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The Necessity of Quantizing the Gravitational Field[1]

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The assumption that a classical gravitational field interacts with a quantum system is shown to lead to violations of either momentum conservation or the uncertainty principle, or to result in transmission of signals faster than c. *A similar argument holds for the electromagnetic field.*

1. INTRODUCTION

It has been suggested that it is unnecessary to apply the quantum theory to the gravitational field(1) even though it interacts with quantum fields, if the gravitational field interacts with the expectation value of the energy tensor of these fields. It has also been proposed that one does not have to quantize the electromagnetic field to get the results of quantum electrodynamics(2); e.g., the photoelectric effect has been consistently explained by Lamb and Scully(3) within the framework of semiclassical radiation theory. There thus seems to be no compelling argument against the thesis that a classical field can in some manner interact with quantum particles and fields. We shall show, however, that the assumption that a classical field interacts with quantum systems in any physically reasonable fashion leads to violation of either momentum conservation, the uncertainty principle, or relativistic causality in the form of signals traveling faster than c.

Briefly, we show that if a gravitational wave of arbitrarily small momentum can be used to make a position measurement on a quantum particle, i.e., to "collapse the wave function into an eigenstate of position," then the uncertainty principle is violated. If the interaction does not result in collapse

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of the wave function, it is then possible to distinguish experimentally between superposition states and eigenstates. We show that this ability allows one to send observable signals faster than c when applied to a state consisting of two spatially separated particles with correlated spins.

2. MEASURING THE POSITION OF A QUANTUM PARTICLE WITH GRAVITATIONAL WAVES

The idea that the gravitational field is classical seems quite elementary, but we must specify precisely what we mean by this concept before we can consider how it might interact with a quantum system. First, for the gravitational field to be "classical," all of its components must simultaneously possess precise values. Second, the field must satisfy the usual wave equation, at least for weak amplitudes, i.e., the field possesses wave excitations or arbitrarily small amplitude and wavelength which may be superimposed to form wave packets. Further, the momentum and energy flux are proportional to the square of the amplitude. These statements imply that the position and momentum of a wave packet can be simultaneously well defined.

We now consider an experiment in which the position of a quantum particle is measured by scattering a classical gravitational wave from it. To utilize the classical gravitational field as a reliable probe of a quantum system, we must show that it is possible to determine the properties of the field (such as the momentum and localization of its wave excitations) to arbitrary precision, using only quantum systems as preparing equipment and as detectors. If this is impossible, then, at least on an operational level, it is doubtful if one even has a classical field, since the fact that it must be measured with quantized matter imposes an observational limit on the precision of the field variables. We analyze this question in detail in Appendix A, but briefly describe here how it is possible to perform such measurements.

Our method is to prepare and detect gravitational waves with quantum matter acting in the classical limit, so quantum uncertainties will result in negligible perturbation of the classical wave. While we wish our preparation and detection apparatus to operate in this limit, we desire the opposite limit to be in effect during the probing of a quantum system with a classical wave. That is, we wish the gravitational wave to leave the quantum system unperturbed. It is possible to accomplish these apparently conflicting aims. The fluctuations the quantum theory imposes on matter are inescapeable, but the magnitudes of quantum parameters such as level spacings, binding energies, zero-point oscillations, etc., scale in a continuous and unrestricted fashion with the masses and coupling strengths of quantum systems. To prepare the initial state of the classical wave, we use extremely massive shutters, collimator slits, etc., which can be localized arbitrarily well for finite times. We also make the amplitude and wavelength of the classical wave so small that diffraction effects during the preparation process and momentum perturbation from the wave to the probed quantum system are negligible. For detectors we use extremely weakly bound systems or harmonic oscillators of very low frequency. Then any sudden excitation of finite energy will cause detectable transitions from the ground state. Thus by scaling masses and binding energies, amplitudes, and wavelengths, we can use the wave as a precise probe of the quantum system.

Let us now consider the sort of interactions possible between a gravitational wave and a quantum particle. In Fig. 1 we show a particle prepared in a state of highly well-defined momentum and thus of very poorly defined position. If a gravitational wave enters the localization region of this particle and interacts with it, the wave will be scattered from its initially well-defined trajectory. Any such interaction must fall into one of two exclusive categories. Either the scattering event constitutes a position measurement of the particle with a consequent collapse of its position wave function to a smaller region of localization, or else it is not a position measurement and does not result in collapse. In the latter case, the only spatial attribute the particle possesses to act as a source for scattering of the gravitational wave is the probability amplitude of the quantum wave packet. That is, the source term for the scattered gravitational waves at the point $\mathbf{x}$ must be constructed in some fashion (not necessarily local) from $\psi(\mathbf{x}, \sigma)$. For example, the source term might be a function of the probability density $|\psi(\mathbf{x})|^2$ (scattering from a continuous, extended source), or possibly pointlike scattering from the spatially

