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A MAGICAL MODEL OF HAPPINESS

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ABSTRACT. A model of happiness is described that is based on nonlinear mathematics. Happiness at time $t + 1$ is given by: $H\text{next} = M(1 - H)\ H + I$, where H is happiness at time t, M is a person parameter, and I is an environmental impact term comprised of a prevailing component (Ip) and an episodic component (dI). Eight properties of the model are detailed and its utility illustrated for reconciling problematic issues in the literature that include the positive skew in the distribution of psychological well being (PWB) scores, the stability and change in PWB, and the relationships of emotionality with global PWB and its indicators. The operationalization of the model is described from the population level down to that of the single case.

Wilson (1967) and later Diener (1984) pointed to deficiencies in our theoretical understanding of psychological well-being (PWB), a construct referred to in philosophy and lay usages as happiness (Brandt, 1967, McGill, 1967; Stones and Kozma, 1980). Although progress has been made since Wilson's (1967) review in the definition, measurement, and knowledge about the correlates of PWB (McNeil *et al.*, 1986), Diener's (1984) plea for a greater sophistication in model building has yet to stimulate major new initiatives, despite some impressive progress on specific issues (Larsen and Deiner, 1987, Michalos, 1985). Our intent in this article is to draw attention to some well-replicated phenomena and paradoxes in the PWB literature, to present a model that both speaks to these issues and contains the propositional rigor that Diener (1984) advocates, and to show how this model can be operationalized to generate new and risky hypotheses. The adjective 'risky' is chosen to indicate that the predictions cannot easily be derived from existing models.

Phenomena, paradoxes, and the treatment of complexity in PWB

Probably the most robust findings in PWB research is the positive skew to the distribution of scores. This finding is so familiar that it receives scant or cursory treatment by researchers, rather than being considered

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an important phenomenon in need of explanation (Heady and Wearing, 1988). Heady and Wearing (1988) write that 'In all developed countries in which QOL [quality of life] studies have been conducted almost all sections of the community rate themselves above the mid-point of scales.' (p. 497). With the scores expressed as a proportion of the distance between the minima and maxima on the respective scales, examples include means approaching 80% for community residents (Fordyce, 1988; Heady and Wearing, 1988; Michalos, 1985, 1986; Stones and Kozma 1989a). Heady and Wearing's data (1988) further suggest that scores not much greater than 85% may represent the normative upper limit to the distribution (i.e., because this level was the mean for several items that index 'the best you could realistically hope for').

Another robust set of findings indicates covariation among the various attributes of PWB. Whether the indexes measure global happiness or life satisfaction, satisfaction with specific life domains, or mood, the intercorrelations are substantial (Kozma *et al.*, 1990; Kozma and Stones, 1980; Michalos, 1985, 1986; Stones and Kozma, 1985, 1986).

A third category of phenomenon that has been replicated in numerous studies is the relative invariance of PWB across cohorts and with age. Costa *et al.* (1987) showed minimal cohort and age effects on PWB in nearly 9 000 persons aged 24—75 years who were followed-up over nine years. Similarly, longitudinal studies over multi-year intervals have yielded stability coefficients of 0.5 and higher (Costa and McCrae, 1984; Costa *et al.*, 1987; Kozma and Stones, 1983; Recker and Wong, 1984). These findings provide strong evidence for an enduring property to global PWB.

Arrayed against the findings of invariance are hypotheses of cohort differences in PWB and a low stability due to environmental change. The former is based on evidence that old age is a time of multiple losses, indications that the perturbation associated with emotional reactions persist longer in the elderly (Schultz, 1985), and beliefs concerning a high prevalence of depression among seniors (cf. Cappeliez, 1988). With respect to stability, we have evidence that PWB indexes are reactive to life circumstances and life change, particularly negative change (Atkinson, 1982; Michalos, 1985, 1986). Atkinson's (1982) data showed that the absolute change in global PWB associated with negative life change was approximately three times that associated with positive life change. The preceding generate expectations that run counter to the findings of a consistency and enduring stability to PWB.

A final puzzle concerns the relationship between emotionality and global PWB. Larsen and Diener (1987) describe a temperament parameter of affect intensity that integrates much that is known about mood reactivity. Persons with high affect intensity react strongly to both positive and negative mood inducing stimuli. What is puzzling are the relationships of affect intensity to PWB and its main indicators. Affect intensity is noncorrelated with global PWB but positively correlated with both positive and negative indicators of PWB (e.g., extraversion, physical symptomatology). How can it be that emotionality is orthogonal to global PWB yet it correlates in the same direction with both the positive and negative predictors of overall happiness?

