

# THE EFFECTS OF OROGRAPHY ON PRECIPITATION

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(Received in final form 20 January, 1995)

**Abstract.** In this paper, a model simulating the effects of topography and altitude on precipitation is presented. Topography has its maximum effect on precipitation when the angle  $\sigma$  which the wind makes with the slope direction approaches zero and the inclination  $\alpha$  of the slope is near  $45^\circ$ . The smaller the angle  $\sigma$ , the greater the influence of slope on precipitation. When  $\alpha < 45^\circ$ , the larger the inclination, the greater the influence of slope on precipitation and the less the difference in precipitation between the windward and the leeward slopes. When  $\alpha > 45^\circ$ , the reverse holds. But for  $\sigma$  in the range of  $0^\circ$ – $45^\circ$  and  $\alpha$  in the range  $30^\circ$ – $60^\circ$ , differences in precipitation on both the windward and leeward slopes are not so well marked and can be neglected in general. In condition of uniform slope inclination, precipitation on the windward slope increases with altitude at first and then decreases after attaining a height ( $H_m$ ) of maximum precipitation; also  $H_m$  is greater, the drier the air mass. When the terrain on the windward side is stepped in shape, it is possible that more than one height of maximum precipitation will occur.

## 1. Introduction

Precipitation over a mountainous region is complicated, and depends on local macroclimate, as well as topography and elevation. Many investigators (such as Storebo, 1976; Barry, 1981; Ozawa and Yoshino, 1965; Chanashva and Subotina, 1983; Carruthers and Choularton, 1983; Wang *et al.*, 1984; Fu, 1983; Li *et al.*, 1987; and Jiang, 1988) have tried to calculate the precipitation over a mountainous region by means of empirical, semi-empirical and theoretical methods, but most studies mainly refer to the relation between precipitation and altitude or to the mechanism of orographic rain. In this paper, we try to establish a practical mathematical model of how precipitation changes with topography and altitude based on theoretical and physical considerations.

## 2. A Model of the Distribution of Precipitation on the Windward Side of a Mountain

### 2.1. IN TERRAIN WITH UNIFORM SLOPE

Considered from a climatological point of view, the difference in precipitation between a mountainous region and flat ground in the same area is primarily due to the effect of orographic lifting. Hence, to begin, we seek to estimate the orographic lifting velocity. Suppose  $\theta$  is the dominant wind direction when rain occurs,  $\beta$  is the orientation of the windward slope, and  $\sigma = \theta - \beta$  is the acute angle between

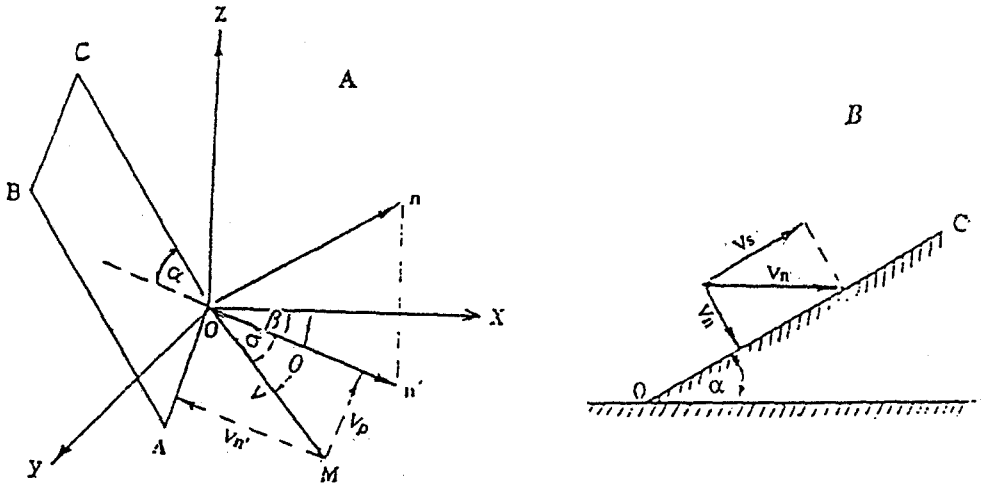


Fig. 1. Sketch map for calculating the orographic lifting velocity.

wind direction and slope orientation. As shown in Figure 1A, let the  $X$  axis point to the north, the  $Y$  axis to the east, and the  $Z$  axis to the zenith. Assume that the inclination of the slope is  $\alpha$ , and that  $\beta$  is the angle from  $ox$  to the projection  $on'$  on to a level plane, i.e., it is the slope orientation expressed by the azimuth calculated clockwise from north. Vector  $MO$  denotes wind velocity  $V$  while the component of  $V$  which is parallel to  $on'$  and which points toward the slope is  $V'_n = V \cos \sigma = V \cos (\theta - \beta)$ . The component parallel to the slope is  $V_p = V \sin \sigma$ . It is obvious that the slope does not affect the component velocity  $V_p$  blowing parallel to it. As shown in Figure 1B,  $V'_n$  can be decomposed into a component  $V_s$  along the slope and  $V_n$  perpendicular to the slope. Only the component  $V_s$  can produce ascending motion. This component is the orographic lifting velocity  $V_g$ , which can be expressed as

$$V_g = V_s \sin \alpha = V'_n \cos \alpha \sin \alpha = \frac{V}{2} \cos \sigma \sin 2\alpha. \quad (1)$$

When a precipitating air current ascends a slope, the cloud water content becomes less and less as a result of rainout, and rainfall intensity decreases with distance up the slope. However, orographic lifting also transports vapour upward from the air layer below the cloud base (or condensation level) increasing the moisture content in the cloud and hence the rainfall intensity. The net result depends on the combined effect of these two factors.