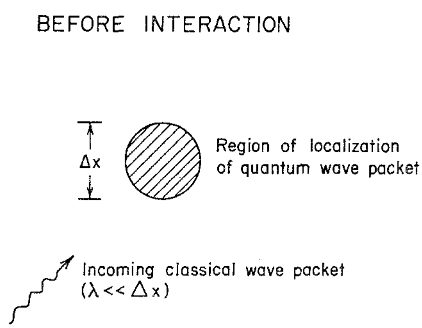

Fig. 1a. A quantum particle is prepared in a state of highly defined momentum and poorly defined position. Gravitational wave packets localized to a region $\lambda \ll \Delta x$ are directed at the region of localization.

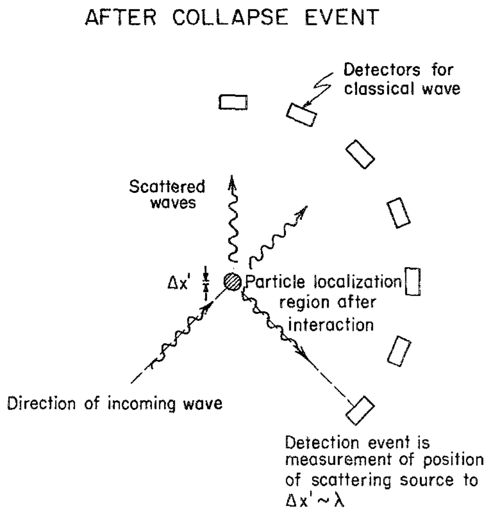

Fig. 1b. If the interaction results in wave function collapse, the particle is now localized to a region of size λ, which is determined by observation of the scattered waves and the known trajectory of the incoming wave packet. Since there was negligible momentum perturbation, both position and momentum of the particle are now highly defined.

averaged position. Roughly speaking, a localized test probe must "see" a quantum object either as a particle or as an extended wave function. The strictly yes or no character and irreversible nature of wave function collapse in quantum measurement theory permit no mixing of these possibilities.[3]

Consider the first possibility, in which a position measurement is performed with the consequent collapse of the wave function into a small localization region. In principle we can make the position localization as fine as we wish if the wavelenth λ of the gravitational wave is short enough (because the final Δx is of order λ). We can also ensure that arbitrarily little momentum is introduced to the quantum system from the gravitational wave by making its amplitude vanishingly small. If we used a quantum object, such as a photon, as a probe, it is of course impossible to simultaneously satisfy both conditions. If the classical probe gives the particle a very good position localization, then quantum mechanics implies that the particle is now in a state of very high momentum. If the quantum description of the

[3] The peculiar assumption that the wave function undergo only a partial collapse would result, as our analysis will show, in violating both relativity and the uncertainty principle!

particle is valid, then momentum is not conserved, since the momentum of the initial quantum state was very well defined and the classical probe imparted negligible momentum. Conversely, if momentum is conserved, the uncertainty principle is violated, since the ability to detect the scattered gravitational wave and thus determine the source of the scattering event to an arbitrarily good localization means that the particle is in a state whose position and momentum are simultaneously arbitrarily well defined. This outcome directly contradicts observation, for such a particle must be regarded as having a classical trajectory. A beam of such particles sent through an arbitrarily narrow slit would show no diffraction, contrary to fact. We conclude that if both momentum conservation and the uncertainty principle are valid, we must reject the possibility that a gravitational wave of vanishing momentum can collapse the wave packet of a quantum particle.

3. SENDING SIGNALS FASTER THAN LIGHT WITH TWO-PARTICLE CORRELATION STATES

Now consider the other possible type of interaction: scattering of the gravitational wave from the wave function of the quantum particle with no collapse. In this case there is no measurement of the particle's position, and thus no violation of either momentum conservation or the uncertainty principle. A problem remains, however. A measurement made on a system in a superposition state of some observable results in a collapse of the wave function into an eigenstate of that observable. If the wave function extends over a large spacelike interval, an influence produced at one point in space would seem to be propagated across this spacelike interval, which is forbidden by relativity. Because there is no way to observe the wave function without collapsing it by using ordinary quantum mechanical measurements, the apparent communication of the collapse event over a spacelike interval does not actually contradict relativity, which really only demands that no operationally well-defined signals be communicable at speeds faster than c. If, however, there exists any experimental method for observing the wave function without collapsing it, then there exists a direct way of viewing the collapse event when it is produced by an ordinary measurement. This would lead to the possibility of sending signals faster than c. Consider an apparatus similar to that of Einstein, Rosen, and Podolsky[(4)] (see Fig. 2). We have taken a π^0 which decays to two oppositely directed photons,[4] whose total spin must

