JOURNAL OF MATERIALS SCIENCE LETTERS 1 (1982) 77–79

Mode II fracture toughness of wood measured by a mixed-mode test method

D. G. HUNT, W. P. CROAGER
Department of Mechanical and Production Engineering, Polytechnic of the South Bank, Borough Road, London SE1, UK

It has been established that in a strongly anisotropic fibrous material such as wood, Mode II or mixed-mode fracture can be important in certain loading situations [1–3]. The mixed-mode fracture test methods used by Wu [1] and by Williams and Birch [3] involved combined tension and shear of large flat plates containing cracks at suitable orientations. The practical difficulties of obtaining the theoretical stresses with such test methods is well known, and Barrett and Foschi [2] suggested a Mode II test method using the three-point bending of a beam in which the crack was located on the neutral plane at one end of the beam. Friction error resulting from crack closure by the beam support was avoided by making the machined crack-mouth wide enough to allow the insertion of two metal rollers between thin metal plates. Whilst this method is much easier to perform than the flat-plate method, there is the possibility that if the crack mouth is slightly too wide the crack would still suffer from closure friction and if it was too narrow the insertion of the rollers would superimpose a positive Mode I component. The previous mixed-mode results on wood [1, 3] and those of Fig. 4 in this paper suggest that the error in the K_{II} value caused by a superimposed K_I could be considerable.

In order to avoid these problems it was proposed to make mixed-mode tests on beams in which the K_I and K_{II} values were known, in order to obtain a pure Mode II fracture toughness by extrapolation or interpolation. The test method is shown in Fig. 1. The relative proportions of support given by P_0 and P_c can be varied by means of shims and measured by means of load cells.

To assess the results from this test it was necessary to determine the values of K_I and K_{II} for various relative values of P_0 and P_c and, in particular, the P_c/P_0 ratio for which $K_I = 0$. Since experimental compliance methods are not applicable to mixed-mode loading, a finite-elements calculation was required.

Sih, Paris and Irwin [4] demonstrated on theoretical grounds that the stress-intensity factors for isotropic and orthotropic materials are identical for a cracked infinite plate. Such a conclusion is often assumed to apply to finite-sized test-pieces. However, Walsh [5] showed, using both an approximate "strength of materials" approach and finite-elements analysis, that the above conclusion

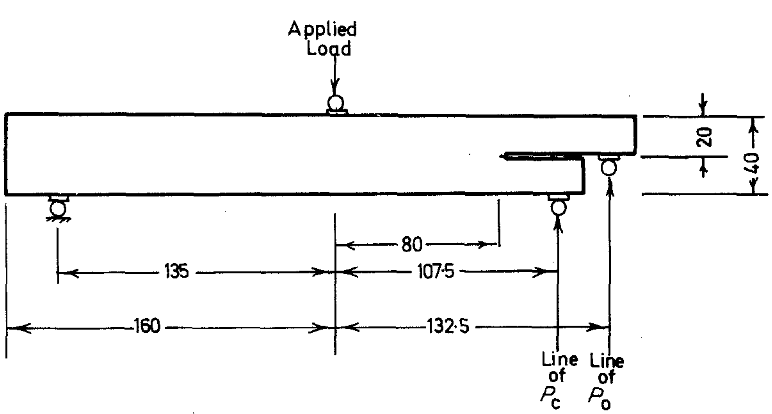

Figure 1 Test-piece for mixed-mode fracture of wood.

0261–8028/82/020077–03$02.30/0

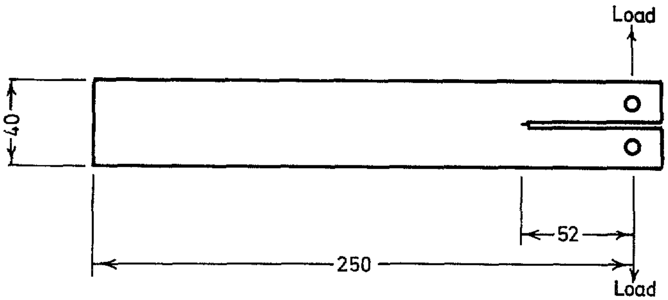

Figure 2 Long test-piece for Mode I fracture.

is only valid for certain test-piece shapes; for very long cleavage test-pieces, for instance, the K_I value of the orthotropic material is only about a third of that of the isotropic material. Differences in the K_I values between the isotropic and the anisotropic cases were also observed by Mandell *et al.* [6]. Others have avoided the problem by determining the K_I or K_{II} values directly with orthotropic finite elements [7].