According to Magnus' formula, the relation between saturated vapour pressure  $E_s$  and temperature  $T$  ( $^{\circ}\text{C}$ ) can be expressed as

$$E_s = 4.6 \exp \left( \frac{17.15T}{235 + T} \right) (\text{mm}). \quad (2)$$

Let  $T_0$  denote the temperature at the foot of mountain and  $r$  the lapse rate of temperature, Then the saturated vapour pressure  $E_s$  at any height  $z$  above the foot of the mountain may be approximately written as

$$E_s = E_{s_0} \exp[-\mu_1 r z - \mu_2 (T_0 - r z) r z], \quad (3)$$

where  $E_{s_0}$  is the saturated vapor pressure at the foot of the mountain,

$$\mu_1 = \frac{17.15}{235 + T_0} \quad \text{and} \quad \mu_2 = \frac{17.15}{(235 + T_0)^2}.$$

When  $r = 0.6 \text{ }^\circ\text{C}/100 \text{ m}$ ,  $T_0 = 0\text{--}30 \text{ }^\circ\text{C}$  and  $z = 3000 \text{ m}$ , we have  $\mu_1 r z = 1.3136\text{--}1.1649$ ,  $\mu_2 (T_0 - r z) r z = 0.1006\text{--}0.0527$ ; thus  $\mu_1 r z \gg \mu_2 (T_0 - r z) r z$ .

In the free atmosphere, the variation of vapor pressure  $E$  with height  $z$  may be expressed by the following empirical formula:

$$E = E_0 e^{-cz}, \quad (4)$$

where  $E_0$  is the vapor pressure at the foot of mountain and  $c$  is a parameter.

Accordingly, the variation of relative humidity  $R$  with height can be written as follows:

$$\begin{aligned} R &= \frac{E}{E_s} = \frac{E}{E_s} \exp[(\mu_1 r - c)z + \mu_2 (T_0 - r z) r z] \\ &= R_0 \exp[(\mu_1 r - c)z + \mu_2 (T_0 - r z) r z] \end{aligned}$$

or

$$R \approx R_0 e^{(\mu_1 r - c)z}, \quad (5)$$

where  $R_0$  is the relative humidity at the foot of the mountain.

The condensation level  $Z_c$  at the foot of the mountain may be obtained by setting  $R = 1$  in Equation (5):

$$Z_c = \ln R_0^{-1/(\mu_1 r - c)}. \quad (6)$$

When air ascends in the region below the condensation level, its temperature decreases dry adiabatically, i.e., when air at height  $Z_0$  with temperature  $T = T_0 - r z$  ascends to height  $h$  (i.e., when it is lifted to height  $Z = Z_0 + h$ ), its temperature decreases by  $r_d h$  ( $r_d$  is the dry adiabatic lapse rate), and its temperature drops to  $T = (T_0 - r z_0) - r_d h = T_0 - r(z - h) - r_d h = T_0 - r z - (r_d - r)h$ . Therefore, according to Equation (2), its saturated vapor pressure will become

$$E_s \doteq E_{s_0} \exp[-\mu_1 r z - \mu_1 (r_d - r)h]. \quad (7)$$

The new condensation level when air ascends to height  $h$  is obtained by letting  $E = E_s$ :

$$Z'_c = \frac{\delta}{c} \ln R_0^{-1} - \frac{s}{c} h, \quad (8)$$

where

$$\delta = \frac{c}{\mu_1 r - c}, \quad s = \delta \mu_1 (r_d - r).$$

The vapor pressure at height  $Z'_c$  is

$$E_c = E_0 e^{-cz'_c} = E_0 R_0^\delta e^{sh};$$

hence the absolute humidity at height  $Z'_c$  can be obtained as follows:

$$a_c = a_0 R_0^\delta e^{sh}, \quad (9)$$

where  $a_0$  is the absolute humidity at the foot of mountain.

Partially differentiating Equation (8) with respect to time  $t$ , we can obtain the velocity of descent of the condensation level:

$$V_c = \frac{\partial z'_c}{\partial t} = -\frac{s}{c} \frac{\partial h}{\partial t} = -\frac{s}{c} V_g. \quad (10)$$

This corresponds to rising air below the condensation level. In fact, velocity  $V_z$  of ascending air is equal to  $V_c$  in absolute value, but is opposite in sign, namely

$$V_z = -V_c = \frac{s}{c} V_g. \quad (11)$$

Hence the flux density of moisture transported upward from the air layer below the condensation level (or cloud base) to the cloud as a consequence of the lowering of the condensation level is

$$F = a_c V_z = \frac{sa_0 R_0^\delta}{c} V_g e^{sh}. \quad (12)$$

But, after all the ascending air has reached saturation, and the cloud base has lowered to the ground of mountain slope, the moisture flux from the air layer below the condensation level is very small or near zero. Hence the general flux density  $Q(h)$  of vapour transported from the air layer under the condensation level to cloud on the windward slope should meet the following conditions:

$$\left. \begin{aligned} Q(h) &\rightarrow F & \text{as } h < h_* \\ Q(h) &\rightarrow 0 & \text{as } h > h_* \end{aligned} \right\} \quad (13)$$

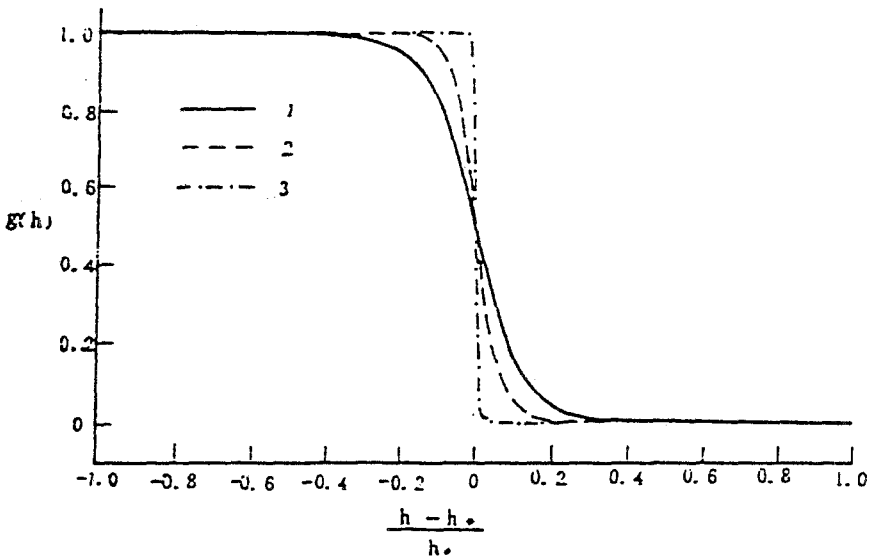


Fig. 2. Variation of the function  $g(h)$  with  $h$  (expressed by the nondimensional quantity  $(h-h_*)/h_*$ ) (assuming  $b = 1 \times 10^{-4} \text{ m}^{-1}$ ) 1,  $m = 100$ ; 2,  $m = 200$ ; 3,  $m = 2000$ .