[4] Our previous discussion considered the scattering of classical gravitational waves from massive particles. There is, however, no reason why gravitational waves should not also scatter from the massless particles, such as photons, which also carry energy and momentum. The second thought-experiment of our paper could, in fact, be done just as well with

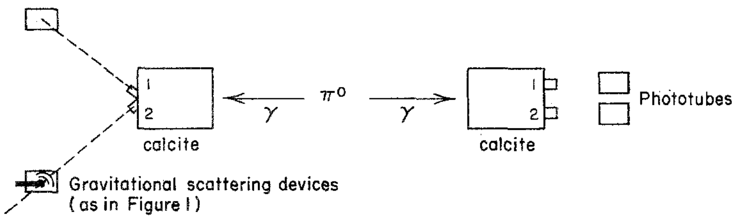

Fig. 2. Apparatus for sending signals faster than c if gravitational wave packets do not collapse the wave function. Case I. When no measurement is made on photon 1, scattered gravitational waves will be observed along both possible paths for photon 2. Case II. When photon 1 is measured to be in a definite channel, scattered gravitational waves will be observed only along the opposite channel for photon 2.

be zero. Before a measurement is made of either particle's polarization, the state of the two particles is a superposition of both possible helicity states. Experimentally, as shown by Freedman and Clauser,[(5)] this superposition results in polarization correlations of a nonlocal nature, i.e., measurement of one photon's circular polarization causes the second photon's polarization state to be collapsed to an eigenstate. Now we set up an apparatus which splits photons into different spatial trajectories according to their polarization in a manner analogous to a calcite crystal with optical photons. If neither particle's polarization is measured,[5] then the fact that they exist in a superposition state means that the second photon possesses equal probability to be found in each polarization channel of the splitter. The situation changes when we measure the polarization of photon 1. Then photon 2 must have the opposite polarization. The opposite channel of the second beam splitter

massive particles. For example, consider a polarized Σ^0 decaying to a Λ and a γ. The z components of their spins are correlated. We send both through beam splitters (for the Λ, this would be a Stern–Gerlach type of device). We can certainly scatter gravitational waves from the Λ, while the γ is detected by conventional devices. Then the conclusions of the thought-experiment are exactly the same as those in our paper for the $\pi^0 \to \gamma + \gamma$ decay.

[5] A beam splitter alone does not constitute a measuring device. A measurement of polarization also requires a device that detects which channel the particle travels through by undergoing some sort of irreversible change. An example is a phototube or a photographic plate. Without such a device, it is possible to reconstruct the original polarization state by recombining both channels with the inverse of the beam splitter. An example of this process is linearly polarized light sent through a calcite circular polarization analyzer. If the beam splitter alone were a measuring device, the recombined beam would be an incoherent, rather than a coherent sum of the two circular polarization states. Then the recombined beam would be a mixture of both linear polarizations, rather than a single one, as is the case.

will contain a photon; the corresponding channel of this splitter will contain no photon. Thus the spatial distribution of the probability amplitude ψ for photon 2 depends upon whether or not an operation is performed on the first photon, which is separated by a spacelike interval. The two cases are portrayed in Fig. 3. (A more detailed discussion of the probability amplitude for the elctromagnetic field is given in Appendix B.) Thus if we possess an experimental method to distinguish these two cases, we could send signals across arbitrarily large spacelike distances. This clearly violates relativity. A gravitational wave which is scattered by the particle's position wave function without collapsing it provides this experimental tool. No matter what specific coupling is assumed (see Appendix B for a detailed discussion),

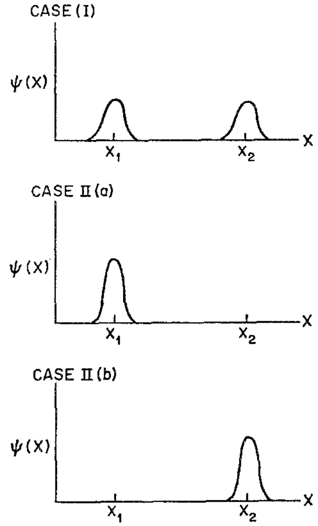

Fig. 3. The wave function of the photon field for photon 2 after passing through the beam splitter. Case I. When no measurement is performed on either photon's polarization there is equal amplitude for the photon to be in channel 1 or channel 2 of the beam splitter. Case II. (a) When photon 1 is observed to be in channel 2, there is nonzero amplitude for photon 2 only in channel 1. (b) When photon 1 is observed to be in channel 1, there is amplitude for photon 2 only in channel 2.

we will always be able to arrange the two trajectories so that the resultant scattering of gravitational waves we direct into this region will be different, depending on whether one or both channels possessed nonzero probability amplitude. We conclude that the principles of special relativity forbid the existence of a classical field which couples to a wave function without collapsing it.