The preceding phenomena illustrate a complex and paradoxical pattern of trends. A model of PWB must take account not only of the positive skew in the distribution of scores, but also findings of covariation, stability, and change, and an ambiguous relationship with mood reactivity. Given this complexity, it is not surprising that models have emphasized either the mediated effects of life circumstances on PWB (Michalos, 1985, 1986) or the constancy in PWB due to personality (Costa and McCrae, 1984; Stones and Kozma, 1986), but have failed to integrate both in readily refutable models. It is not our intent to evaluate again the models previously reviewed by Diener (1984). The addition of further complexity to these models is unlikely, in our view, to balance greater breadth with the propositional rigor that Diener (1984) demands. Instead, we will consider a new approach to model building.

A nontraditional approach to the treatment of complexity was pioneered in the physical sciences and mathematics (Gleick, 1987), with subsequent introduction to the life sciences in the mid-1970s (May, 1976). This approach relies on nonlinear mathematics that permit the generation of complex data from simple equations. A powerful advantage that nonlinear models have over linear deterministic models is that an accumulation of parameters is not necessary to account for the complexity in data. With data that can be represented by a single point, an equation with a single parameter can model conditions ranging from a steady state to apparent chaos. Furthermore, different equations within this class were shown to display universal properties (Fiegenbaum, 1978). In the present application of a nonlinear approach to the modelling of PWB, a form of the logistic difference equation is utilized. We will show that an expanded form of the equation models properties of PWB that include a dependence of the subsequent state on the preceding condition; the stability, change, and covariation within and across different classes of index; the positive skew to the distribution of scores; the relationship of mood reactivity to global PWB and its indicators; as well as providing a clear demarcation between the personality and environmental influences on PWB. Because variation to the modulus in nonlinear equations (i.e., the nonlinear term that is the person parameter) results in outcomes that our colleague Dr. Michael Rabinowitz describes as magical (cf. the *Oxford Dictionary* that defines magic as a 'remarkable influence producing surprising results'), we came to think of the model as the *magical model.* In the following sections, we will describe a basic then the expanded form of the magical model and detail its properties.

BASIC PROPERTIES OF THE MAGICAL MODEL

PWB is a multi-faceted construct that can be accessed using several types of instrument. The more common types include (1) measures of global PWB, comprising items with a prolonged time frame (e.g., "During the past month have you felt . . . upset because somebody criticised you . . . happy because things were going your way?"); (2) mood indexes with an immediate time frame (e.g., "Right now do you feel . . . happy? . . . blue?"); (3) domain satisfaction indexes (e.g., "How satisfied are you with your . . . state of finances? . . . health?"). Because these indexes typically intercorrelate and/or load on a common factor (Kozma *et al.*, in press; Michalos, 1985; Stones and Kozma, 1985), a person parameter (M) is postulated that affects all types of PWB index. If we disregard, for the moment, the impact of external sources on

PWB, the relationship between PWB and M in the logistic difference equation is as follows:

$$H\text{next} = M(1 - H)H, \tag{1}$$

where Hnext represents a PWB rating at time $t + 1$, H represents PWB at time t, and M is the person parameter. Hnext and H are expressed as the proportionate distance on the index (i.e., $0 < Hnext, H < 1$), with the person parameter nominally allowed to vary within a range $1 < M < 4$ (outside of which the Hnext values go out of range).

The dynamics of the model are only minimally revealed through an inspection of the form of Equation 1. To illustrate its properties, the equation must be iterated multiple times, with the value of Hnext on any one iteration becoming the value of H on the subsequent iteration. Three basic properties are revealed through this procedure.

PROPERTY 1. *PWB is a self-correcting process that tends toward stability over a wide range of M values. If the value of H deviates markedly toward either extreme, the subsequent value (Hnext) is likely to rebound from that extreme. This rebound occurs because Hnext is corrected by the difference value (1 − H)H.*

PROPERTY 2. *The person parameter affects the type of equilibrium attained after multiple iterations. The equilibrium can be either static (i.e., a steady state) or dynamic (i.e., multiple values for Hnext). With values of* $1 < M < 3$, *Hnext attains a steady state that varies positively with M. The maxima for the steady state distribution of Equation 1 is approximately* $Hnext = 0.66$ *with* $M = 2.9$. *With values of M progressively greater than 2.9, the Hnext distribution bifurcates, then doubly bifurcates, eventually to become chaotic. Consequently, the person parameter encapsulates the distinction between normal and pathological affective conditions. Because both low PWB and marked fluctuations in its level are pathological (DSM III, 1980), low values and values of* $M > 2.9$ *are likely to connote a psychopathology-prone personality.*

PROPERTY 3. *Indexes relevant to different attributes of PWB are correlated. This correlation occours because M is common to the ratings over a range of measures (e.g., global PWB, mood, domain satisfactions).*