Here  $h_*$  denotes the height at which the surface air over the slope reaches saturation. The result can be obtained by setting  $Z'_c = h$  in Equation (8) so that

$$h_* = \ln R_0^{-1} / (\mu_1 r_d - c). \tag{14}$$

In order to satisfy condition (13), we suppose

$$Q = F \cdot g(h) \tag{15}$$

where

$$g(h) = [1 + f e^{mb(h-h_*)}]^{-((1/m)+1)}, \tag{16}$$

where  $b$  and  $m$  are parameters to be determined. The dimensionless parameter  $f = 1$  (on the windward slope) or  $0$  (on the leeward slope) is introduced temporarily to distinguish between the windward and the leeward slopes.

Obviously, the function  $g(h)$  has the following characteristics as long as  $m$  is large enough (see Figure 2)

$$\left. \begin{aligned} g(h) &\rightarrow 1, \text{ as } h < h_* \\ g(h) &\rightarrow 0, \text{ as } h > h_* \end{aligned} \right\} \tag{17}$$

Hence function  $Q(h)$  expressed by Equation (15) satisfies condition (13).

Substituting the expressions for  $F$  and  $g(h)$  into Equation (15), we can obtain:

$$Q(h) = \frac{sa_0 R_0^\delta V_g}{c [1 + f e^{mb(h-h_*)}]^{(1/m)+1}} e^{sh}. \tag{18}$$

In precipitating cloud, the water expenditure per unit area of horizontal surface in unit time is the precipitation intensity  $P$  and the water income is the flux density  $Q(h)$  of vapour transported upward through the cloud base by orographic lifting. Thus, according to the law of mass conservation, the equation for the water budget in the cloud may be written as follows:

$$\frac{dw}{dt} = Q - P, \quad (19)$$

where  $dw/dt$  represents the changing rate of water content ( $W$ ) in a cloud column of unit cross-sectional area.

Considered from a climatological point of view, we may suppose that, in identical macroclimatic conditions, the ratio of precipitation intensity  $P$  to cloud water content  $W$  in a mountainous region and the ratio of  $P_0$  to  $W_0$  over flat ground are basically identical, namely

$$\frac{P}{W} = \frac{P_0}{W_0},$$

or

$$P = kW, \quad (20)$$

where  $k = P_0/W_0$  depends only on climatic conditions in the area; the larger the value of  $k$ , the more that climatic conditions favor precipitation. For given area and period,  $k$  may be considered as a constant and determined from observational data.

From Equation (20), we have

$$\frac{dw}{dt} = \frac{1}{k} \frac{dP}{dt}.$$

If  $t$  is the time taken for the airflow to climb the slope from the mountain foot to height  $h$ , since  $h = V_g t$ , we have

$$\frac{dP}{dt} = \frac{dp}{dh} \frac{dh}{dt} = V_g \frac{dp}{dh}.$$

Thus

$$\frac{dw}{dt} = \frac{V_g}{k} \frac{dp}{dh}. \quad (21)$$

Substituting (21) and (18) into (19), we get

$$\frac{dp}{dh} + \frac{k}{V_g} P - \left( \frac{k s a_0 R_0^\delta}{c} \frac{e^{sh}}{1 + f e^{mb(h-h_*)}} \right) = 0. \quad (22)$$

The solution of Equation (22) is

$$P = e^{-qh} \left\{ P_0 + M \left[ \frac{e^{sh}}{(1 + fe^{mb(h-h_*)})^{1/m}} - \frac{1}{(1 + fe^{-mbh_*})^{1/m}} \right] \right\}, \quad (23)$$

where

$$q = \frac{k}{V_g}, \quad b = q + s, \quad M = \frac{ksa_0R_0^\delta}{bc}. \quad (24)$$

Equation (23) shows that precipitation in a mountainous region depends on many factors: wind velocity  $V$ , wind direction  $\theta$ , slope orientation  $\beta$ , and inclination  $\alpha$  (these are contained in the orographic lifting velocity  $V_g$  expressed by Equation (1)), air temperature  $T_0$  (contained in constants  $\mu_1$ ,  $s$  and  $b$ ), absolute humidity  $a_0$  and relative humidity  $R_0$  at the foot of the mountain, height  $h$  and the distribution of temperature and water vapour in the free atmosphere (reflected in  $r$  and  $c$ ).

From (1) and (23) we find that the greater the wind velocity  $V$  and the smaller the angle  $\sigma (= \theta - \beta)$ , the greater the orographic lifting velocity  $V_g$  and hence the greater the precipitation intensity  $P$  on the windward slope; when  $\alpha$  increases,  $V_g$  and  $P$  increase for  $\alpha < 45^\circ$ , but decrease for  $\alpha > 45^\circ$ , so the maximum influence of the inclination of mountain slopes on  $V_g$  and hence  $P$  is displayed near  $\alpha = 45^\circ$ .

Equation (23) also shows that the moister the air (i.e., the higher the absolute humidity  $a_0$  and the relative humidity  $R_0$  of surface air at the foot of mountain), the greater the orographic enhancement of precipitation.

Differentiating partially Equation (23) with respect to  $h$ , and putting  $\partial P/\partial h = 0$ , we can obtain an equation to determine the height ( $h_m$ ) of maximum precipitation on the windward side as follows:

$$\begin{aligned} & \left[ 1 - \frac{b}{q[1 + fe^{mb(h_m-h_*)}]} \right] e^{bh_m} \\ & = \left[ \frac{1}{1 + fe^{-mbh_*}} - \frac{p_v}{M} \right] [1 + fe^{mb(h_m-h_*)}]^{1/m}. \end{aligned} \quad (25)$$

After further calculations, we can deduce that

$$\frac{\partial h_m}{\partial R_0} < 0,$$

which means that  $h_m$  decrease with increasing  $R_0$ , i.e. the greater the relative humidity at the foot of the mountain, the lower the height of maximum precipitation. When  $R_0 \rightarrow 1$ , the height of maximum precipitation will occur at the foot of the mountain.

