16) one sees that this is the region where, at constant density, the dependence of a_c (and of τ_c) on the mass changes from a slow increase to a fast one with decreasing mass. This is exhibited in Figs. 1 and 2 and in Table I. We shall call the values of the various parameters in the region $R \approx a_c$ "transition values." For $\rho \approx 1$ g/cm^3 one finds

$$R^{\mathrm{tr}} \approx a_c^{\mathrm{tr}} \approx 10^{-5}\,\mathrm{cm}, \qquad m^{\mathrm{tr}} \approx 10^{-14}\,\mathrm{g} \tag{20}$$

$$\tau_c^{\mathrm{tr}} \approx 10^{4}\,\mathrm{sec} \tag{21}$$

So, for $m > m^{\mathrm{tr}} \approx 10^{-14}$ g the center of mass is localized within 10^{-5} cm, whereas already for $m < 10^{-16}$ g the center-of-mass wave funtion can expand coherently over obviously macroscopic distances $a_c > 10$ cm.

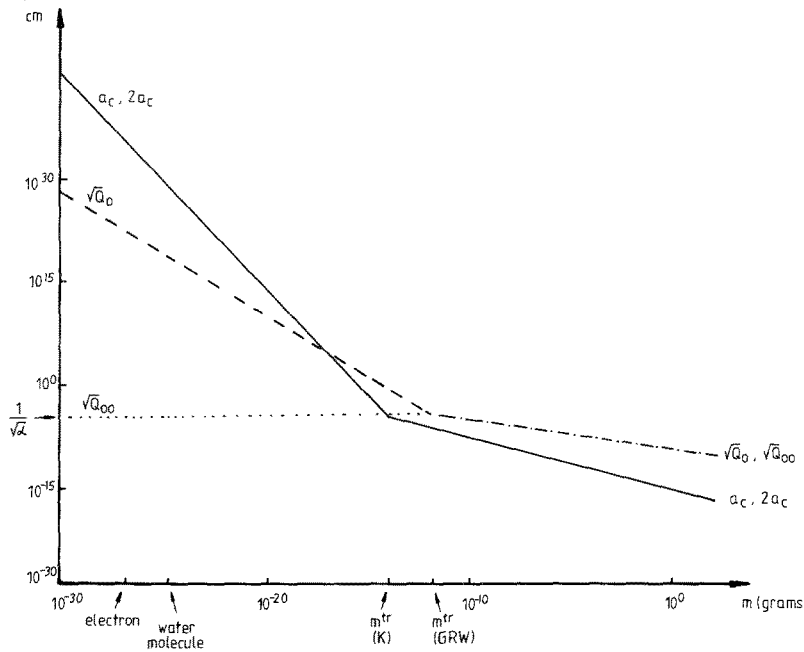

Fig. 1. The mass dependence of $\sqrt{Q_{00}}$, $\sqrt{Q_0}$, and a_c.

Colloidal grains and dust particles fall into the transition region. Therefore we shall call them submacroscopic, rather than macroscopic. Notice that while the Planck length enters the equations determining the transition mass, this mass is much smaller than the Planck mass $m_P = \hbar/cA_P \approx 10^{-5}$ g.

The transition region can be characterized also by the mass dependence of the uncertainty in the velocity of the center of mass between two successive self-reductions. This uncertainty obviously is $\Delta v \approx a_c/\tau_c$, and from Eqs. (12), (13), and (16) it is easy to see that as a function of the mass Δv has its maximum (10^{-9} cm/sec) at the transition mass. As stated already, a_c/τ_c gives at the same time the order of magnitude of the stochastic velocity jumps $\Delta v'$ at each reduction. Indeed, for minimal wave packets

$$\Delta v' \approx \frac{\hbar}{2a_c m} - \frac{\hbar}{4a_c m} = \frac{\hbar}{4a_c m} \approx \frac{1}{4}\frac{a_c}{\tau_c} \tag{22}$$

4.2. The Transition Region in the GRW Model

In order to find the transition region in the GRW model, we first express the position spreads $\sqrt{Q_0}$, $\sqrt{Q_{00}}$ through α and ε. For macro-objects ($\varepsilon \ll 1$) we find

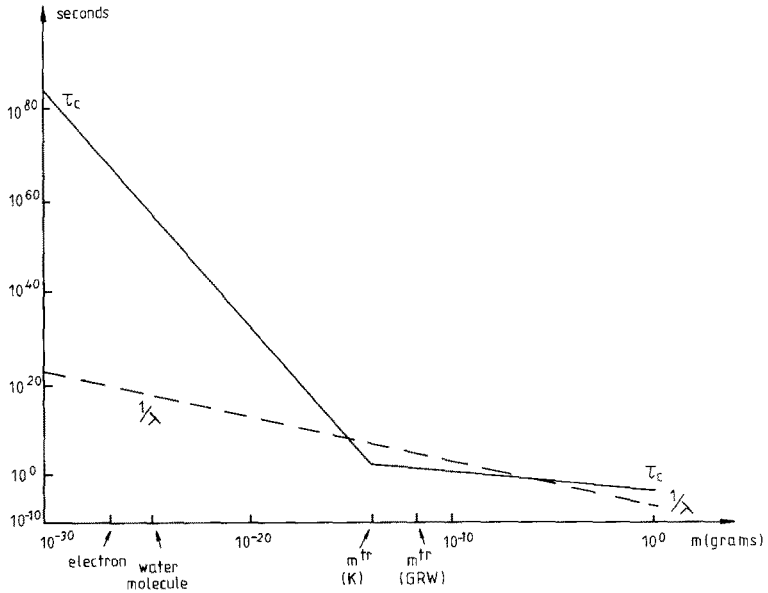

Fig. 2. The mass dependence of $1/\lambda$ and of τ_c.