An illustration and implications

Properties 1 and 2 of the model are illustrated in Figure 1, which shows the outcome of 20 iteratations of Equation 1 with M set a 1.5, 2.3, and 3.1, respectively, and with arbitary values of H at the origin (i.e., iteration zero). Property 1 is illustrated by the trend away from the extreme PWB levels with $M = 1.5$ and $M = 2.3$. Property 2 is shown by the steady state equilibrium for Hnext at the lower two M values but the dynamic equilibrium with $M > 2.9$. The steady state levels increase with M. The implications of Properties 1 through 3 for modelling

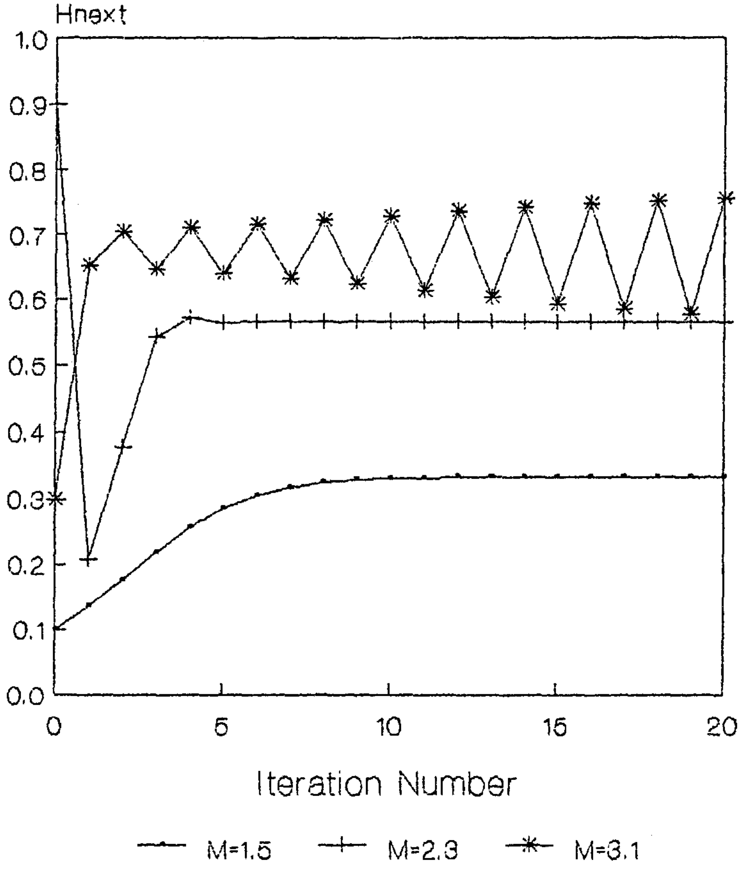

Fig. 1. *H*next over 20 Iterations at Different Levels of *M*.

PWB are that, in the absence of perturbation due to external input, a cohort consistency and a longitudinal stability to PWB are predicted over a range of person parameter values (i.e. persons with $1 < M < 3$), with the steady state level positively related to M and correlated across different types of index.

THE MAGICAL MODEL EXPANDED

Equation 1 is deficient as a model of PWB in that it is fully self-contained, allowing for no impact on Hnext via external sources. A term I must be added that represents the impact of the environment at time t. A premise made is that the environmental impact on Hnext is dependent on the nature of the PWB prompt. Global PWB is sensitive to those changes in life circumstance that affect the prevailing environment, rather than to a transient disruption of that environment, whereas mood indexes have a short-term sensitivity to environmental change (Atkinson, 1982; Kozma *et al.*, 1990). Consequently, it is convenient to partition the environmental impact term into two additive components: the prevailing environment component (Ip) represents the prevailing impact of the environment to time t; the episodic component (dI) represents the deviation from that impact due to an episode at time t. The revised model is given as follows:

$$H\text{next} = M(1 - H)\,H + I \qquad (2)$$

where $I = I\text{p} + \text{d}I$.

We now examine properties of the model that are revealed through manipulation of the external impact term (I). In the *probe paradigm*, I is used as a probe to investigate the perturbation in PWB due to an episodic change from the prevailing level of I (i.e., manipulation of dI). The *prevailing impact* paradigm examines the change to the equilibrium level of Hnext occasioned by varying the Ip component of I. For convenience, values of $I\text{p} < 0$, $I\text{p} = 0$ and $I\text{p} > 0$ are refered to as negative, zero, and positive environments, respectively.