One might suppose that a similar experiment could be used to send signals even without using a classical field as a probe, but this is not the case, for a measurement performed on a quantum system in a superposition state by definition causes the wave function to collapse to an eigenstate of that observable. Thus in our experiment any measurement with an ordinary measuring device that detects a photon in either channel causes a collapse of the wave function. This collapse cannot be distinguished from the collapse into a single channel caused by a measurement of the other particle's polarization. Thus, measuring which trajectory the photon went through tells us nothing about whether or not a measurement was made on the other photon. Likewise, attempts to distinguish the superposition state from the eigenstate by creating interference between the two trajectories fail, because these trajectories represent orthogonal quantum states.[6] Thus there is no incompatibility on the operational level resulting from the Einstein, Rosen, Podolsky paradox between the postulates of quantum mechanics and those of special relativity.

It has been argued that a correct analysis of these phenomena must involve quantum field theory since we are mixing relativity and quantum mechanics. This objection is not valid because of the hierarchic structure of relativity, quantum mechanics, and field theory. Field theory assumes that the ideas of relativity and quantum mechanics are true as limiting cases. Clearly any contradiction between the axioms of relativity and those of quantum mechanics would also result in a contradiction built into the axiom base of field theory. Relativity and quantum mechanics separately, however, each permits certain phenomena which are not allowed by the other theory. Field theory is more than a simple union of quantum mechanics and relativity, for it insists that all phenomena obey the postulates of both theories, and as a result makes predictions in certain contexts which do not follow from either theory separately. The special context of field theory involves lengths of the order of the Compton wavelength, energies of the order of particle rest masses, and other factors associated with particle creation and annihilation. Our thought-experiment does not involve any of these phenomena and is thus suited to the non-field-theoretic description we have given. The fact that the ordinary quantum mechanical description

[6] Pointed out by Claude Swanson in a private communication.

of just this sort of nonlocal correlations has been experimentally verified confirms our view that a field-theoretic description would not modify our conclusion.

We conclude that the gravitational field cannot be a classical field without violating accepted principles of physics. We therefore conclude that this field must satisfy the principles of quantum mechanics. (We also note that a similar argument holds for the electromagnetic field.) Semiclassical theories may of course be valid in various limits, but there must be circumstances for which their predictions become incorrect. We conclude that the world cannot be half classical and half quantum.

APPENDIX A. PREPARATION AND DETECTION OF CLASSICAL GRAVITATIONAL WAVES WITH QUANTUM MECHANICAL DEVICES

We assume a classical gravitational field which satisfies the usual linear wave equation and which is coupled to quantum matter in such a manner as to allow the generation of wavelike disturbances. As with a quantized field, we can produce a monochromatic wave train which corresponds to a state of well-defined wave vector. The intensity of such a wave is continuously variable, by the attenuation with distance of a spherical wave, if by no other means. We must now select a segment of this wave by means of timed shutters and direct its trajectory precisely if we are to use it as a reliable test probe. Timed shutters and collimators constructed out of quantum matter will of course obey the uncertainty principle. We need to make the position uncertainty of these devices arbitrarily small for a finite time. Because of the momentum uncertainty of a quantum system, the position uncertainty will grow with time as

$$\Delta x(t) \simeq \{\Delta x(0)^2 + [ht/m\, \Delta x(0)]^2\}^{1/2} \tag{A1}$$

Thus if we allow m to be arbitrarily large, we can make $\Delta x(t)$ as small as we wish for any finite time. We now have a very large Δp. It might therefore be thought that the trajectory of the classical wave passing through such a collimator would be disturbed by sideways momentum transfer. However, the slits are very massive, and have vanishingly small velocity. By conservation of energy and momentum (scattering a very light object from a very heavy one) the momentum transfer to the wave packet must be of the order of its initial momentum, which was also arbitrarily small. Thus we can select from an originally monochromatic wave train a short packet whose amplitude, wavelength, and momentum are simultaneously as small as we wish.