$$\sqrt{Q_0} \approx \left(\frac{\varepsilon}{\alpha}\right)^{1/2} \approx 10^{-11}\ \text{cm} \tag{23}$$

$$\sqrt{Q_{00}} \approx \sqrt{Q_0}\left(1 - \frac{\varepsilon}{\sqrt{2}}\right) \tag{24}$$

while for microobjects ($\varepsilon \gg 1$)

$$\sqrt{Q_0} \approx \frac{1}{\sqrt{\alpha}}\varepsilon^2 \approx 10^{19}\ \text{cm} \tag{25}$$

$$\sqrt{Q_0} \approx \frac{1}{\sqrt{\alpha}} \tag{26}$$

The numerical values in (23) and (25) refer to a solid of 1 g and to an atom (or molecule) with $m \approx 10^{-23}$ g respectively.

Table I. The Mass Dependence of the Cycle Parameters in CGS Units

GHIRARDI, RIMINI, WEBER		$m < 10^{-12}$	$m > 10^{-12}$
	$\sqrt{Q_{00}}$	10^{-5}	$\frac{10^{-11}}{\sqrt{m}}$
	$\sqrt{Q_0}$	$\frac{10^{-29}}{m^2}$	$\frac{10^{-11}}{\sqrt{m}}$
	$1/\lambda$	$\frac{10^{-7}}{m}$	$\frac{10^{-7}}{m}$
KÁROLYHÁZY		$m < 10^{-14}$	$m > 10^{-14}$
	a_C	$\frac{10^{-47}}{m^3}$	$\frac{10^{-16}}{m^{7/9}}$
	τ_C	$\frac{10^{-66}}{m^5}$	$\frac{10^{-4}}{m^{5/9}}$

To arrive at (23) and (25) it is convenient to solve Eq. (8.17) in Ref. 1 for ε^2:

$$\varepsilon^2 = 2x_0^2(1+x_0)^{-1}(1+2x_0)^{-1/2} \tag{27}$$

Here x_0 is the fixed-point value of the first of the two dimensionless coordinates x, y introduced in (8.13) of Ref. 1:

$$x = \alpha Q, \qquad y = \frac{\alpha}{\lambda^2 m^2} P \tag{28}$$

(Let us remark that in (8.17) of Ref. 1, x stands in fact for x_0, because (8.17) is valid only if $x' = x = x_0$ and $y' = y = y_0$.) Taking into account that ε and x_0 are positive by definition, (27) gives

$$\varepsilon = \sqrt{2}\, x_0, \qquad \text{if} \quad \varepsilon \ll 1 \tag{29}$$

$$\varepsilon = (2x_0)^{1/4}, \qquad \text{if} \quad \varepsilon \gg 1 \tag{30}$$

With $x_0 = \alpha Q_0$ one arrives at (23) and (25) after the omission of unimportant numerical factors of the order of 1.

Equations (24) and (26) are easily obtained with the help of (8.9a) of Ref. 1:

$$Q_{00} = Q_0(1+2\alpha Q_0)^{-1} \tag{31}$$

Here we substituted Q_0 for Q_i and Q_{00} for Q_f as required "under the regime condition," that is, at the fixed point.

In Ref. 1 the cycles were investigated only for $\varepsilon \ll 1$. The general case has been considered in Ref. 7. In particular, the stability of the cycles under small perturbations has been proven there[7] for any positive value of ε.

We shall need presently the mass dependence of λ and of ε. With $\mathcal{M} = m/m_{\text{micro}}$, Eqs. (3), (4), and (7) give

$$\lambda = \frac{\lambda_{\text{micro}}}{m_{\text{micro}}} m \approx 10^7\, m \tag{32}$$

$$\varepsilon \approx \frac{10^{-12}}{m} \tag{33}$$

where $1/\lambda$ should be expressed in seconds and m in grams. For the sake of definiteness the numerical factors in (32) and (33) are given for ice ($m_{\text{micro}} \approx 10^{-23}$ g, the mass of a water molecule), but (32) and (33) are valid also for other solid bodies of normal density within a factor of 10.

[7] I regret that I became acquainted with this important work only after the distribution of the preprint of a previous version[15] of the present paper.

From (33) one sees that obviously macroscopic (microscopic) objects indeed fall into the region $\varepsilon \ll 1$ ($\varepsilon \gg 1$).