Properties of the model revealed by the probe paradigm. As stated, the probe paradigm examines the effects on mood due to an episodic disruption to the prevailing environment. The procedure involves, (1) iteration of Equation 2 until an equilibrium pattern is obtained for *H*next, (2) insertion of the probe (d*I*) for one iteration, (3) resumption of the former level of *I* on the subsequent iterations. This procedure is repeated with different values for the person parameter, under different prevailing environments, and with different levels of the *I* probe. The index of perturbation to *H*next is the number of iterations after termination of the probe on which *H*next deviates from its baseline equilibrium. The main property illustrated by the probe paradigm is both powerful and theoritically intriguing.

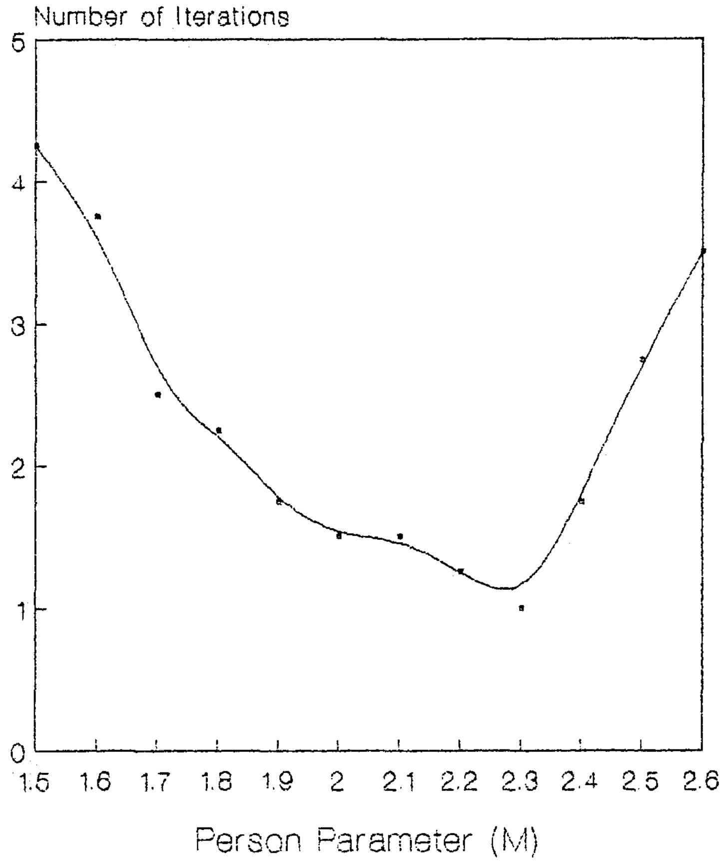

Fig. 2. Mean Probe Perturbation at Different Levels of *M*.

PROPERTY 4. *The perturbation caused by a probe generally has a U shaped relationship with the person parameter, M. This form of relationship is obtained over the full range of the M that tolerates steady states of Hnext. The property is illustrated in Figure 2, with the perturbation from a neutral environment averaged over a range of I probes. Although the M value associated with the floor of the U curve shifts to the right with increasing value of dI and Ip, the mean perturbation attains its lowest level near M = 2.2. We also note that, under any given combination of Ip and dI, the Hnext distribution during the period of perturbation differs depending on whether M is below or above that level corresponding to the floor of the U curve: with M below, the Hnext scores either fail to cross or cross only minimally to the side of the steady state baseline opposite from the probe: with M above, Hnext oscillates around that baseline. The presence or absence of appreciable oscillation provides an alternative criterion for identifing the value of M associated with minimal perturbation.*

Implications of Property 4. The property that mood reactivity achieves an overall minimum with a midrange M of near 2.2 has implications for systems efficiency and functioning. The use of U curves to model systems efficiency is novel in the PWB literature, although precedents exist in other areas of psychology (Tomporowski and Ellis, 1986). But examples abound that make low mood reactivity a condition favourable to optimal functioning. Meditation, relaxation techniques, and aspects of behavioral and traditional psychotherapy all aim to reduce emotional reactivity in the interests of overall functioning capability. Consider also the connotation of the ancient and ironic curse, "May you live in interesting times!" The model is consistent with findings that a single parameter equally affects the perturbation associated with positive and negative mood inductions (cf. Larsen and Diener, 1987). It is economical in that the same parameter has implications for both mood and global PWB. We will discuss subsequently how the model clarifies paradoxical findings on the relationship of mood reactivity to global PWB and its indicators.

Properties of the model shown by the prevailing impact paradigm. The prevailing impact paradigm has bearing on the stability and change in global PWB. The procedure is to retain a constant *I*p term over one set of iterations, followed by a different *I*p level over a subsequent set. We index the equilibrium of *H*next at each level of *I*p and monitor the number of iterations required for the transition from the preceding to the subsequent steady state. This transition index measures the readiness with which adaptation is accomplished to a new prevailing environment. The properties of the model revealed by prevailing impact paradigm have relevance to intervention research and clinical practice.

