A COMPARISON OF TWO METHODS OF SOLUTION OF THE PROBLEM OF OBTAINING THE SPECIFIED CARBON DISTRIBUTION IN THE MINIMUM TIME

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Mathematical models of the gas carburizing process are used at present for calculation of the optimum condition providing, for example, the minimum process time, the best approach to the desired carbon distribution, etc. [1-9].

Such optimal problems were accurately formulated for the first time in [3].

Subsequently solutions of a number of problems obtained with use of the method of successive linearization were published [4-6]. In foreign works [8, 9] a basically different approach based on approximation of the original model of teh process by a polynomial has been used. The purpose of this work is to compare the above two approaches to calculation of the optimum carburizing conditions using the specific examples given in [9]. This is even more necessary since from the reference in [9] on our work [4] it follows that the author of [9] obviously underestimates the broad capabilities and effectiveness of the method used by us.

Let us present the formulation of the problem solved in [9]. This concerns the minimum total time for a two-stage carburizing cycle providing the required carbon concentration gradient in a steel plate. The center of the plate must contain more carbon than the surface (such a profile is characteristic of steel strip for the production of blades). To obtain the specified carburization profile a standard two-stage (impregnation + diffusion) carburizing process is used. The following process parameters are subject to selection: the carbon potential of the atmosphere C_{atm1} and C_{atm2}; the temperature t_1 and t_2; the time τ_1 and τ_2 (the subscript 1 refers to the impregnation stage and the subscript 2 to the diffusion stage). The values of all of the parameters are limited:

$$\begin{aligned} &0.5 \leqslant C_{atm} \leqslant 1.5, \\ &830\ ^\circ\mathrm{C} \leqslant t_i \leqslant 1000\ ^\circ\mathrm{C}, \\ &0 \leqslant \tau_1 \leqslant 90 \quad \text{min} \\ &0 \leqslant \tau_2 \leqslant 30 \quad \text{min}. \end{aligned} \tag{1}$$

Here and subsequently the specific data of [9] is given.

It is required to select the values of the six treatment parameters restricted by conditions (1) so that the carbon contents on the surface and in the center of the plate are equal to the specified contents C^{spec}_{surf} and C^{spec}_{cent} and the total time of the process $\tau_0 = \tau_1 + \tau_2$ is a minimum.

The mathematical model of the carburizing process has the form:

$$\begin{aligned} &\frac{\partial C(x,\tau)}{\partial \tau} = D\frac{\partial^2 C(x,\tau)}{\partial x^2}, \\ &C(x,0) = C_0, \\ &D\frac{\partial C(x,\tau)}{\partial x}\Big|_{x=\delta} = K[C_{atm}(\tau) - C(\delta,\tau)], \\ &D\frac{\partial C(x,\tau)}{\partial x}\Big|_{x=-\delta} = -K[C_{atm}(\tau) - C(-\delta,\tau)], \\ &0 \leqslant \tau \leqslant \tau_0,\ -\delta \leqslant x \leqslant \delta, \end{aligned} \tag{2}$$

where $C(x, \tau)$ is the carbon content at the point x at the moment of time τ, δ is half the plate thickness, C_0 is the initial carbon content in the steel, D is the diffusion coefficient of the carbon, and K is the coefficient of mass transfer of carbon to the surface.

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TABLE 1

Example	$C_{atm\,1}$	$C_{atm\,2}$	t_1	t_2	τ_1	τ_2	C_{surf}	C_{cent}
	%		°C		min		%	
1	1,1	0,6	955	955	90	10	0,64314 0,63959	0,64548 0,64108
2	1,4	0,6	955	845	90	30	0,74770 0,75164	0,77796 0,78447
3	1,4	0,6	955	955	90	30	0,74489 0,73982	0,76987 0,76228

Explanation. The upper figures are C_{surf} and C_{cent} obtained in [9] and the lower obtained in this work.

TABLE 2

Example	$C_{atm\,1}$	$C_{atm\,2}$	t_1	t_2	τ_1	τ_2	C_{surf}	C_{cent}
	%		°C		min		%	
1	0,98	0,68	997,9	977,2	86,6	5,1	0,66765	0,65102
2	1,34	0,50	999,9	897,6	74,4	7,2	0,78957	0,81346
3	1.47	0.55	963.4	971.9	78.3	4.7	0.78297	0.79473

TABLE 3

Example	$C_{atm\,1}$	$C_{atm\,2}$	t_1	t_2	τ_1	τ_2	C_{surf}	C_{cent}
	%		°C		min		%	
1	1,5	0,5	1000	1000	40,75	3,99	0,64457	0,64450
2	1,5	0,5	1000	885	57,15	12,45	0,74631	0,77756
3	1.5	0.5	1000	905	55.8	9,9	0,74630	0,76958

TABLE 4

Example	Initial conditions			Conditions obtained in [9]			Conditions obtained in this work		
	τ_0, min	$\lvert\Delta C_S\rvert$, %	$\lvert\Delta C_C\rvert$, %	τ_0, min	$\lvert\Delta C_S\rvert$, %	$\lvert\Delta C_C\rvert$, %	τ_0, min	$\lvert\Delta C_S\rvert$, %	$\lvert\Delta C_C\rvert$, %
1	100	0	0	91,7	0,02451	0,00554	44,74	0,00143	0,00098
2	120	0	0	81,6	0,04187	0,0355	69,60	0,00139	0,00041
3	120	0	0	83,0	0.03808	0,02484	65,70	0,00141	0,00031

The relationship of the coefficients D and K to temperature have the forms:

$$D=D_0 \exp(-Q_D/RT), \quad K=K_0 \exp(-Q_K/RT),$$

where R is the universal gas constant, Q_D and Q_K are the activation energies, and T is absolute temperature.