Equations (23), (25), and (33) show that the mass dependence of the largest allowed expansion $\sqrt{Q_0}$ of the center-of-mass wave function has a similar trend as that of a_c in the K model. This can also be seen in Figs. 1 and 2 and in Table I, where the mass dependence of the cycle parameters of the models are presented. The same reasoning as in the K model shows that the transition region in the GRW model is determined by the condition $\varepsilon \approx 1$. So, from (27) and (28) one gets

$$\sqrt{Q_0^{\mathrm{tr}}} \approx \frac{1}{\sqrt{\alpha}} \approx 10^{-5}\ \mathrm{cm} \tag{34}$$

and from (33) and (32) one finds (for $\rho \approx 1\ \mathrm{g/cm^3}$)

$$m^{\mathrm{tr}} \approx 10^{-12}\ \mathrm{g}, \qquad R^{\mathrm{tr}} \approx 10^{-4}\ \mathrm{cm} \tag{35}$$

$$\lambda^{\mathrm{tr}} \approx 10^{-5}/\mathrm{sec} \tag{36}$$

Comparison of Eqs. (34) and (35) with (20) and of Eq. (36) with (21) shows that the transition regions of the models lie close to each other. Also, it is easy to see that the uncertainty in the velocity of the center of mass during a cycle

$$(\Delta v)_{\mathrm{GRW}} = \sqrt{Q_0}\,\lambda \tag{37}$$

is again maximal (10^{-10} cm/sec) in the transition region. Notice also that in the GRW model the relative breathing

$$B = \frac{\sqrt{Q_0} - \sqrt{Q_{00}}}{\sqrt{Q_{00}}} \tag{38}$$

is much larger (smaller) than 1 in the microscopic (macroscopic) region and is of the order of 1 in the transition region, whereas in the K model the relative breathing is always equal to 1.

5. THE ADAPTATION OF EQ. (1) TO THE K MODEL

In the K model the effect of the self-reductions on the propagation of the wave function has been evaluated in terms of the elongation of the anomalous Brownian motion [see Eq. (50) below], but no time evolution equation for $\hat{\rho}$ has been given. In this section we show how Eq. (1) can be

used to describe the nonselective evolution of the wave function in the K model. The mass dependence of α, λ, and of some other parameters will now be dictated by the K model. This will be indicated by a tilde over those quantities.

As mentioned previously, in Ref. 1 the cycles were constructed in the approximation in which the successive self-reductions are equally spaced in time, just as in the K model. Calculating therefore the value of a_c for a given solid body of mass m from Eqs. (12) or (13) and taking $\tilde{Q}_{00} = a_c^2$, $\tilde{Q}_0 = 4a_c^2$, we easily obtain the values of $\tilde{\alpha}$ and of $\tilde{\lambda}$.

However, one can achieve more than that. With the help of the formalism used in Ref. 1 one can also refine the somewhat bumpy character of the propagation of the wave function in the K model. Such a refinement is desirable because, as noticed in Section 3, the randomized wave function may develop incoherent parts in 10^{-11} sec, that is, long before it reaches the linear size $2a_c$. So, self-reductions could occur at time intervals substantially shorter than the cycle period τ_c given in (16). Then the self-reduction should go to one of the largely overlapping cells in the domain occupied by the wave function, this domain being only a little larger than a single cell. A method of construction of the density operator arising after reduction to a non-orthogonal overcomplete set of states has been given in the measurement theory based on "effects" and "operations."[8] A typical example of an operation is the T operation used in Eq. (1).

In order to express the parameters $\tilde{\alpha}$ and $\tilde{\lambda}$ belonging to the original and to the refined K cycles through a_c and τ_c, one should obviously require

$$\tilde{Q}_{00} = a_c^2, \qquad \tilde{Q}_0 = (\kappa a_c)^2 \tag{39}$$

where the refinement parameter κ satisfies

$$1 \leqslant \kappa \leqslant 2 \tag{40}$$

Equations (31) and (39) immediately give

$$\frac{1}{\sqrt{\tilde{\alpha}}} = \frac{\sqrt{2}\,\kappa}{\sqrt{\kappa^2 - 1}}\, a_c \tag{41}$$

while for $\tilde{\lambda}$ one obtains from (7)

$$\frac{1}{\tilde{\lambda}} = \frac{\tilde{\varepsilon}^2 m}{\tilde{\alpha}\hbar} \tag{42}$$

Inserting (27) into (42) with $\tilde{x}_0 = \tilde{\alpha}\tilde{Q}_0 = \tilde{\alpha}(\kappa a_c)^2$, and excluding $\tilde{\alpha}$ and a_c with the help of (41) and (16), one arrives at

$$\frac{1}{\tilde{\lambda}} = 2\kappa \frac{\kappa^2 - 1}{\kappa^2 + 1} \tau_c \tag{43}$$

The original K cycles are generated if $\kappa = 2$. Then

$$\frac{1}{\sqrt{\tilde{\alpha}}} = \left(\frac{8}{3}\right)^{1/2} a_c \approx a_c \tag{44}$$

$$\frac{1}{\tilde{\lambda}} = \frac{12}{5} \tau_c \approx \tau_c \tag{45}$$

The refined cycles correspond to values of $\kappa < 2$. Let us show that the refinement does not alter sensibly the diffusion of the mean values q, p of the position and of the momentum of the center of mass. Since the diffusion of q and the elongation of the anomalous Brownian motion are closely related, the refinement leaves practically unchanged the elongation of the anomalous Brownian motion, too.