PROPERTY 5. *A negative environment has more potent effects on PWB than a positive environment. This property is illustrated in Figure 3 by the low steady state for Hnext with Ip = −0.1, but the relatively high levels with both Ip = 0 and Ip = 0.1. The values were obtained with M = 2.2 and H = 0.5 at origin, athough the outcome generalizes to other levels of the person parameter and initial state. This property is consistent with findings by Atkinson (1982) that negative life change impacts more strongly on global PWB than positive life change.*

PROPERTIES 6 AND 7. *Clinically relevant properties are that the transition from a negative to a neutral prevailing environment is accomplished more rapidly with M near 2.2 than at the extremes of that distribution, although the greater enhancement in global PWB is obtained with lower levels of M. These properties are shown in Figure 4, that illustrates the transition (beginning at iteration 6) from a negative (Ip = −0.1) to a neutral (Ip = 0) prevailing environment. The transition from the preceding to the subsequent steady state is attained most rapidly with M = 2.2, indicating that a brief intervention aimed to promote PWB is likely be successful in persons at the midrange of the M distribution. This property is consistent with findings by Stones and Kozma (1989b) on individual differences in the response to group discussion intervention among elderly nursing home residents. In a low M person, Figure 4 shows that the greater gain in PWB is offset by the relatively low equilibrium for Hnext even after the environmental change.*

PROPERTY 8. *A change from a neutral to a nonzero prevailing environment results in modification to the range of M values that tolerate steady states of Hnext. With a positive prevailing environment of*

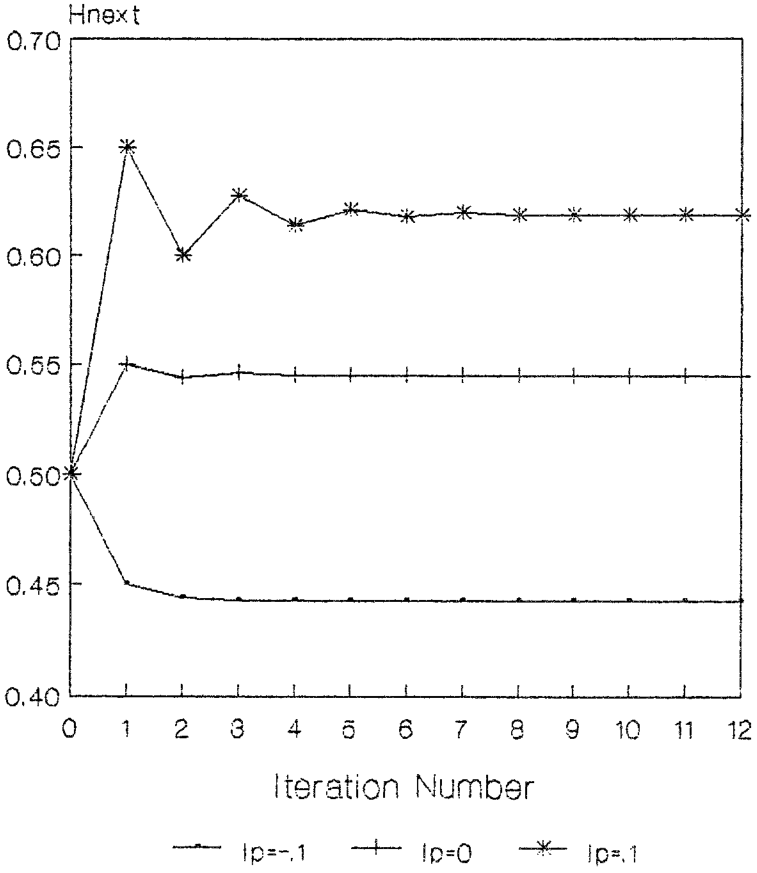

Fig. 3. *H*next over 10 Iterations with $M = 2.2$ and Three levels of *I*p.

I = 0.1, the equilibrium distribution of Hnext bifurcates with M as low as 2.7. With a negative environment, the Hnext scores go out of range at levels of $M > 1$ *(e.g., with* $M < 1.9$ *at* $Ip = -0.1$*). The implications for psychpathology are that a negative environment promotes depression in low M persons, whereas a positive environment contributes to affective instability in high M persons.*

Implications of Properties 5—8. Clinically relevant implications are that a negative prevailing environment promotes depression, and that the