A more realistic but less conceptually simple method of generating a pulse of gravitational radiation is to drive a massive quadrupole oscillator for a few cycles. The wave will be spherical rather than localized to a small region, but it is still possible to measure the particle position to uncertainty λ. As with the slits, Δx of the generator is negligible compared to λ for large enough mass. The Δp of the generator is not important because the total energy carried by the gravitational wave is very small, due to the weak coupling between matter and gravitation, and the maximum momentum perturbation to the probed system can still be vanishingly small. Note that the only assumption we have made about the way gravitational waves interact with quantum objects is that the correspondence principle applies for systems, such as our generator, that act in the classical limit.

The probing of a quantum particle with our gravitational wave has been previously discussed. By making the amplitude vanishingly small, the momentum disturbance of the wave on the quantum particle is negligible. Making the wavelength very short also allows an extremely fine localization of the scattering center.

Using ordinary scattering theory,[7] the result of scattering a classical wave which is a superposition of plane waves is an outgoing wave packet and a scattered spherical wave. Since the coupling between the scattering source and the incident gravitational wave is very weak, the energy carried by the scattered wave is extremely small compared to that of the incident wave. However, conservation of energy and momentum in the center-of-mass frame implies that the maximum momentum transfer to the particle is not the momentum of the incident waves, but is of the order of the momentum carried by the scattered waves. Thus the intensity of the incident wave can be made strong enough to produce appreciable scattering, while still measuring the particle position and momentum to better than the uncertainty principle, as long as $p\lambda \ll \hbar$ holds for the scattered waves.

To measure the position of the scattering source we must detect the time of arrival of the scattered waves at points off the original trajectory. As detectors we can use massive harmonic oscillators or simple bound systems in the ground state, which will absorb energy from the gravitational wave and can undergo transitions to excited states. Although the energy carried by the scattered wave is small, there is no restriction in principle on the transition energy of such detectors. By making this energy small enough (i.e., by making the mass of the oscillator large and thus its frequency small) the scattered waves will have enough energy to excite transitions.

[7] As usual, we can represent the scattered wave by $f^{\mu\nu}(\mathbf{x})\,(e^{i\omega r}/r)\,e^{-i\omega t}$, where $f_{\mu\nu}$ depends on the details of the scattering but has no effect on the kinematics of determining the position of the scattering center.

Since the coupling between the scattered waves and the detectors is weak, the probability of transition is small, even though the wave possesses sufficient energy to induce a transition. However, there must be some finite cross section for absorbing energy and undergoing transition at any frequency ω of the wave. By using a very large number of detectors we can make the probability that one will undergo a transition of order unity. Those that do not undergo a transition absorb no energy from the wave.

Since the detectors are massive, their position uncertainty can easily be made to order λ or better. It is also necessary to know the time at which the transition occurs to order λ/c if the position of the scattering center is to be known to order λ. At first sight this seems difficult because the frequency ω_0 is small, and the characteristic oscillation time $1/\omega_0$ is much longer than λ/c. However, to observe a transition we do not need to measure the energy of the final state to an uncertainty ω_0, which would in fact require a time of order $1/\omega_0$. We only need to know if the energy of the detector is above some level. We can ask experimentally if the energy is in some broad band much wider than ω_0, but with its lower edge above the ground state. Then the time at which the transition occurred can be determined to much better than $1/\omega_0$. Alternately, we could use timed shutters to expose the detectors to the wave only for a time λ/c, and then examine whether the detectors had made transitions.

One might reasonably ask whether the gravitational field generated by the massive objects that we must use to observe the scattered gravitational wave does not in some way interfere with the measuring process and prevent us from doing better than the uncertainty principle. Since there are quantum fluctuations in the positions of these detectors, presumably there are uncertainties in the gravitational field which they produce. If one demands as an a priori definition of a classical field that it have no uncertainties in its attributes, then this question does not arise. One can, however, regard the uncertainties as being uncertainties of knowledge (as Bohr did), in which case it is conceivable to have a field which may be treated classically, but which possesses this sort of uncertainty. These two points of view essentially correspond to the distinction we made of whether or not the classical field could be used to make measurements on a quantum system with which it interacts. When the uncertainty is one of knowledge, then it is clear that a measurement on the classical field removes the uncertainty and thus also determines the corresponding attributes of a quantum system with which the field has interacted. If, however, there are no uncertainties allowed in the classical field, the only interaction possible with a quantum system is of the wave function, which is precisely defined.