From (11) we see that the diffusion coefficient d_p is proportional to $\tilde{\alpha}\tilde{\lambda}$, while d_q is independent of $\tilde{\alpha}$ and of $\tilde{\lambda}$. The third, q–p mixing coefficient $d_m = \tilde{\lambda}\tilde{a}\tilde{b}$ is proportional to $\sqrt{\tilde{\alpha}\tilde{\lambda}}$. Now, from (41) and (43) one obtains

$$\tilde{\alpha}\tilde{\lambda} = \frac{\kappa^2 + 1}{4\kappa^3} \frac{1}{a_c^2 \tau_c} = \begin{cases} \dfrac{5}{32} \dfrac{1}{a_c^2 \tau_c} & \text{if} \quad \kappa = 2 \\ \dfrac{1}{2} \dfrac{1}{a_c^2 \tau_c} & \text{if} \quad \kappa = 1 \end{cases} \tag{46}$$

While the relative breathing of the wave function decreases from unity ($\kappa = 2$) to zero ($\kappa = 1$), d_q remains unchanged and d_m (d_p) increases only by a factor of about $\sqrt{3}$ (3).

So, Eq. (1) describes, with α, λ replaced by $\tilde{\alpha}$, $\tilde{\lambda}$, the original ($\kappa = 2$) and the refined ($\kappa < 2$) nonselective evolution of the wave function of the K model. As far as the selective time evolution is concerned, any of the histories of the center-of-mass wave function can be obtained following the procedure outlined in Section III of Ref. 7. The connection between the notation in Ref. 7 and in the present paper is: $\sigma = 1/\tilde{\alpha}$ $\tau = 1/\tilde{\lambda}$, $\Delta = 2a_c^2$, $C = \kappa^2$.

The no-breathing limit ($\kappa = 1$) is of special interest. In this limit $\tilde{\alpha}$ tends to zero and $\tilde{\lambda}$ tends to infinity, while from (46)

$$\lim_{\kappa \to 1} \tilde{\alpha}\tilde{\lambda} = \frac{1}{2} \frac{1}{a_c^2 \tau_c} \equiv \tilde{\gamma} \tag{47}$$

Also, from (7) one obtains

$$\lim_{\kappa \to 1} \tilde{\varepsilon} = 0 \tag{48}$$

As noted in Ref. 1, in the no-breathing limit Eq. (1) degenerates into the simpler equation

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}] - \frac{\tilde{\gamma}}{4}[\hat{q}, [\hat{q}, \hat{\rho}]] \tag{49}$$

This equation has been extensively investigated in the last decade from various point of views.[3,7,9–11] According to Eqs. (47), (16), (18), (13), and (12), in the K model $\tilde{\gamma}$ depends, apart from the constants of nature $\hbar$, c, and Λ_P, only on the mass in the case of microparticles and of solid grains with mass below the transition region, and also on the geometrical size in the case of solid bodies with mass above the transition region. For microparticles $\tilde{\gamma}$ is so small (for an electron $\tilde{\gamma} \approx 10^{-140}/\text{cm}^2\,\text{sec}$) that for them the non-Hamiltonian term in (49) can be safely ignored. We recovered the result that microparticles obey the Schrödinger equation. On the other hand, for a solid body with $m \approx 1$ g, $R \approx 1$ cm, one finds $\tilde{\gamma} \approx 10^{37}/\text{cm}^2\,\text{sec}$. The presence of the non-Hamiltonian term in Eq. (49) leads then to the emergence of the very nearly classical behavior of the body.

From (16) and (47) one sees that a_c and τ_c are the characteristic length and time associated with $\tilde{\gamma}$ and m. In the case of solid bodies a_c gives the width of the Gaussian center-of-mass wave function, the nonselective propagation of which is described by Eq. (49) in the no-breathing limit. The quantity τ_c determines the time at which the diffusion of the mean momentum reaches the value $\hbar/a_c$ of the coherent momentum spread of the wave function.[7] Of course, at the same time a_c and τ_c are the coherence length and the nonrefined cycle period of the K model.

We close this section with a remark concerning the well-known mathematical ambiguity of the decomposition of $\hat{\rho}$ into projection operators projecting on an overcomplete system of states. The "additional principle which goes beyond the density matrix formalism" when choosing one or the other decomposition, required in Ref. 12, is the assumption, common to both models, that the self-reductions are the only instances of the reductions of the wave functions. This means that the center of mass of an isolated solid body is always in a pure state corresponding to a (breathing) wave packet of width a_c ($\sqrt{Q_0}$). It follows that the physically relevant decomposition of $\hat{\rho}$ is the decomposition into projectors on these pure states.