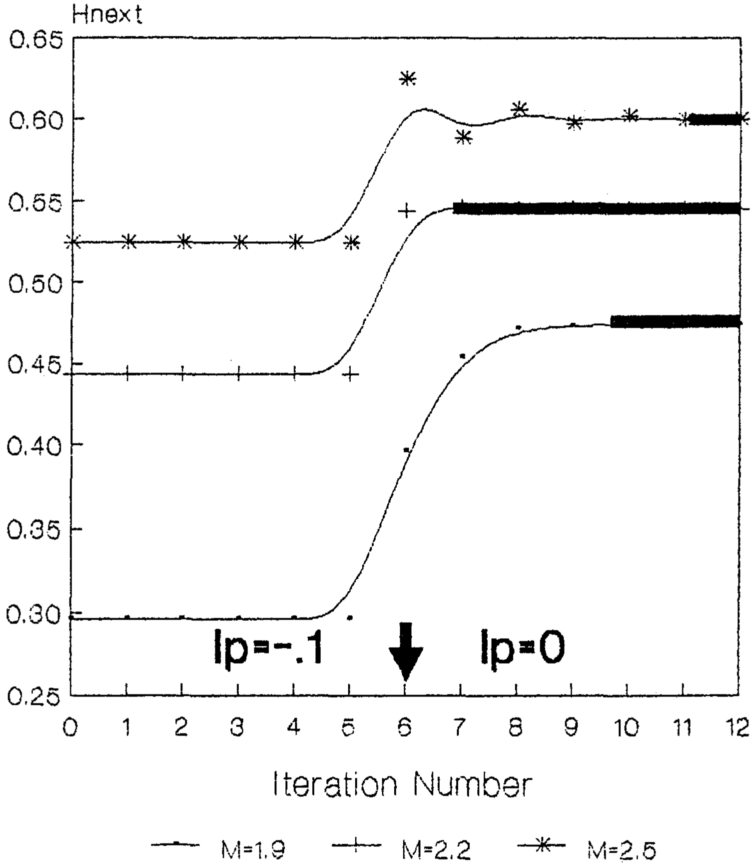

Fig. 4. *H*next with Change from *I*p = −0.1 to *I*p = 0 at Iteration Number 6.

transition to a positive shift from this environment is accomplished most rapidly with *M* near 2.2, the value previously shown to be associated with lowest overall mood reactivity. Persons with lower *M* are hypothesized to remain moderately or mildly depressed even under a neutral environment, although the global PWB of persons with $2.2 < M < 3$ is expected to obtain near maximal steady states.

IMPLICATIONS AND APPLICATIONS OF THE MODEL

The most striking implication is that a midrange value on the person parameter (approximately $M = 2.2$) represents an optimal systems property in that mood reactivity is relatively muted and the transition from a negative prevailing environment is accomplished rapidly. Were humans to be designed in accordance with the model, an accomplished engineer would orient production toward a product with M near 2.2. Evidence to be reviewed in this section permits a derivation that an M near 2.2 represents the central tendency in the population, a proposition that clarifies the paradoxical relationships of mood reactivity to global PWB and its main indicators. We also discuss the operationalization of the model at the level of the single case, for purposes of applied and clinical research.

Known trends and paradoxes resolved

The positive skew in the PWB distribution. By fitting M to the empirical estimates of global PWB, the model is shown to anticipate the positive skew and the central tendency estimates obtained in survey research. The central tendency estimate in most studies is approximately 80% of the distance along the useable range of the scale, with the latter identified as the range that encompasses the distribution of scores. With 20-point rating scales, Heady and Wearing (1988) found that the range reported to be usable over an array of scales averaged between 7.5 and 17 points. Scores outside of this range were either below the worst level reported or beyond a limit judged to be attainable. The mean PWB of 15 points is approximately 80% of the distance along the usable range. The useable portion on the multi-item Memorial University of Newfoundland Scale of Happiness (MUNSH) ranges from −16 to 24 points (Kozma and Stones, 1980; Stones and Kozma, 1989a). Only a handful of persons among the thousands we have tested scored below −16. The grand mean on the MUNSH is approximately 16 points, or 80% along the useable range dimension. The same estimate of approximately 80% is obtained from the model with $M = 2.2$ and a neutral environment. The useable range of the Hnext distribution that encompasses the steady state values (i.e., with $1 < M < 3$) lies between 0.09 and 0.66. The steady state of $H\text{next} = 0.55$ with $M = 2.2$ is approximately 80% of the distance along the useable range. Because the central tendency estimates obtained from empirical research correspond to that generated by the model with $M = 2.2$, we infer that 2.2 is the central tendency of the M distribution. This inference is consistent with our previous deduction that an M near 2.2 is an optimal property from the perspective of systems design.

The stability of PWB. The model anticipates a stability to global PWB due to the influence of the person parameter, M. But because changes in Ip also affect global PWB, the stability is predicted to be relative rather than absolute. These hypotheses are borne out empirically (Costa and McCrae, 1984; Stones and Kozma, 1986). Furthermore, the absence of cohort effects on global PWB (Costa *et al.*, 1987) suggests that changes in Ip are not systematic with age: Ip varies within rather than across cohorts.