Since no direct K_{II} comparison between isotropic and orthotropic materials appears to have been made, it was decided to pursue this matter by comparing the results of finite-elements calculations of both K_I and K_{II} for isotropic and orthotropic materials for the test-piece shapes used in this study. In addition to the mixed-mode test-pieces shown in Fig. 1, this included also the standard compact-tension and single edge-notched bend test-pieces [8] and a K_I test-piece shown in Fig. 2, whose proportions are roughly similar to those of the mixed-mode test-piece shown in Fig. 1.

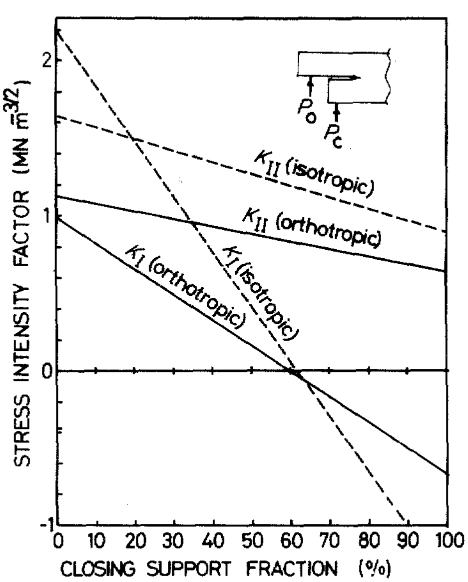

Figure 3 Stress-intensity factors for mixed-mode test-piece calculated by finite elements for an isotropic and an orthotropic material. Closing support fraction = $P_c/(P_o + P_c)$.

The finite-elements calculations were made with the PAFEC [9] program using 8-noded orthotropic plane elements on a DEC-10 computer. The stress-intensity factors were calculated from the computed plane-stress displacements near the crack tip from

$$K_i = E_i u(2\pi/r)^{1/2}/4, \tag{1}$$

where i is the fracture mode, u is the displacement in the appropriate direction at a distance r along the crack face from the crack tip, and the values of E_i are given by

$$E_I = \left\{\left(\frac{a_{11}a_{22}}{2}\right)^{1/2}\left[\left(\frac{a_{22}}{a_{11}}\right)^{1/2} + \frac{2a_{12}+a_{66}}{2a_{11}}\right]^{1/2}\right\}^{-1} \tag{2}$$

and

$$E_{II} = \left\{\frac{a_{11}}{\sqrt{2}}\left[\left(\frac{a_{22}}{a_{11}}\right)^{1/2} + \frac{2a_{12}+a_{66}}{2a_{11}}\right]^{1/2}\right\}^{-1}, \tag{3}$$

where a_{ij} are the anisotropic compliances. The modular ratios for the orthotropic materials were taken from [10] for sitka spruce at a moisture content of about 12%. The values of $u/r^{1/2}$ were plotted and extrapolated back to the limiting value at the crack tip.

The mixed-mode finite-elements results are given in Fig. 3, both for isotropic and orthotropic calculations, as a function of the "closing support fraction", $P_c/(P_0 + P_c)$. These were scaled for a total support of $P_0 + P_c = 1$ kN. It can be seen that for both types of material, K_I is zero at a value of closing support fraction of about 60%. In other respects the orthotropic and isotropic materials are very different.

TABLE I Comparison between computed K_I values for isotropic and orthotropic materials

Type of test-piece	Type of material	K_I (MN m$^{-3/2}$)		
		Displacement method	Compliance method	Formula
Long test-piece	Isotropic	3.922	3.970	–
(see Fig. 2)	Orthotropic	1.742	1.744	–
Compact tension	Isotropic	2.425	2.447	2.400
specimen	Orthotropic	2.165	2.164	
Single edge-notched	Isotropic	2.616		1.653
bend specimen	Orthotropic	2.662		

Notes: (1) Compact tension specimen and single edge-notched bend specimens according to [8] with $a = 20$ mm, $W = 40$ mm, $B = 20$ mm. (2) Formulae for test-pieces are given in [8].

