INTRODUCTION TO OBUKHOV'S PAPER ON 'TURBULENCE IN AN ATMOSPHERE WITH A NON-UNIFORM TEMPERATURE'*

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In 1943 at the age of 25, A. M. Obukhov finished the remarkable paper that follows this brief introduction. Because World War II ravaged around the globe at that time this manuscript was not published until 1946. Unfortunately, however, it appeared in the very limited first issue of the Journal: *Trudy Instituta Teoreticheskio Geofiziki AN SSSR* (Works of the Institute of Theoretical Geophysics, Acad. Sci. USSR, No. 1). So without the aid of modern duplicating techniques, the paper was doomed to obscurity right from the start. Very few scientists outside the Soviet Union were aware of it and even in the U.S.S.R. not many realized its real importance.

At present, nearly 25 yr after its publication, most of the information it contains is known to the scientific community, partly from the original source and partly by independent rediscovery. Nevertheless, it seems fully justified to give this paper new exposure because it is a truly classical contribution and every serious student of the atmospheric boundary layer should have the opportunity to study it.

Probably the major contribution of the paper is the introduction of the 'length scale of the dynamic turbulence sublayer', L_1. This length scale was later introduced independently by Lettau (1949), and at present it is commonly known as the Monin-Obukhov length. Its fundamental role in the whole field of boundary-layer meteorology was most clearly explained in the well-known paper by Monin and Obukhov (1954). In this last paper the presentation is based on purely dimensional considerations which imply that the mean wind and temperature profiles must be determined by some universal function of the uniquely defined (to within a numerical factor) dimensionless height $\zeta = z/L_1$. (Let us note that Obukhov included the numerical factor $1/k$, where k is the von Karman constant, in the definition of L_1 which does not follow from dimensional considerations. The same factor was also included in the definition of length scale by Monin and Obukhov in 1954 and is used in practically all subsequent works although its presence produces some difficulties in comparison of the data

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from various sources.) Obukhov's 1946 paper differs considerably from the paper by Monin and Obukhov, because it presents mainly a semi-empirical approach; nevertheless the idea of describing the effect of thermal stratification by some universal functions of the dimensionless parameters *Ri* or ζ is already well developed.

The second important achievement of this paper is connected with the study of the asymptotic states of boundary-layer turbulence in the case of strongly stable or strongly unstable flows. It was shown for the first time that if the stratification is stable then the wind and temperature profiles are approximately logarithmic at small heights $z \ll L_1$. However, at sufficiently large heights $z \gg L_1$ both of them are approximately linear. The corresponding shape of the profiles is described by a special interpolation formula which we shall discuss below. The simplest interpolation equation with the correct asymptotic behaviour is provided in the stable case by the log-linear relation introduced by Monin and Obukhov (1954). It has since become known that this last relation fits the data quite well in the stable range (Webb, 1970; Businger *et al.*, 1971).

The asymptotic state of turbulence in an unstably stratified boundary layer was in fact given in an early remarkable paper by Prandtl (1932) which was considerably ahead of its time and was forgotten in the early forties. Obukhov obtained the same result with the aid of the interpolation equation proposed by him. Namely, he pointed out that the asymptotic behaviour of this equation, i.e., in the free convection regime, requires that

$$\frac{\partial \theta}{\partial z} \propto z^{-4/3}, \qquad \frac{\partial u}{\partial z} \propto z^{-4/3}$$

which can be easily verified with Obukhov's Equation (40): $K(z) \propto z^{4/3}$. The temperature profile in the free convection regime was also independently obtained by Priestley (1954). Observations of truly free convection conditions are still very scarce and experimental confirmation of the Prandtl-Obukhov-Priestley predictions is until now somewhat in doubt (Dyer, 1965; Elliot, 1966; Businger *et al.*, 1971). A careful study of the free convection case in the atmosphere is still needed. Deardorff's (1970) recent numerical experiments may give new insights into this problem.

Finally Obukhov proposed in the 1946 paper an interpolation formula which embraces the entire range of stabilities. The functional form is the same as the so-called KEYPS formula. (The word KEYPS is formed by the initials of a number of authors who subsequently proposed and justified this formula; see Lumley and Panofsky, 1964. However, all these authors published long after Obukhov so that O'KEYPS may be more appropriate.) This equation is usually written in the form

$$\phi^4 - \gamma \zeta \phi^3 = 1$$

where $\phi = kz/v_* \, \partial v/\partial z$, $\zeta = z/L_1$, and $\gamma = \text{const.}$ (For further information concerning the symbols see list of symbols at the beginning of Obukhov's paper.) It is not difficult to transform Obukhov's Equation (39) to the same form.

Obukhov assumes that the ratio of eddy transfer coefficients K_M/K_T can be considered in the first approximation as a constant, different from unity but independent of stability, whereas Lumley and Panofsky assume this constant to be unity and Zilitinkevich and Chalikov (1968) present even some data in support of Lumley and Panofsky's assumption. However, there is presently considerable evidence that this ratio is a fairly strong function of stability for unstable conditions (Swinbank, 1964; Laikhtman and Ponomareva, 1969; Businger *et al.*, 1971). This fact clearly does not influence the wind velocity profile if it is considered as an interpolation equation with proper asymptotic behavior. It also does not change significantly the semi-empirical derivation of the profile equation although some of Obukhov's formulations must be slightly transformed when K_M/K_T is not constant (See Monin and Yaglom, 1971, subsection 7.4). However, wind and temperature profiles are not similar when K_M/K_T is a function of stability and hence in this case both of them can not be simultaneously described by the same equation. The possibility of the simultaneous proportionality of wind and temperature gradients to $z^{-4/3}$ requires only that K_M/K_T tends to a constant in the free convection regime. There are no quite satisfactory data at present to show whether this last statement is valid in the lower atmosphere or not.

An effort has been made to make the English translation follow the original as closely as possible, except when obvious misprints were encountered. A few footnotes are added where some additional information may be helpful, especially references.

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