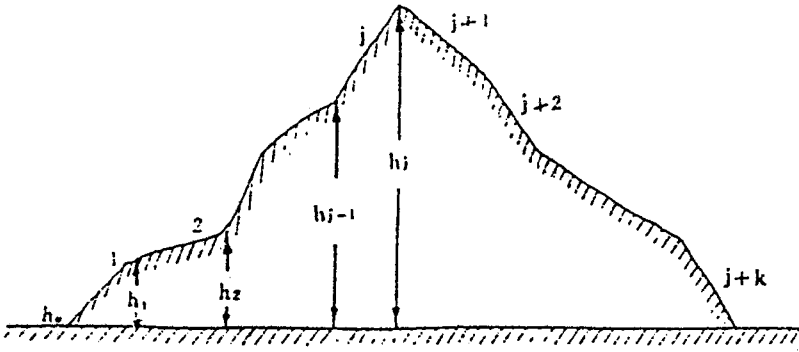


Fig. 3. Sketch map of topographic variation with elevation.

2.2. IN TERRAIN WITH VARYING SLOPE

When the orientation and inclination of terrain changes with height, the distribution of precipitation with elevation may be derived as follows:

As shown in Figure 3, if the average orientation ( $\beta$ ) and inclination ( $\alpha$ ) of the terrain from elevation  $h_0 (= 0)$  to  $H_1$  (layer 1) are  $\beta_1$  and  $\alpha_1$  (where  $V_g = V_{g1}$ ), from  $h_{i-1}$  to  $h_i$  are  $\beta_i$  and  $\alpha_i$  (where  $V_g = V_{gi}$ ), where  $h_{i-1}$  and  $h_i$  are the lower and upper heights of layer  $i$ , respectively, the solution of Equation (23) for evaluating the precipitation  $P$  at an arbitrary height  $h$  in layer  $n$  will be:

$$P = \exp \left[ - \sum_{i=1}^n q_i (h_i - h_{i-1}) \right] \left[ P_0 + \sum_{i=1}^n M_i (X_i - Y_i) \right], \tag{26}$$

where

$$X_i = \frac{e^{b_i h_i}}{[1 + f e^{m b_i (h_i - h_*)}]^{1/m}}, \quad Y_i = \frac{e^{b_i h_{i-1}}}{[1 + f e^{m b_i (h_{i-1} - h_*)}]^{1/m}}, \tag{27}$$

$$q_i = k/V_{gi}, \quad b_i = q_i + S, \quad M_i = S k a_0 R_0^\delta / b_i c, \tag{28}$$

with  $h_0 = 0, h_n = h$ .

As  $a_0, R_0, s$ , and  $h_*$  are dependent only on atmospheric conditions and not on topography, in identical climatic conditions they may be regarded as constants; orographic factors then affect the lifting velocity  $V_g$  and parameters  $b, q, M$ , which again relate to  $V_g$ . Hence, in identical climatic conditions, if the parameters  $b_k, q_k, M_k$  and  $h_{*k}$  for given terrain  $K$  (or certain  $V_{gk}$ ) are known, the corresponding



parameters  $b_i, q_i, M_i$  and  $h_{*i}$  for other terrain  $i$  (or other  $V_{gi}$ ) can be deduced from Equation (28)

$$\left. \begin{aligned} q_i &= \frac{V_{sk}}{V_{gi}} q_k = \frac{L_k}{L_i} q_k \\ b_i &= b_k - q_k + q_i = b_k - q_k + \frac{L_k}{L_i} q_k \\ M_i &= \frac{b_k M_k}{b_i} = b_k M_k / \left( b_k - q_k + \frac{L_k}{L_i} q_k \right) \\ h_{*i} &= h_{*k} = h, \end{aligned} \right\} \quad (29)$$

where  $L_j = \cos \sigma_j \sin 2\alpha_j$  ( $j = i, k$ ).

If  $P_{n-1}$  is the precipitation at the lower height  $h_{n-1}$  in layer  $n$ ,  $t_i = (h_i - h_{i-1})/V_{gi}$  is the time taken for the air to cross layer  $i$ , and

$$\begin{aligned} \tau_{n-1} &= t_1 + t_2 + \dots + t_{n-1} \\ &= \frac{h_1 - h_0}{V_{g1}} + \frac{h_2 - h_1}{V_{g2}} + \dots + \frac{h_{n-1} - h_{n-2}}{V_{g(n-1)}}, \end{aligned} \quad (30)$$

is the total time for the air to ascend the mountain from its foot to the base of layer  $n$ , as

$$\sum_{i=1}^n q_i (h_i - h_{i-1}) = \sum_{i=1}^n k \frac{h_i - h_{i-1}}{V_{gi}} = K \tau_{n-1} + q_n (h - h_{n-1}),$$

the following recurrence formula can be deduced from Equation (26)

$$P = e^{-q_n(h-h_{n-1})} [P_{n-1} + M_n (X_n - Y_n) e^{-K \tau_{n-1}}]. \quad (31)$$

### 3. A Model of Precipitation on the Lee Side of a Mountain

On the lee side, as the air descends down the slope of mountain, the vertical velocity caused by orography is negative. If the inclination  $\alpha'$  of the lee side is regarded as negative, the descending speed of the air current on the lee side can be expressed by Equation (1), namely

$$V'_g = \frac{V}{2} \cos \sigma \sin(-2\alpha') = -\frac{V}{2} \cos \sigma \sin 2\alpha' = -V_g. \quad (32)$$

On the lee side, due to adiabatic warming, the condensation level rises (i.e., the cloud base rises). Vapour is, therefore, transported downward through the cloud base. In the case of  $h_t < h_*$ , when the surface air is not saturated, the vapour

flux  $Q$  transported downward through the cloud base on the lee side may still be expressed by Equation (12) in which the ascending speed  $V_g$  should be replaced by the descending speed  $V'_g$ , and  $f = 0$ . Therefore, the precipitation on the lee side can still be calculated by using Equation (26) or Equation (31). When  $h_t > h_*$ , as the cloud base between heights  $h_*$  and  $h_t$  on the windward slope has fallen to the ground, Equation (7) expressing the relation between the saturated vapour  $E_s$  at the condensation level and the height  $h$  is invalid. The vapour flux transported downward through the bottom of the cloud on the lee slope obviously can not be expressed by Equation (12). In this case, however, since the change in air temperature below the condensation level is dry adiabatic when the air descends, the descending air on the lee side may be thought of as the opposite of airflow ascending and the vapour flux transported downward through the bottom of the cloud during descent can be thought of as the reflection of that transported upward during ascent. Therefore, using expression (12) for the vapour flux transported by ascending airflow, we can get an expression for vapour flux transported downward through the bottom of cloud as follows:

$$Q' = F' = -\frac{s'a'_0R_0^{\delta'}}{c'}V_g e^{S'h} = \frac{s'a'_0R_0^{\delta'}}{c'}V'_g e^{S'h},$$

where  $a'_0$  and  $R'_0$  are the absolute and relative humidities of surface air at the foot of the mountain on the lee side, respectively,  $s'$ ,  $c'$  and  $\delta'$  are parameters related to the vertical distributions of temperature and humidity in the free atmosphere at the foot of the mountain on the lee side. Thus, when  $h_t > h_*$ , we can obtain a formula for calculating the precipitation at an arbitrary height on the lee slope as follows:

$$P' = \left[ P_0 + \sum_{i=1}^n M_i(X_i - Y_i) \right] \exp \left[ -\sum_{i=1}^n q_i(h_i - h_{i-1}) \right], \quad (33)$$

or

$$P' = e^{-q_n(h-h_n)} [P_{n-1} + M_n(X_n - Y_n)e^{-K\tau_{n-1}}], \quad (34)$$

where

$$X_i = \begin{cases} e^{b_i h_i} [1 + e^{m b_i (h_i - h_*)}]^{-1/m}, & \text{as } i \leq j (\because f = 1) \\ e^{b'_i h_i}, & \text{as } i > j (\because f = 0) \end{cases},$$

$$Y_i = \begin{cases} e^{b_i h_{i-1}} [1 + e^{m b_i (h_{i-1} - h_*)}]^{-1/m}, & \text{as } i \leq j \\ e^{b'_i h_{i-1}}, & \text{as } i > j \end{cases},$$

$$M_i = \begin{cases} k s a_0 R_0^\delta / b_i c, & \text{as } i \leq j \\ k s' a'_0 R_0^{\delta'} / b'_i c', & \text{as } i > j \end{cases},$$

$$b'_i = q_i + S',$$

where  $j$  is the total number of terrain layers on the windward side.

If the terrain on both sides of the mountain is rather uniform after appropriate smoothing, an average orientation and inclination can be defined, being  $\beta$ ,  $\alpha$  on the windward side, and  $\beta'$ ,  $\alpha'$  on the lee side; the height of the mountain top is  $h_t$  on average, over which the mean precipitation is  $P_t$ . The general formula for calculating the precipitation on the slope may be written as follows:

$$P' = e^{-q'(h-h_t)} [P_t + \hat{M}_b (e^{\hat{b}h} - e^{\hat{b}h_t}) e^{-qh_t}], \quad (35)$$

$$P_t = e^{-qh_t} \left[ P_0 + M_b \left( \frac{e^{bh_t}}{[1 + e^{mb(h_t-h_*)}]^{1/m}} - \frac{1}{[1 + e^{-mbh_*}]^{1/m}} \right) \right],$$

where

$$\hat{M} = \begin{cases} ksa_0 R_0^s / bc & h_t \leq h_* \\ ks'a'_0 R_0^{s'} / b'e' & h_t > h_* \end{cases},$$

$$\hat{b} = \begin{cases} b & h_t \leq h_* \\ b' & h_t > h_* \end{cases}.$$

#### 4. Applications

We take July precipitation data in the Qinling Mountains area in Shanxi province as an example to study precipitation patterns in a mountainous area and to test our models.

The Qinling Mountains run east to west. For the whole mountainous area, the average terrain inclination  $\alpha$  is  $1.2^\circ$  in the south and  $2.7^\circ$  ( $\alpha' = -2.7^\circ$ ) in the north. In July, the prevailing wind direction is southeast ( $\theta = 135$ ); the average wind velocity  $V$  on the windward side ( $\beta = 180$ ) is 1.4 m/s. Thus, from Equation (1), we get  $V_g = 0.0311$  m/s. According to the observations for July 1961–1970, we determine  $b = 1.7 \times 10^{-4} \text{ m}^{-1}$ ,  $q = 1.6 \times 10^{-4} \text{ m}^{-1}$ ,  $h_* = 1060$  m,  $M = 329.6$  mm/month;  $b' = -0.976 \times 10^{-4} \text{ m}^{-1}$ ,  $q' = -1.067 \times 10^{-4} \text{ m}^{-1}$ , and  $M' = -515.2$  mm/month. Thus, by using Equations (23) and (35), the variation of precipitation with elevation in July on the southern and northern sides of the Qinling Mountains can be computed; see Figure 4. The theoretical curves are in good agreement with observations. As the mean height of the ridge in the area where the observations were made is lower on the northern than on the southern side, curve 2 does not cross curve 1 at the top of the figure.

In the following Figures 5–7, we show families of curves of the precipitation ratio  $P/P_0$  as a function of  $\sigma$ ,  $\alpha$  and height on the windward and leeward sides.