Changes in PWB. A change in mood or global PWB reflects the respective impact of a change in the dI or Ip component of I, with the person parameter moderating the change on the index. Compared to persons more extreme, those with midrange values of M are anticipated to obtain a low perturbation index but a high transition index value in the relevant paradigms, hypotheses supported by the findings of Stones and Kozma (1989b). Persons with midrange M are also anticipated to be less susceptible to the onset of depression or cyclic affective disorders with changing environmental conditions. Unfortunately, few studies outside of the clinical area have examined the longitudinal effects of enduring environmental change. Studies that do so support the model with findings that a negative environmental shift has a greater impact on global PWB than positive change (Atkinson, 1982).

The model also has implications for the design of interventions aimed to promote global PWB. Because episodic change has an impact restricted to mood, enduring gains in PWB are anticipated only in response to an enduring positive shift in the prevailing environment, with the extent of gain being greater against a negative baseline. Two main principles have guided such implementation. First, the mastery and control over the environment can be enhanced by the acquisition of new skills and opportunities (Rodin, 1986). Second, the environment can be modified to impact more positively on the person (Sherman, 1987). Both principles have been used to good effect in clinical endeavours (Gatz *et al.*, 1985).

Relationships among indexes of PWB. Correlations among the three main types of index, global PWB scales, mood scales, and domain satisfaction indicators are anticipated because of a common influence due to M. Although correlational findings are sometimes interpreted to indicate the mediated effects of the environment on global PWB (e.g., Ip affects the domain satisfaction indicators that in turn affect global PWB), the model dictates that causal inferences based on the correlation matrix are inappropriate because M affects all categories of index.

Emotionality and PWB. Larsen and Diener (1987) describe a temperament of affect intensity that relates to PWB and its indicators in a puzzling way: no correlation was reported between global PWB and affect intensity, yet the latter correlated positively ($0.3 < r < 0.5$) with both positive and negative indicators of global PWB. These findings become comprehensible with the Affect Intensity Measure (AIM) interpreted as an absolute index of the deviation of M from the population central tendency estimate (i.e., $m = \text{ABS}\ [M - 2.2]$, where m represents the AIM). The m estimate is hypothesized to correlate at zero with PWB because high m is associated with both extremes of the global index. Furthermore, m is hypothesized to correlate positively with both the positive and negative indicators of PWB for the following reasons: the positive and negative indicators relate specifically to the positive and negative subregions of a global PWB scale (Costa and McCrae, 1984); persons with high scores in the positive subregion are expected to show elevation on both m and the positive indicator distribution; similarly, persons scoring high in the negative subregion are expected to have high levels on both m and the negative indicators; consequently, m is hypothesized to correlate positively with both the positive and negative indicators of PWB. Because the empirical findings are consistent with these expectations, the model clarifies our understanding of the relationship among mood reactivity, global PWB, and its indicators.

Operationalization of the model. A full operationalization of the model requires a procedure to estimate M and I, given that Hnext and H can be measured directly. The estimation of the prevailing environmental *impact* (Ip) and its change with intervention, has greater utility for clinical and applied purposes than the estimation of environmental change *per se*. M is estimated from the steady state value of Hnext in a neutral environment, such that $H\text{next} = H$ and $I = 0$. Under such conditions, Equation 2 can be written as:

$$H = M(1 - H)H,$$

$$1 = M(1 - H), \text{ and}$$

$$M = 1/(1 - H). \qquad (3)$$

A reliable index of global PWB (e.g., the MUNSH) can be used to estimate M because of its high test-retest stability over a prolonged interval and intransigence to episodic environmental impact (Kozma and Stones, 1983; Kozma *et al.*, 1990). The scaling of PWB for this purpose requires a transformation of the PWB scores to approximate the range of steady states that are tolerated by the model. These levels were previously stated to range from 0.09 to 0.66, corresponding to $M = 1.1$ and $M = 2.9$. Errors in the estimation of M with a normal population obtain from two sources: random error associated with the PWB scale, and nonsystematic deviations in Ip around a population mean of zero. On the assumption that the latter are uncorrelated over a prolonged time interval, the total error in estimating M is obtained from the test-retest stability of the scale (Stones, *et al.*, 1991). The MUNSH has a long-term stable variance of 70% (Kozma and Stones, 1983), indicating a total error of approximately 30% in estimating M with this instrument.