6. PROSPECTS AND DIFFICULTIES OF FALSIFYING THE MODELS

Both models bar all hopes of observing the breakdown of the superposition principle by letting the wave function of an isolated elementary particle (or of a system composed of a few particles) expand over macroscopic spatial domains, because according to both models the distant components of the wave function remain coherent for practically arbitrarily large separations. However, at least in principle, a new possibility emerges. Namely, one may try to observe the anomaly in the motion of the center of mass of solid bodies, caused by the frequent self-reductions interrupting the Schrödinger evolution.

Let us consider first the unrealistic case of an ideally isolated solid body of 1 g. As we know already, its center-of-mass wave function is a tiny wave packet–soliton: although it is breathing while it propagates (except in the no-breathing limit), it does not expand without limits like a free Schrödinger wave packet does. Instead, it is its "trajectory" which "spreads out." More precisely, the trajectory—a tube of thickness $\sqrt{Q_{00}}$ to $\sqrt{Q_0} \approx 10^{-11}$ cm in the GRW model (of thickness a_c to $2a_c \approx 10^{-16}$ cm in the K model)—is zigzagging with increasing latitude around the Hamiltonian trajectory, due to the tiny stochastic kicks received by the center of mass at each self-reduction.

The propagation of the center-of-mass wave function of an isolated solid grain in the transition region can be pictured in a similar manner, the thickness of the trajectory being then of the order of 10^{-5} cm and the successive self-reductions less frequent.

As mentioned already, in the K model the deviation from the Hamiltonian evolution has been expressed in terms of the elongation e_K of the anomalous Brownian motion of the center of mass with respect to the trajectory which would be followed by the mean value of the center-of-mass coordinate under pure Schrödinger evolution. By Ehrenfest's theorem, this would be the classical trajectory.

After $n \gg 1$ self-reductions, for an isolated solid body e_K is given, in leading order, by the n^3 term of Eq. (4.3.15a) in Ref. 2. In our notations it reads

$$e_K \approx \frac{1}{10} a_c \left(\frac{t}{\tau_c}\right)^{3/2} \tag{50}$$

(The empty space-time of Károlyházy can be viewed as a "medium" of zero viscosity in which a stochastic force acts. In this case the elongation of the Brownian motion is proportional to $t^{3/2}$, instead of the usual $t^{1/2}$ dependence.)

The anomalous Brownian motion is fuelled by small portions $m(\Delta v')^2/2$ of kinetic energy received by the center of mass at each self-reduction. In the K model this energy production amounts to 10^{-13} erg/year for a mass of 1 g, while in the GRW model it is still much smaller, 10^{-30} erg/year (see Eq. (7.2) in Ref. 1). Actually this tiny gain in energy by macroscopic bodies is converted into heat and unfortunately remains undetectable even in the K model, except perhaps in refined experiments designed to observe the anomalous Brownian motion. As far as microparticles are concerned, we repeat that practically they do not undergo self-reductions, so that for them and for the interactions between them all the usual laws hold, energy conservation included.

From Eq. (50), one finds for masses $m > m^{\mathrm{tr}}$ (in CGS units)

$$e_{\mathrm{K}} \approx 10^{-11} m^{1/18} t^{3/2} \approx 10^{-11} t^{3/2} \tag{51}$$

Here we took into account that $m^{1/18} = O(1)$ in the mass interval where (51) is relevant. This interval goes from the transition mass 10^{-14} up to 10^6 g. The upper limit is imposed because for higher values the center of mass of the body gradually loses its leading role in determining the conditions of the self-reductions. This is due to the fact that at normal density too large bodies are not rigid enough.[2]

According to Eq. (51) the elongation of the center of mass of a perfectly isolated solid body would reach the respectable value of 10^{-5} cm within 10^4 sec. Notice that, under pure Schrödinger evolution, during this time the width of a Gaussian wave packet would increase from the initial value $a_c \approx 10^{-16}$ cm to only 10^{-7} cm. So, the prediction of the model differs not only from the classical, but also from the orthodox quantum mechanical prediction.

How will the above picture change in realistic situations when the isolation is not perfect? It turns out[2] that in the case of frictionless suspension, the anomalous elongation of a ball of radius $R \gtrsim 1$ cm suspended in a gas would supersede the elongation of the normal Brownian motion due to the gas molecules by a few orders of magnitude. (Of course, the elongations refer now to the equilibrium position of the suspended ball.) However, under realistic conditions the "microfrictions" in the suspension would probably absorb too much of the energy, so that no effect will be seen. To eliminate the suspension, it has been proposed[5] to put the gas container with the ball—more exactly, with a "dumbbell"—on a spacecraft. The angular anomalous elongation of the axis of the dumbbell is larger than the elongation of the normal Brownian motion, and *per se* is large enough to be observed. However, only a careful study of the noises arising in space laboratories could show whether the experiment is feasible or not.