For our first thought-experiment we must therefore consider whether or not the effect of the measuring devices upon the probed particle will allow us to violate the uncertainty principle. We must assume a specific type of scattering of the gravitational wave from the particle. For simplicity consider the deflection of the trajectory of the wave in the particle's own gravitational field. The change in the angle of the trajectory is of order (Ref. 6, Chapter 4)

$$\Delta\theta \sim Gm/rc^2 \tag{A2}$$

where m is the particle mass and r is the distance of closest approach. To detect a change $\Delta\theta$ the detector must be at a distance R large enough to that the transverse deflection is larger than the size of the wave packet, i.e.,

$$R \gtrsim \lambda/\Delta\theta \sim \lambda rc^2/Gm \tag{A3}$$

If the position uncertainty of the detector is initially of order λ and remains of order λ during the experiment time $T = R/c$, then from (A1) its mass M is at least

$$M \gtrsim \hbar T/\Delta x(0)\, \Delta x(T) \sim \hbar R/c\lambda^2 \tag{A4}$$

The gravitational field of this mass at the particle's location is

$$\phi = GM/R^2 \tag{A5}$$

Since the uncertainty in R is of order λ, the uncertainty in ϕ is

$$\Delta\phi \sim GM\lambda/R^3 \tag{A6}$$

Thus the uncertainty in the particle's momentum after time T is

$$\Delta p \sim \Delta f \cdot T \sim m\, \Delta\phi\, T \sim GMm\lambda/R^2c \tag{A7}$$

From (A4) this implies, since $\Delta x \sim \lambda$ for the particle,

$$\Delta p\, \Delta x \sim (Gm/Rc^2)\, \lambda\hbar \tag{A8}$$

which is negligible for ordinary matter.

However, Salecker and Wigner[7] have pointed out that there are also fundamental quantum limitations on the accuracy of clocks constructed from quantum materials. They found that for a clock of characteristic size l, which is to measure intervals to accuracy τ after a running time T, the mass M must be at least $T^3/\tau^2 l^2$. For our experiment the required τ is λ/c. Putting this into (A7), and using (A3), we obtain

$$\Delta p\, \Delta x \sim (r\lambda/l^2)\, \hbar \tag{A9}$$

There is no reason why we cannot make $l^2 \gg r\lambda$ (the clocks may be distinct from the detectors). We observe that in the limit in which the wave becomes more and more like a classical particle ($\lambda \to 0$), this limitation on the accuracy to which we can measure the particle's position and momentum becomes negligible.

We have shown that the existence of the classical limit allows us to use classical waves as probes of arbitrary precision for quantum particles, even when they are prepared and detected by quantum devices.

APPENDIX B. DETECTING WAVE FUNCTION COLLAPSE BY SCATTERING OF GRAVITATIONAL WAVES

With appropriate choice of gauge, the wave equation for linearized gravitational waves in vacuum is[8]

$$\Box^2 h_{\mu\nu} = 0 \quad (h_{\nu\mu} \equiv g_{\mu\nu} - \eta_{\mu\nu}) \tag{B1}$$

which has plane wave solutions of arbitrarily small amplitude. The scattering of gravitational waves by matter is given by the equation (in units with $G = c = 1$)[8]

$$\Box^2 h_{\mu\nu} = -16\pi S_{\mu\nu} \tag{B2}$$

in the same gauge, where

$$S_{\mu\nu} \equiv T_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu} T \tag{B3}$$

and $T_{\mu\nu}$ is the energy–momentum tensor for the scattering source. If the source is a single nonrelativitstic particle, then the only important component of $T_{\mu\nu}$ will be T_{00}, which has the expectation value at $\mathbf{x}$ of $m \mid \psi(\mathbf{x})|^2$. Whatever the source term $S_{\mu\nu}(x)$ is, it must be some functional of $\psi(\mathbf{x}')$, because the wave function $\psi(\mathbf{x})$ completely describes the spatial distribution of the particle.

Strictly speaking, $\psi(\mathbf{x})$ is not well defined for a photon, because if one tries to localize a photon to too small a region, one will create new photons. However, the amplitude to observe a photon in a region large compared to its wavelength certainly exists. One can be more rigorous and describe the electromagnetic field completely in terms of the field variables $F_{\mu\nu}(\mathbf{x})$. The state of the field is given by $\psi(F_{\mu\nu}(\mathbf{x}))$, i.e., the amplitude to observe the field to have the values $F_{\mu\nu}$ at the point $\mathbf{x}$. For a field which contains a single photon of well-defined trajectory, the amplitude to observe this photon in a region corresponds to the amplitude to observe an intense distribution of electric and magnetic fields in this region.

The source term in (B12), $S(\mathbf{x})$ (suppressing indices), must be some functional of $\psi(F_{\mu\nu}(\mathbf{x}))$, since ψ completely describes the state of the field.

Figure 3 shows the spatial distribution $\psi(\mathbf{x})$ to observe the photon (more precisely, to observe an intense electromagnetic field) after it has passed through the beam splitter in the second part of our thought-experiment. In case I no polarization measurement has been performed; in case II a measurement has been made.