The numerical values of the parameters are: $2\delta = 0.05$ cm; $C_0 = 0.18\%$ (16NC6 steel); $K_0 = 0.019$ cm/sec; $D_0 = 7.92$ cm^2/sec; $Q_K = 87{,}504$ J/mole; $Q_D = 175{,}008$ J/mole; $R = 8.32022$ J/(mole·K).

In [9] Eq. (2) was solved by the finite-difference method [10]. The algorithm of solution of the optimal problem was based on construction of the second order approximating polynomials relative to the selected parameters in Eq. (1) for a carbon content on the surface of $C_{surf} = C(C(\pm\delta, \tau_0)$ and in the center of $C_{cent} = C(0, \tau)$. The approximation was made on the basis of processing of the resutls of 32 numerical experiments on integration of Eq. (2) with various combinations of the following values of the parameters: $\tau_1 = 60$ and 90 min; $\tau_2 = 10$ and 30 min; $t_1 = 845$ and 955°C; $t_2 = 845$ and 955°C; $C_{atm1} = 1.1$ and 1.4%; $C_{atm2} = 0.6$ and 0.9%.

The coefficients of the approximating polynomials were determined with the use of a standard program of statistical approximation.

Table 1 gives examples of three solutions obtained in [9] in conducting numerical experiments on integration of Eq. (2).

The values of C_{surf} and C_{cent} obtained for the conditions given were used subsequently as the required in formulation of the optimal problems and the conditions themselves as the initial approximations. Table 1 gives the results of solution of Eq. (2) obtained in our calculations with the use of a program based on an implicit finite-difference scheme with the use of a trial run. The accuracy obtained of agreement with the calculation resutls given in [9] must be considered satisfactory keeping in mind that the calculations were made by different numerical methods, with different grid dimensions, etc.

After construction of the simplified model of the process the following problem of nonlinear programming was solved: to minimize $\tau_0 = \tau_1 + \tau_2$ with limitations (1) and with fulfillment of the equalities

$$C_{cent}^{spec} C_{cent} = 0, \quad C_{surf}^{spec} C_{surf} = 0. \tag{3}$$

Table 2 gives the conditions obtained in [9] in solution of the optimal problem. These conditions are not the optimal since the values of the parameters t_i and $C_{atm\ i}$ are far from their boundary values.

These same examples were solved by us with use of the method of successive linearization [7]. The results of optimization obtained in 15 steps of the iteration process of improving the initial conditions are given in Table 3. Table 4 shows a comparison of the conditions obtained in solution of the optimal problem in [9] and in this article for the values of the total process time τ_0 and for the accuracy of fulfillment of conditions (3):

$$|\Delta C_{surf}| = |C_{surf}^{spec} - C_{surf}|, \quad |\Delta C_{cent}| = |C_{cent}^{spec} - C_{cent}|$$

Therefore, the method of successive linearization provides a significant advantage both in the value of the optimized functional (for example, in example 1 of Table 4 according to our data the time τ_0 is half that obtained in [9]) and in the accuracy of fulfillment of conditions (3) (the accuracy of fulfillment of conditions (3) under our conditions is more than an order of magnitude more than in those obtained in [9]).

The high effectiveness of this method is related to the use of information on the derivatives, which, as is known [7], has a very favorable influence on the effectiveness and accuracy of solution of an optimal problem. The method of [9] leads to solutions far from the optimal as the result of use of a rough approximation of the original carburizing process. In addition the method proposed and approved in [9] may be useful in solution of problems of optimization of complex heat treatment processes the mathematical models of which do not allow such a simple method of calculation of the derivatives as in the above described situation such as when it is necessary to take into consideration structural tranformations, thermal and residual stresses, etc.

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AUTOMATION OF GAS CARBURIZING AND CARBONITRIDING PRODUCTION OPERATIONS WITH THE USE OF THE PAU-1 EMISSION OPTICAL QUANTOMETER

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The obtaining of a high and stable level of strength properties of steel parts raises the requirement of full automation of the production operations of chemicothermal treatment with the use of progressive monitoring and control systems, microprocessor technology, and computers.

For example, the Degussa Company (Federal Republic of Germany) has developed automated systems for gas carburizing of tapered and cylindrical gears and camshafts, the Hayes Company (USA) for vacuum carburizing of gears and sleeves, and the David Brown Gear Industries Company (Great Britain) for production of high-quality carburized gears.

The leading direction in the development of modern automated gas carburizing, carbonitriding, and nitriding systems is the development and produciton use of mathematical models of the production operation [1].

However, such factors as the instability and complexity of the physicochemical processes occurring in gaseous atmospheres in chemicothermal production cycles, the use of indirect methods of measurement of the carbon and nitrogen potentials in impregnation and also of slow and difficult to automate destructive test methods (chemical, metallographic, local x-ray

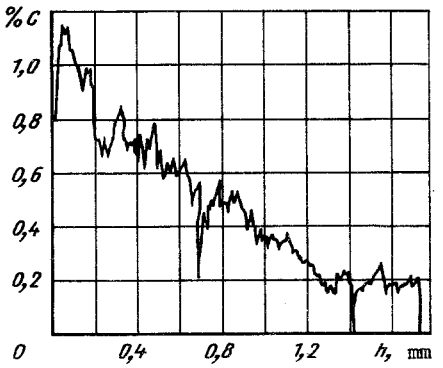

Fig. 1. Scanogram of the wt. % carbon distribution across the depth of the carburized case of a tooth of a whole gear of 10KhGNR steel; h is the depth of the measured portions of the case.

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