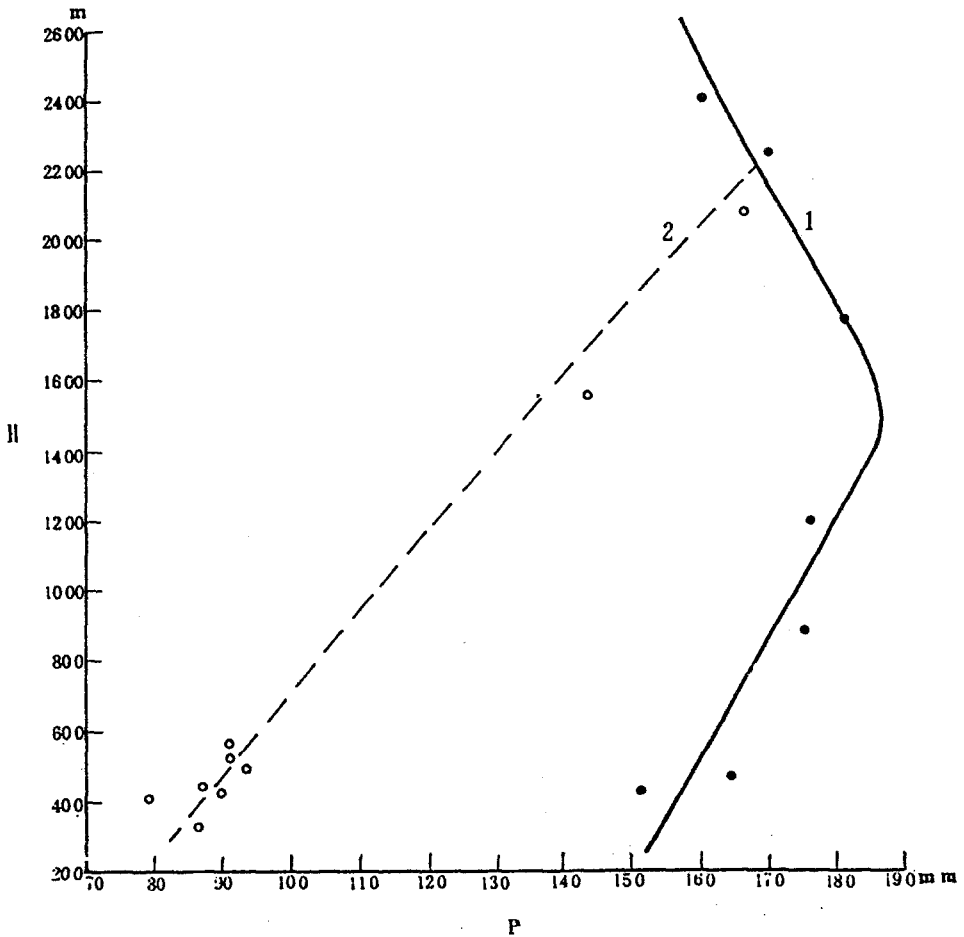


Fig. 4. Variation with elevation ( $H$ ) of precipitation ( $P$ ) on both sides of the Qinling Mountains in July. 1, Computed curve on the southern side; 2, Computed curve on the northern side. The dots and open circles denote observed data.

In these figures, too many variables are involved to test model performance with observation.

Figure 5 shows the variation of precipitation with altitude and inclination on both sides of a mountain in the Qinling Mountains area in July for  $h_t = 3000$  m. The precipitation on the windward side is more than on the lee side, and the difference becomes larger toward the foot of the mountain. When the inclination changes when  $\alpha < 45^\circ$ , precipitation on both the windward and the lee sides increases with increasing  $\alpha$ , and the reverse holds when  $\alpha > 45^\circ$ . When  $\alpha < 45^\circ$ , the smaller the  $\alpha$ , the greater the variation of precipitation with  $\alpha$  and the larger the difference in precipitation between the windward and the lee sides, but it is just the reverse when the inclination is above  $45^\circ$ . However, the differences in

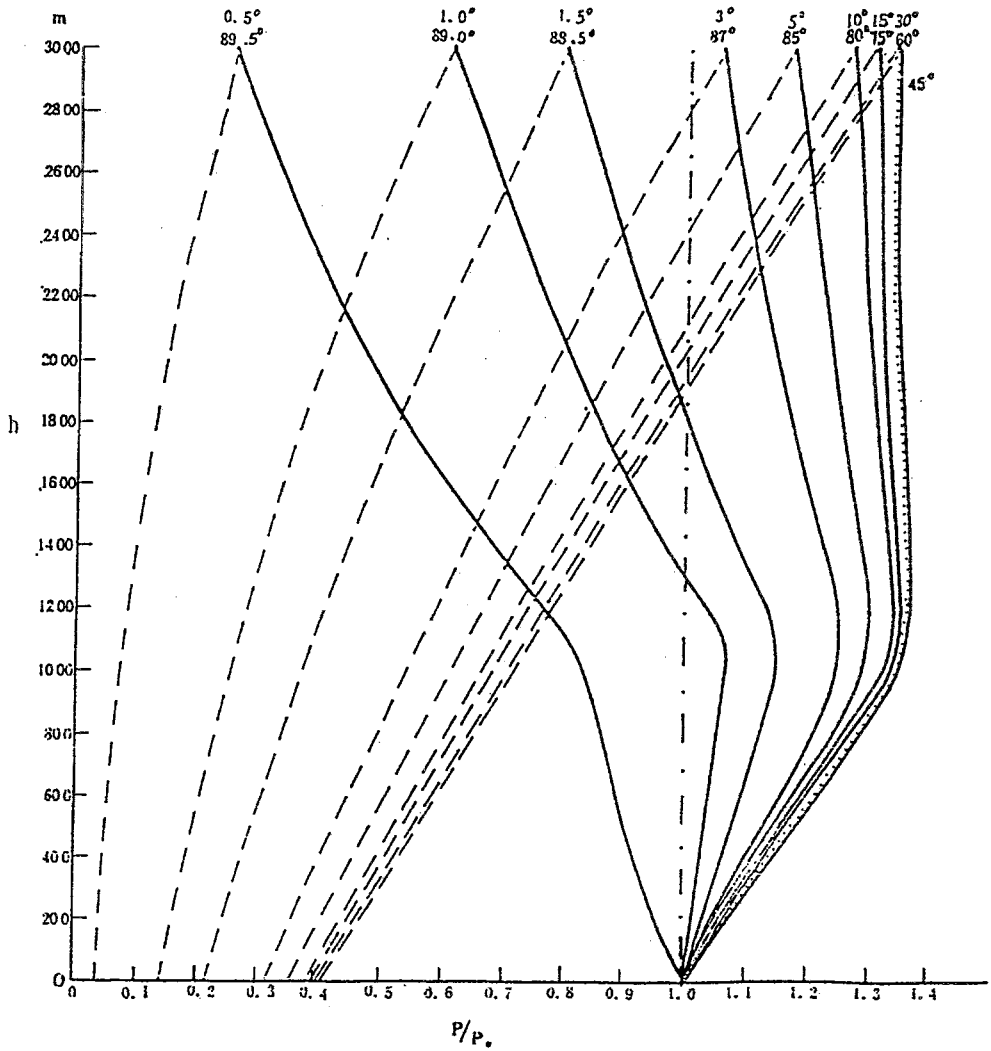


Fig. 5. Variation of the precipitation ratio  $P/P_0$  with relative height  $h$  on the windward side (solid and dotted lines) and the lee side (dash lines) at different inclinations under the climatic regime of the Qinling Mountains area in July. It is assumed that the inclinations of both sides are identical.

precipitation on both the windward and the lee slopes with  $\alpha$  in the range  $15^\circ-75^\circ$ , especially in the range  $30^\circ-60^\circ$  are small. From Figure 5 we can also see that if the terrain slope is rather uniform, the precipitation on the windward side increases, in general, with increasing altitude at first, and then decreases after reaching a height of maximum precipitation. On the lee side, however, precipitation always increases with increasing altitude.