Since an interaction does occur, we know that $S(x)[\psi(\mathbf{x}')]$ is not identically zero in either case I or case II. The correspondence principle also tells us that in the classical limit S must becomes $S_{\mu\nu}(\mathbf{x})$, as defined by (B3), for the classical matter distribution.

We have shown in Appendix A that the scattered radiation is detectable. Thus $S(\mathbf{x})$ is experimentally observable. We wish to use the measurement of $S(\mathbf{x})$ to distinguish case I from case II(a) or II(b), as shown in Fig. 3. Any $S(\mathbf{x})$ that depends locally on $\psi(\mathbf{x})$ and its derivatives at x is clearly different for case I and case II. Suppose $S(x)$ is any functional of $\psi(\mathbf{x}')$ for which case I cannot be distinguished from either case II(a) or II(b), i.e., $S(\mathbf{x})[\psi_{\mathrm{I}}] = S(\mathbf{x})[\psi_{\mathrm{II(a)}}] = S(\mathbf{x})[\psi_{\mathrm{II(b)}}]$. Obviously we then cannot distinguish case II(a) from case II(b) either. But we know from the correspondence principle that this is false, since the peaks in cases II(a) and II(b) can be separated by any arbitrary distance. In this limit the radiation scattered from a single, peaked source distribution must be centered about this source, and not some other point.

We conclude that the existence of a field which scatters from quantum particles without collapsing their position wave functions allows us to distinguish experimentally between a superposition state and an eigenstate of position.

APPENDIX C. NUMERICAL ESTIMATES

We now make order-of-magnitude calculations to see what would be needed to perform the conceptual experiment of measuring the position of a quantum particle with a classical gravitational wave. We want to show that the experiment is possible in principle, in the sense that it does not require any masses, lengths, or times greater than those of the universe.

We can generate a pulse of gravitational radiation by colliding two objects of mass m and size λ initially at velocity $v \sim c$. Gravitational radiation is emitted only during the period of deceleration, so the pulse length is also of order λ. In units with $G = c = 1$, the energy of gravitational radiation emitted is of order (Ref. 6, Chapter 37)

$$E_{Gw} \sim \gamma^2 m_1{}^2/\lambda \tag{C1}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$.

The outgoing wave pulse travels a distance r to the particle whose position we wish to measure. This particle, of mass m_2, has a gravitational scattering cross section of order m_2^2. Thus the scattered wave carries off energy of order

$$E_{\text{scatt}} \sim \begin{pmatrix}\text{energy incident}\\ \text{per unit area}\end{pmatrix} m_2^2 \sim \frac{\gamma^2}{\lambda}\frac{m_1^2}{r^2}m_2^2 \tag{C2}$$

The scattered wave at a distance R carries a density of energy of order

$$E_{\text{det}} \sim E_{\text{scatt}}/\lambda R^2 \qquad \text{(energy per cm}^3\text{)} \tag{C3}$$

This energy can also be written in terms of the amplitude A of the difference of the metric from the flat-space value,

$$E_{\text{det}} \sim A^2/\lambda^2 \tag{C4}$$

This radiation acts on the detector. We model the detector by two masses m_3 bound elastically at distance L with frequency ω_0. When a classical oscillator of this type is driven by a gravitational wave with $\lambda \gtrsim L$, $\omega \gg \omega_0$ ($\omega = 2\pi c/\lambda$), and amplitude A, it will oscillate with energy (Ref. 6, Chapter 37)

$$E_{\text{osc}} \sim m_3\omega^2A^2L^2 \sim m_3L^2E_{\text{det}} \sim m_3L^2E_{\text{scatt}}/R^2\lambda \tag{C5}$$

If these detectors are quantum oscillators, initially in the ground state, the scattered wave can induce transitions to excited states. We can estimate the transition probability for these oscillators from their known behavior in the classical limit. In this limit, the expectation value of the energy which the oscillator absorbs from the wave is given by (C5). If the transition energy is $\hbar\omega_0$, the probability of undergoing a transition must then be

$$P_{\text{trans}} \sim (m_2L^2/\omega_0)\, E_{\text{scatt}}/R^2\lambda \tag{C6}$$

To observe a transition we need the probability that at least one oscillator is excited be of order unity. For this we require $N \sim 1/P_{\text{trans}}$ oscillators. The total mass of detectors required to detect the scattered waves is

$$M_{\text{tot}} \sim \frac{m_3}{P_{\text{trans}}} \sim \frac{\hbar\omega_0}{L^2}\frac{R_2\lambda}{E_{\text{scatt}}} \sim \frac{\hbar\omega_0\lambda^2}{L^2\gamma^2}\frac{r^2R^2}{m_1^2m_2^2} \tag{C7}$$