We know of two cases in which the anomalous Brownian motion of

macroscopic bodies can perhaps be observed as an anomaly in the functioning of planned or existing devices. One is the gravitational gyroscope experiment with cylindrical gyroscopes.[13] (For spherical ones the K model predicts no anomaly, because the rotation of a homogeneous sphere does not lead to macroscopically distinct mass distributions.) The other is to be able to observe the anomalous Brownian motion as a tiny stochastic disturbance while monitoring the proof mass of a drag-free satellite.[14]

In the GRW model the non-Hamiltonian contribution to the elongation of the center-of-mass coordinate of an isolated solid body is given by the second term on the right-hand side of Eq. (3.16a) in Ref. 1

$$e_{\mathrm{GWR}} = \left(\frac{\alpha\lambda}{6}\right)^{1/2} \frac{\hbar}{m} t^{3/2} \tag{52}$$

With (3) and (32) we find, in cgs units,

$$e_{\mathrm{GRW}} \approx 10^{-18} \frac{t^{3/2}}{\sqrt{m}} \tag{53}$$

Comparison of (53) with (51) confirms that in the region of macroscopic masses $e_{\mathrm{GRW}} \ll e_{\mathrm{K}}$, so that according to the GRW model the departure from Hamiltonian behavior escapes detection in this domain. Let us point out that e_{GRW} cannot be made equal to e_{K} by a new choice of the parameters α and λ_{micro}. As an example, consider the case of a solid body of 1 g. In order to reach the value of e_{K}, one should then require $\alpha\lambda = \tilde{\gamma} = 10^{37}$, instead of the actual value $\alpha\lambda = \alpha\mathcal{N}\lambda_{\mathrm{micro}} \approx 10^{17}$. Now, α can be increased at most by a few orders of magnitude, because one should keep $1/\sqrt{\alpha} \gg 10^{-8}$ cm. Indeed, this is the condition which guarantees that the relative coordinates of the constituents of the body practically do not undergo self-reductions.[1] So, it is λ_{micro} which should be increased by almost 20 orders of magnitude. This would lead to the obviously unacceptable result that the free wave function of an atom (or of a molecule) udergoes about 100 self-reductions per second.

On the other hand, in the transition region $e_{\mathrm{GRW}} \approx e_{\mathrm{K}}$. This had to be expected, since the cycle parameters of the two models are so close to each other in this region. The observation of the anomalous Brownian motion in the transition region may turn out to be still more difficult than in the macroscopic region, because for a colloidal grain in a gas the normal Brownian motion is considerably larger than the anomalous one[2].[8] In

[8] To make this statement plausible, note that according to the textbook formula for the elongation e of the normal Brownian motion of a solid grain of 10^{-14} g one finds $e^2 \approx 10^{-6}t$ in a gas (with viscosity 10^{-4}) and $e^2 \approx 10^{-8}t$ in a liquid (with viscosity 10^{-2}). Comparison with the anomalous elongation $e_{\mathrm{K}}^2 \approx 10^{-22}t^3$ given in (51) shows that $e_{\mathrm{K}} \ll e$ during reasonable times of observation.

order to carry out an experiment, either a high degree of isolation of the grain must be achieved, or one should be able to evaluate theoretically the influence of the surroundings with high precision.

Since the anomaly appears as a small stochastic force, it would seem advantageous to look for its effect in systems with (dynamical) instabilities. Indeed, such systems are very sensitive to fluctuations when they are close to the point of instability. However, the estimates carried out by Károlyházy for a few realistic cases show that the location of the unstable point (with the anomaly disregarded) cannot be theoretically determined with a precision sufficient to separate the tiny effect of the hypothetical anomaly. Of course, this discouraging result does not mean that work on this line should be given up; suitable systems may well exist.

The above discussion shows that verification of the existence of the anomalous Brownian motion is not easy. At present, observation of the anomaly predicted by the K model for macroscopic solid bodies seems the most promising. With the advance of technology, both models may become falsifiable in the transition region also.

7. THE SUBMACROSCOPIC BREAKDOWN OF THE SUPERPOSITION PRINCIPLE IN A CLOUD CHAMBER

As we have seen in Sections 2 and 3, in the case of solid bodies the breakdown of the superposition principle is due to the self-reductions of a single degree of freedom—the center-of-mass coordinate of the body. The situation is more complicated in the case of measuring devices, where amplification of microscopic signals takes place. As a typical example, the decay of superpositions during the process of drop formation in a could chamber has been described in detail in the thesis of Károlyházy.[2] Here we shall merely recall the line of thought and the main result.

Consider $N-Z\approx 10^{23}$ vapor molecules and a droplet of $Z\approx 10^{6}$ molecules in a Wilson chamber. (The walls of the chamber are taken into account as boundary conditions.) We shall be interested in the fate of a superposition of two states of the vapor + droplet system, differing in the positions of the droplet. For concreteness, let the distance between the positions of the droplet in the two branches be $d\approx 1$ cm. The uncertainty in the position of the droplet within a single branch is irrelevant, as far as it is much smaller than d.

If one disregards the further growth of the droplet, one would think that our superposition will not break down, because the coherence length a_c corresponding to the mass of the droplet is much larger than the uncertainty d in the position of the droplet. Indeed, $a_c\approx 100$ m.