The finite-elements results for pure Mode I are summarized in Table I. These values are for nominal loads of 1kN, and the same modular ratios were used as for the mixed-mode calculations. It may be seen from Table I that the ratios of K_I (orthotropic) to K_I (isotropic) are:

long test-piece 0.44,

compact tension specimen test-piece 0.88,

single edge-notched bend test-piece 1.02.

The results of the mixed-mode tests on baltic redwood (*Pinus sylvestris*) at 10% moisture content are given in Fig. 4, for crack growth in the RL [3] system. For each test the "closing support fraction" $P_c/(P_0 + P_c)$, was determined at failure. Reference to Fig. 3 then gave values of K_I and K_{II} for a total load of 1 kN, and these values were scaled up for the actual failure load, $(P_0 + P_c)$. As expected, with a variable material such as wood, the results show considerable scatter, although possibly less than those obtained by previous authors using biaxially-stressed plates. Fig. 4 also shows a least-squares fitted curve based on the equation

$$\frac{K_I}{K_{Ic}} + \alpha\left(\frac{K_{II}}{K_{IIc}}\right)^{\beta} = 1.$$

The values of the constants α and β were 1.005 and 3.4, respectively.

If the value of α is taken as 1, the value of 3.4 for β may be compared with a value of 2 obtained by Wu [1] using biaxial stressing of balsa plates. Fig. 4 also gives the ratio K_{IIc}/K_{Ic} of 2.7, which agrees reasonably well with the value of 2.4 obtained by Williams and Birch [3].

Whilst the results from this method depend partly on a theoretical calculation of the stress-intensity factors, it can be shown that the variation in the modular ratios from species to species has a very small effect on the stress-intensity factors.

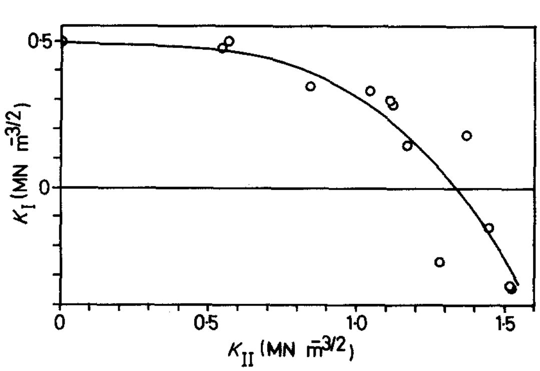

Figure 4 Mixed-mode fracture results for baltic redwood. The fitted curve has the equation $K_I/K_{Ic} + (K_{II}/K_{IIc})^{3.4} = 1$.

References

1. E. M. WU, *J. Appl. Mech.* 34 (1966) 967.
2. J. D. BARRETT and R. O. FOSCHI, *Can. J. Civ. Eng.* 4 (1977) 86.
3. J. G. WILLIAMS and M. W. BIRCH, ASTM Special Technical Publication number 601. (ASTM, Philadelphia, 1976) p. 125.
4. G. C. SIH, P. C. PARIS and G. R. IRWIN, *Int. J. Fracture Mech.* 1 (1965) 189.
5. P. F. WALSH, *Eng. Fracture Mech.* 4 (1972) 533.
6. J. F. MANDELL, F. J. McGARRY, S. S. WANG and J. IM, *J. Comp. Mater.* 8 (1974) 106.
7. R. O. FOSCHI and J. D. BARRETT, *Int. J. Numerical Methods Eng.* 10 (1976) 1281.
8. British Standard number 5447 (1977) "Plane Strain Fracture Toughness of Metallic Materials".
9. Program for Automatic Finite Elements Calculations, PAFEC Ltd, Nottingham, UK.
10. "Wood Handbook", U.S. Department of Agriculture Handbook number 72 (1955).

Received 9 July and accepted 18 August 1981