Figure 6 shows the July distribution of precipitation with height on both the windward and the lee sides for different mountain heights when the inclination

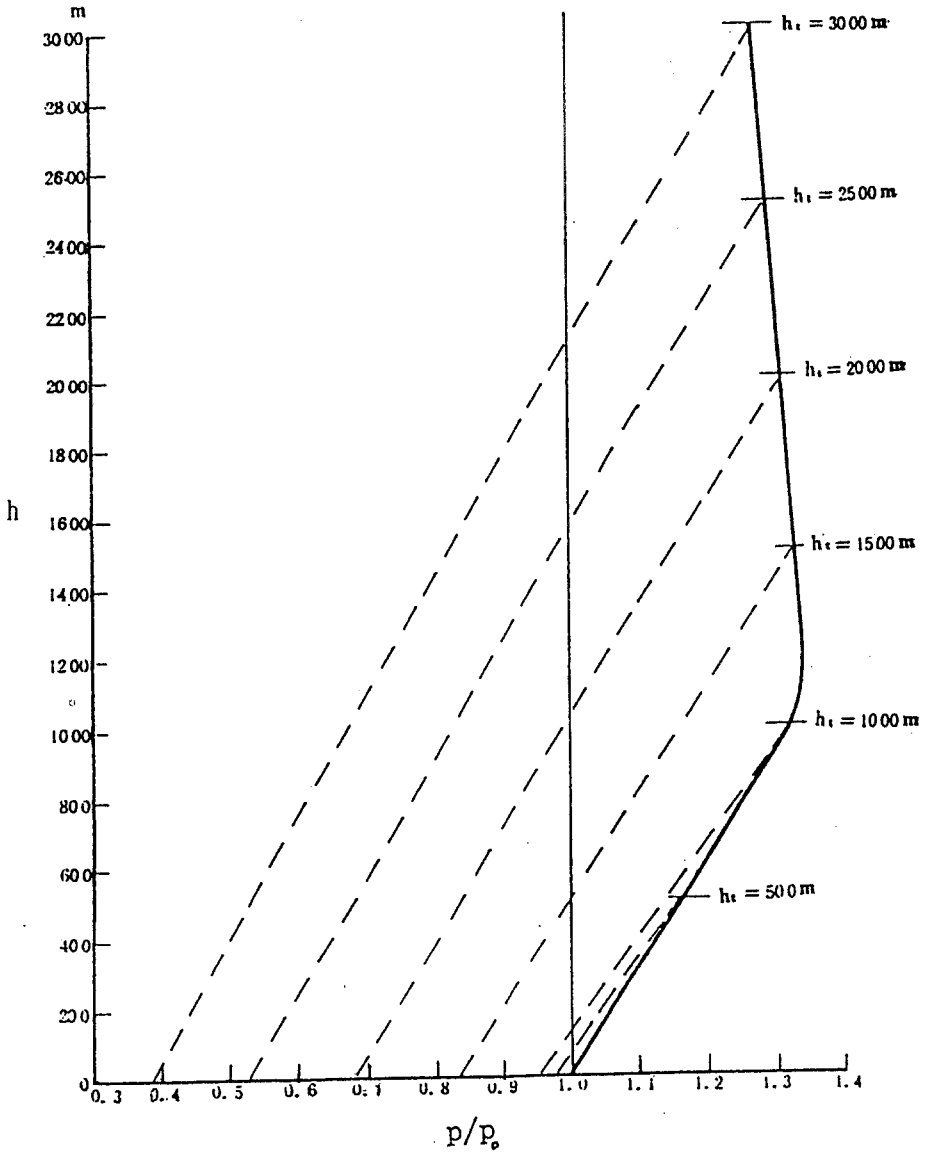


Fig. 6. Variation of precipitation ratio  $P/P_0$  with height  $h$  on the windward side (solid lines) and the lee side (dashed lines) for an inclination of  $10^\circ$ , and different mountain heights  $h_t$  under the climatic regime of the Qinling Mountain area in July.

of both sides is  $10^\circ$ . The figure shows that precipitation on the lee side is less, the higher the mountain height ( $h_t$ ), but the difference between the lee and the windward sides is far smaller when  $h_t < h_m$  than when  $h_t > h_m$  (the height of maximum precipitation).

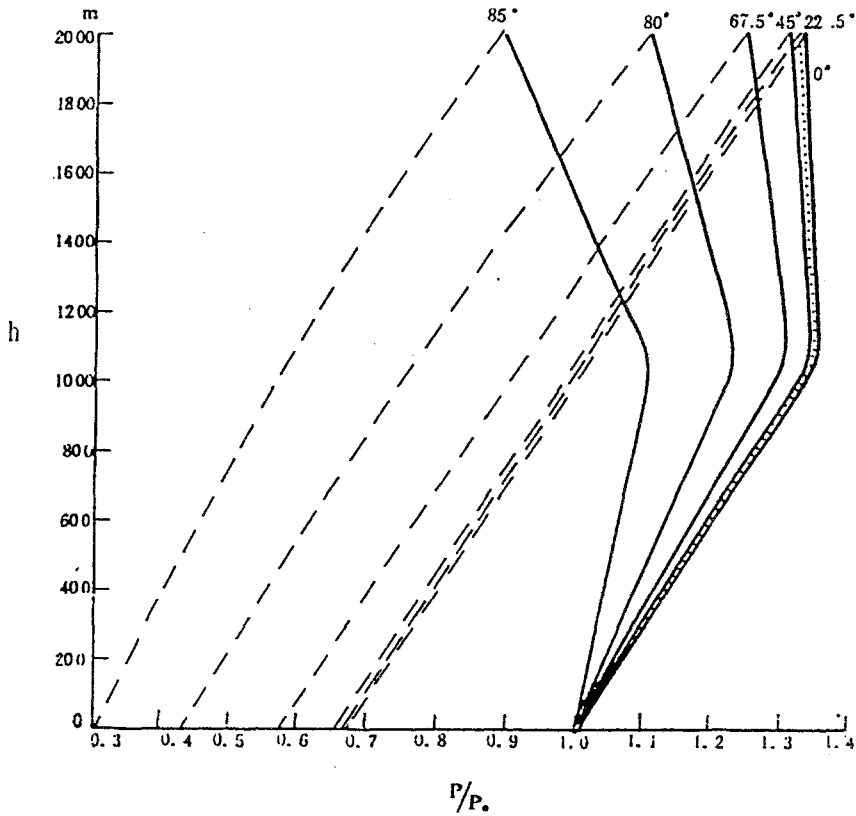


Fig. 7. Variation of precipitation ratio  $P/P_0$  with height  $h$  and angle  $\sigma$  under the climatic regime of the Qinling Mountains area in July for  $\alpha = 10^\circ$  and  $h_t = 2000$  m (the legend is the same as in Figure 5).