Still, the superposition will decay even if the drop does not grow.[2] Indeed, due to the difference in the positions of the droplet in the two components of the superposition, phase spreads of the order of π develop between many points of the multidimensional configuration space of the vapor + droplet system. These spreads entail a sequence of self-reductions of the total system, each of which changes stochastically the relative weight of the branches in the superposition. This win-or-lose game ends with the survival of only one of the branches. The probability for a given branch to win is proportional to its weight in the initial superposition, whereas the time needed for the decay of the superposition is inversely proportional to Z^2. Our superposition will decay through this mechanism in about 10^{-7} sec. Under realistic conditions this is the dominant way of breakdown of the superposition during the process of drop formation in a Wilson chamber. Indeed, the growth of the drops is not fast enough to trigger the self-reduction mechanism discussed in Section 3 by bringing down the value of a_c below that of d in 10^{-7} sec.

We call two positions of a body macroscopically distinct if the separation between the positions is larger than a_c. Since now the breakdown occurs when $d < a_c$, that is, when the separation is not yet macroscopic, we shall say that the breakdown occurs at the "submacroscopic level."[2] Note that the submacroscopic decay of the superposition takes place also if the droplets in the different branches are of unequal size.

Of course, the important point here is not that in this example the submacroscopic breakdown happens to supersede the macroscopic one, but the very existence of a mechanism through which the presence of the vapor—of the surroundings—influences the breakdown of the superposition in the composite system containing the droplet—the local system.

The wave functions in the branches of the initial superposition are entangled with wave functions of the ionizing microparticle, the interactions of which initiate the drop formation. Therefore, as a result of the above breakdown of the superposition, only the wave function of the particle in the winning branch survives.

Most of the macroscopic systems do not possess a spectacular amplification mechanism of microscopic signals. Still, as a rule, they convert the energy brought in by a microparticle into heat, and thereby they also couple many of their degrees of freedom to those which directly interacted with the microparticle. The result again is a relatively slow submacroscopic breakdown of the superposition.

There are cases in which the microparticle does not lose energy and its interactions with the constituents of the macrosystem do not induce self-reductions. Then the coherence of the superposition is preserved, and under suitable circumstances an interference picture can be observed. A

celebrated case is that of an electron beam which passes through a crystal. Of course, in all the branches of the superposition, the center-of-mass wave functions of the crystal undergo their frequent self-reductions to coherence cells with linear size of about 10^{-16} cm, but these self-reductions do not destroy the coherence between the branches containing the wave functions of the electron entangled with the wave functions of the crystal.

It may be of interest to note what the models can say about superconductivity. The relevant remark is that the Cooper pairs are manifestly microobjects which can propagate in the raw material of the superconductor without self-reductions, whereas the piece of solid in which they move behaves like any other solid. This line of thought seems to be applicable in the framework of the GRW model also. Indeed, for a Cooper pair, $\sqrt{Q_0}$ is certainly larger than 10^{19} cm [see Eq. (25)]. (For similar reasons, the limitation of the GRW model to insulating solids[1] can probably be lifted: The wave functions of the nonlocalized electrons in a metal practically do not self-reduce.)

Finally, let us point out that in the framework of the K model monstrous superpositions like that with Schrödinger's cat dead and alive do not develop. To begin with, an exploded cat gets into a different coherence cell than an unexploded one in 10^{-11} sec. Of course, one can kill a cat by gentler methods. Then, as indicated by the example of the cloud chamber, the superposition falls apart slower, but already at the submacroscopic level, long before the death of the cat.

8. SELF-REDUCTIONS AND CONTINUOUS MEASUREMENTS

As we know, in Eq. (49) the non-Hamiltonian term comes from the self-reductions going on in an isolated system. On the other hand, in the theory of continuous measurements the same equation has been derived,[3,10] but there the non-Hamiltonian term describes the impact of the surroundings on the time evolution of a local system. So, while the coefficient Λ used in Ref. 3 in place of our $\tilde{\gamma}/4$ measures the effectiveness of a specific environment in destroying the coherence of the density operator of a local system, $\tilde{\gamma}$ measures the effectiveness of the self-reductions in destroying the coherence of the density operator of an isolated system. We conjecture therefore that the relation $\Lambda \gg \tilde{\gamma}$ means that the influence of the surroundings on the local system is strong.

As an example, let us consider a solid grain in the transition region. From Eqs. (20), (21), and (47) we find $\tilde{\gamma} \approx 10^7$ in cgs units. On the other hand, in Ref. 3 Λ was estimated for dust particles of various sizes and in various surroundings. Dust particles of radius $R \approx 10^{-5}$, called colloidal

grains in Ref. 2, belong to the transition region. As can be seen from Table 2 in Ref. 3, for them $\Lambda \gg \tilde{\gamma}$ in all the cases considered, except for cosmic background radiation. In particular, for air, $\Lambda \approx 10^{32}$. The conclusion is that a gaseous surrounding strongly modifies the behavior of a colloidal grain. As mentioned in Section 6, the same conclusion has been reached in the K model by comparing the normal and the anomalous Brownian motion of the grain. The normal Brownian motion arises due to Hamiltonian interactions. It should be added that this is not the whole story. The presence of the gas also modifies the conditions of the self-reductions.