A modification to this procedure is required with samples believed to reside under nonzero prevailing environments (e.g., institution residents): the H estimates must be corrected to represent the levels postulated under a neutral environment. A simple correction is to adjust the denominator in the transformation of the raw PWB scores to proportionate distance estimates, such that the sample mean of M approximates the population central tendency estimate of 2.2 (Stones and Kozma, 1989b). An assumption made here is that the discrepancy in the mean PWB between the sample and the population reflects a difference in Ip rather than in the distribution of M. With M computed from the adjusted PWB estimates (Ha), the level of Ip pertaining to the unadjusted estimates (Hu) can be obtained:

$$Hu = M(1 - Hu)Hu + Ip,$$

$$Ip = Hu - M(1 - Hu)Hu,$$

$$Ip = Hu(1 - M + HuM) \tag{4}$$

Equation 4 has utility for evaluating the significance of the impact of the prevailing environment on a sample, and for testing the changing impact due to intervention in clinical and applied research. Because a neutral environment is associated with nonsystematic variation in Ip around a sample mean of zero, any overall discrepancy from the expected level can be evaluated by a t text. Similarly, the effects of intervention can be appraised from the change in Ip from before to after the intervention. An application of the latter type was reported by Stones and Kozma (1989b).

Environmental effects on mood (dI) can be estimated using a mood index, given a prior estimation of M and Ip based on a global PWB measure. The scaling of a mood index for this purpose requires a transformation of the raw scores such that the lower and upper limits of the useable range on the scale correspond to $Hm = 0$ and $Hm = 1$. With prior estimation of M and Ip, dI is obtained as follows:

$$Hm = M(1 - Hm)Hm + Ip + dI,$$

$$dI = Hm - M(1 - Hm)Hm - Ip, \text{ and}$$

$$dI = Hm(1 - M + HmM) - Ip \tag{5}$$

The dI estimate is used to evaluate the impact of episodic events. With no systematic deviation from a prevailing environment, $dI = 0$ and the Hm estimate from the mood index is expected to correspond to the H estimate from a global PWB scale. Systematic effects on mood are analyzed by the mean sample deviation of dI from zero, and changes in mood are inferred from systematic differences in dI at two times of measurement.

CONCLUSIONS

We began this article with a reiteration of Diener's (1984) plea for the development of more sophisticated models of PWB. Because the discussion of happiness for centuries was subsumed under the branch of learning known as moral philosophy, it is not surprising that the models now debated fall mainly within a verbal descriptive category, owing little debt to mathematics either in conception or description. Yet we have shown that simple a nonlinear model (Equation 2) can provide the propositional rigor that Diener (1984) advocates while addressing the complexity of the phenomena to be explained. We propose that the criteria against which the utility of Equation 2, or any other encompassing model of PWB, be evaluated include the following: the model should (1) permit empirical differentiation between the effects on PWB due to personality and the environment, (2) integrate the attributes of PWB within a unified framework (e.g., global PWB, mood, domain satisfactions, (3) reconcile paradoxes and puzzles in the existing literature, (4) permit application at the level of the single case, and (5) generate new and refutable hypotheses.

We have attempted to show how the model satisfies the first four of these criteria. The model brings an elegant resolution to some longstanding and current issues in the empirical literature on PWB (e.g., the respective contributions of personality and the environment, stability versus change, the relationships among different types of index, the relevance of emotionality). Furthermore, a powerful property is that the terms contained in model can be operationalized at the levels of the population, the sample, and the single case. Convergent evidence indicates a population central tendency estimate to the person parameter of approximately $M = 2.2$. Because M is related to the level on a global PWB index, the sample discrepancy in $I\mathrm{p}$ from a population estimate of $I\mathrm{p} = 0$ can be computed, and changes to the impact of the prevailing environment monitored in intervention research. Within specified limits of measurement error, the M, $I\mathrm{p}$, and $\mathrm{d}I$ terms can be estimated at the level of the single case, based on the scores obtained from a global PWB and a mood index. Application at this level evidences the potential viability of the model for clinical purposes.

We have dwelled little on risky hypotheses, the primary intent of this article being to specify the model and relate its properties to established phenomena. Yet risky but confirmed hypotheses were reported elsewhere, that we will mention in conclusion. Rattenbury and Stones (1989) reported gains in the global PWB of elderly nursing home residents after a series of eight group discussion sessions (i.e., compared to a no treatment group). A rationale for the success of this type of intervention is the facilitation of *I*p from the negative institutional level (Stones and Kozma, 1989a) due to enhanced peer support (Sherman, 1987). Hypotheses from the model are that persons with *M* near 2.2 are the least subject to episodic changes in mood (i.e., the d*I* shift within sessions) but respond the most readily to the total intervention (i.e., the *I*p shift). These hypotheses are discussed under Properties 4, 6, and 7. Stones and Kozma (1989b) reported that, compared to persons with extreme *M*, those with midrange values had the lower mood response within sessions but the greater *I*p shift over the full intervention. In the absence of the model, the researchers would have been hard put to explain why persons showing the lesser gain on a mood index had the greater gain in global PWB.

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