Figure 7 shows the variation of precipitation with height  $h$  and angle  $\sigma$  on both the windward and leeward sides for a terrain inclination  $\alpha = 10^\circ$  and mountain height  $h_t = 2000$  m. It follows from this figure that the precipitation on both the windward and leeward sides increases with decreasing  $\sigma$ , the difference in precipitation on both sides being less, the smaller the angle  $\sigma$ . When  $\sigma$  changes in the range of  $0^\circ$ – $45^\circ$ , however, the variations of precipitation on the two sides are very small.

### 5. An Example of Complex Terrain

In order to show the distribution of precipitation in complex terrain, we again take the east slope of Taihang Mountain as an example. Taihang Mountain is located to the west of the northern China Plain; its east slope has rugged topography but after smoothing, can be divided into three main terrain regions: the first region ( $i = 1$ ) lies between 300 and 1200 m altitude with a mean inclination  $\alpha_1 = 1.7^\circ$ ; the second

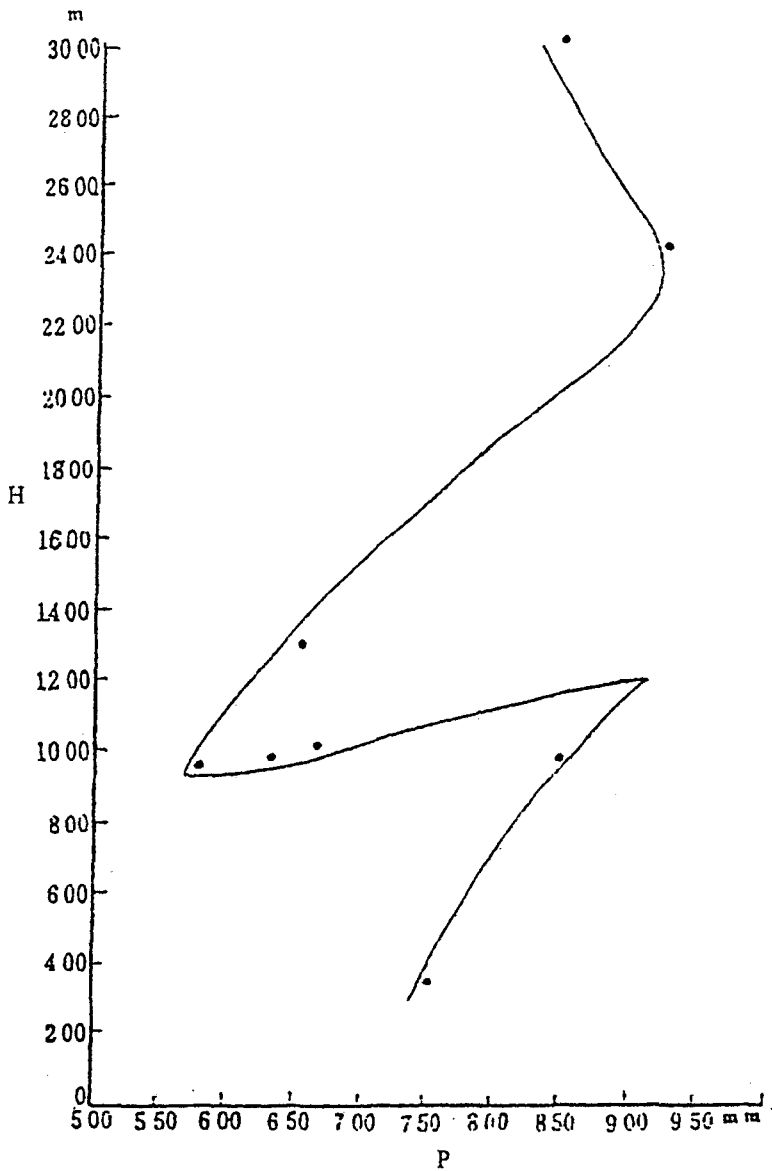


Fig. 8. Variation of annual precipitation  $P$  with elevation  $H$  on the east slope of the Taihang Mountain (theory and observations).

region ( $i = 2$ ) between 1200 and 950 m is a great plateau with mean inclination  $\alpha_2 = -0.6^\circ$ ; and the third region ( $i = 3$ ) lies between 950 and 3000 m with mean inclination  $\alpha_3 = 4.1^\circ$ . The prevailing wind here is southeast, with mean velocity  $V$  of 4 m/s. Consequently the east slope is a windward one with  $\sigma = 45^\circ$ , and it follows from Equation (1) that  $V_{g1} = 0.084$  m/s,  $V_{g2} = -0.029$  m/s,  $V_{g3} = 0.202$  m/s.



Making use of local observations, it is determined that  $h_* = 2040$  m,  $m = 22$ ,  $b = 9.81 \times 10^{-4} \text{ m}^{-1}$ ,  $q = 5.12 \times 10^{-4} \text{ m}^{-1}$  and  $M = 500$  mm/year. Thus the distribution of annual precipitation on the east slope of the Taihang Mountain can be computed using formula (26), which is shown by a solid line in Figure 8. For comparison, the observational data (Guo, 1981) are given in the figure.

Figure 8 shows that the computed results are in full agreement with the observations. Because of the level plateau existing half way up the mountain, the distribution of precipitation on the east slope of the Taihang Mountain is quite unique, with two precipitation maxima appearing at 1200 and 3000 m and a precipitation minimum at 950 m.

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