As far as a macroscopic solid body of 1 g at normal density is concerned, in the K model the size of the coherence cell of its center-of-mass wave function is so small and the frequency of its self-reductions is so high [Eq. (17)] that a gaseous (or a radiative) environment does not modify the conditions of its self-reductions appreciably. This conclusion is not invalidated by the results presented in Ref. 10. Indeed, the excitations of the vibrational states of the tungsten cube considered there do not change the mass distribution of the cube, and therefore they do not affect the self-reductions of the center-of-mass wave function. Also, the 25 sec needed for the impingning atoms to change the state of the cube are much larger than the period of 10^{-4} sec of the self-reductions. In other words, we conjecture that in this case one will find $\Lambda \ll \tilde{\gamma} \approx 10^{37}$. Note that in the GRW model $\gamma = \alpha\lambda \approx 10^{17} \ll \tilde{\gamma}$, so that in this model the conditions of the self-reductions of macroscopic bodies are more sensitive to the surroundings than in the K model.

However simplistic the above comparisons may be, they shed some light on the problem of the relation between models with self-reductions and the theory of continuous measurements raised in Ref. 12.

9. CONCLUSION

In the orthodox quantum theory one needs some external agent (a classical apparatus or the consciousness of an observer) not subject to the Schrödinger equation in order to interpret the result of the Schrödinger evolution of the system under investigation. Furthermore, quantum and classical systems obey different laws of motion. Both these laws are deterministic. Probability comes in "only" in the description of the quantum world in terms of concepts of the classical world.

In models with self-reductions a different picture emerges. All systems obey the same law of propagation, in which deterministic (Schrödinger evolution) and stochastic (self-reductions) aspects are combined. There is

no need for the notion of purely classical and/or purely quantum mechanical systems. Macroscopic systems possess very nearly classical intrinsic properties, and microscopic systems—very nearly quantum mechanical intrinsic properties. The gradual transition between predominantly classical and predominantly quantum behaviors can be quantitatively characterized. The influence of specific environments on various systems can also be evaluated. As a rule, the environment affects strongly the behavior of submacroscopic and microscopic systems not only through the usual Hamiltonian interactions, but also due to the ability of the environment to undergo self-reductions. In particular, when a microobject comes into contact with a macrosystem called a measuring apparatus in the orthodox theory, the Hamiltonian interactions lead to a superposition in which wave functions of the object and of the apparatus are entangled. This superposition undergoes then a sequence of self-reductions, induced by the apparatus, at the end of which only one of the entangled branches remains alive. Thereby the measurement is accomplished: The wave function of the microobject is reduced to a state corresponding to the indication of the "pointer." The probability that a branch survives is proportional to its weight in the initial entangled superposition. This guarantees that the probabilistic predictions of the orthodox measurement theory are reproduced. More precisely, they are reproduced up to small anomalies akin to the anomalous Brownian motion.

According to both models the genuinely stochastic aspect of the propagation of the wave function is as basic a feature of nature as the deterministic aspect. The opinion that genuine (or "objective") probability should be incorporated into our world view[16] is strengthened by the experimental evidence[17] against the existence of simple local hidden variables.[18] Thus, quantum mechanical nonseparability, leading to such striking phenomena when the wave function of microparticles extends over macroscopic domains, is likely to stay with us. The only comfort provided by models with self-reductions is the unification of the picture: Nonseparability and the E.P.R. paradox are present in embryonic form in the macroworld also, and again independently of any observer. Indeed, every time the tiny center-of-mass wave packet of a macroscopic solid body undergoes a self-reduction, its value jumps to zero everywhere except in the coherence cell into which it projects itself. The paradoxical character of this jump remains unnoticed, because even before the jump the wave function is zero everywhere, except for a very small domain barely larger than the minuscule coherence cell itself. However, the accumulated effect of the jumps leads to the anomalous Brownian motion of solid bodies which can perhaps be observed.

Even if the observation of the predicted anomaly remains beyond

the scope of experimental ability, models with self-reductions provide a way of saving the concept of "independent reality."[19] The price to pay is the acceptance of the idea that the Schrödinger wave function, which represents this reality in the theory, obeys a propagation law that has a dual character.

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NOTES ADDED IN PROOF

1. In Eq. (39) a factor of 8 has been overlooked. The correct relations are $8\tilde{Q}_{00} = a_c^2$ and $8\tilde{Q}_0 = (\kappa a_c)^2$. Accordingly, numerical factors appear in some of the subsequent equations. They do not invalidate any of our conclusions. Also, the correct connection between the parameter Δ of Ref. 7. and a_c^2 is $4\Delta = a_c^2$ and not $\Delta = 2a_c^2$.

2. For further developments of the GRW and K models see the Proceedings (in preparation) of the Summer School "Sixty-two Years of Uncertainty...", held at Erice, Italy, in August 1989, to be published by Plenum Press.

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