We minimize M_{tot} by making $L \sim \lambda$ and R as small as possible (for a fixed density of the detectors). We can substitute $M_{\text{tot}} \sim R^3$ into (19) and solve first for R, then M_{tot}, getting

$$M_{\text{tot}} \sim \frac{1}{\rho^2}\left(\frac{\hbar\omega_0}{\gamma^2}\frac{r^2}{m_1^2m_2^2}\right)^3 \tag{C8}$$

(In these units $\hbar = 2.62 \times 10^{-66}$ cm², ω_0 has units cm⁻¹, and m_1 and m_2 have units of cm.) An array of detectors of mass M_{tot} will measure the position of mass m_2 to precision λ.

If we wish to keep the detector mass M_{tot} within limits, we need to make the masses of the generator and the probed particle as large as possible. Now the scattered gravitational pulse will be extended over a distance of the order of the size of this probed particle. Thus the final position uncertainty in our thought-experiment will be of this order. In order to do better than the uncertainty principle, this final Δx must be smaller than the initial Δx of the particle, i.e., the initial Δx must be larger than the particle's size. To accomplish this we must measure the particle's velocity to an uncertainty $\Delta v_x \lesssim \hbar/(m\,\Delta x)$. For example, an object of mass 10 g, of size 1 cm, and Δx of 10 cm needs Δv_x of order 10^{-29} cm/sec. We wish to accomplish this by scattering from it a low-mass particle, such as a proton, whose velocity has been measured to this precision.

We can measure velocities to great precision by using a diffraction grating. The grating consists of a linear array of scatterers, such as atoms in a crystal, separated by a distance d. Diffraction will occur at angles where the Brag condition $n\lambda = 2d \sin\theta$ is met. (The process is precisely the same as that in x-ray diffraction by crystals.) By making the grating curved rather than straight, the diffracted rays from different parts of the lattice can be focused to a point. For a grating of finite extent W, the spread in the wave vector Δk of the diffracted beam from a monochromatic source is of order $1/W$.[9] Thus to resolve velocity components differing by Δk we must have

$$W \gtrsim 1/\Delta k \sim \hbar/(m\,\Delta v) \tag{C9}$$

The difference in diffraction angle for particles with velocities differing by Δv is

$$\Delta\theta \sim \Delta\lambda/d \sim \hbar\,\Delta v/dm\,v^2 \tag{C10}$$

To select velocities to the precision Δv we use an aperture of size D at a distance L from the grating. This will select an angular spread $\Delta\theta \sim D/L$. To distinguish velocities differing by Δv, L must be at least

$$L \gtrsim dDmv^2/(\hbar\,\Delta v) \tag{C11}$$

To measure a proton, with $m \sim 10^{-24}$ g, to a $\Delta v \sim 10^{-29}$ cm/sec, we must have $W \sim 10^{26}$ cm. We must make $v \sim 10^9$ cm/sec if the measurement time $T \sim W/v$ is $\lesssim 10^{17}$ sec. If we make $d \sim D \sim 10^{-13}$ cm, then we need $L \gtrsim 10^{24}$ cm.

The proton exits with $\Delta v_x \sim 10^{-29}$ cm/sec in the direction perpendicular to the exit aperture. (Since its y position was localized to distance D, it will have a Δv_y much larger.) We wish to use this proton to measure the velocity of a 10-g object to the same precision. We assume the object has a flat surface perpendicular to the x direction and we scatter the proton elastically from it. The 10-g mass is effectively infinite for the collision, so the velocity of the proton afterward is

$$v_{fx} = 2V_x - v_{xi} \tag{C12}$$

where v_{xi} is the initial x velocity of the proton and V_x is the x volocity of the 10-g object.

The scattered proton can then be sent back through the same diffraction grating. Now we can use an array of detectors instead of a single exit slit, and observe which detector the particle triggers. This measures v_{fx} to the same precision as that of the proton.

Thus we can prepare a particle of mass 10 g and size 1 cm in a state with $\Delta x \sim 10$ cm. Therefore we can choose m_2 to be 10 g and take λ to be 1 cm.

In order that the momentum uncertainty of the probed particle after the position measurement be less than the quantum value of $\hbar/\Delta x$, we require that the initial pulse of gravitational radiation be weak enough that the momentum carried by the scattered wave be small compared to the quantum value $\hbar/\Delta x \sim \hbar/\lambda$. This condition is easily satisfied. Since the size of the generators is of order $\lambda \sim 1$ cm, their mass will be of order 10 g, or 10^{-26} cm in geometrical units. The smallest r we can take will also be of order λ. If we take $\gamma \sim 3 \times 10^{18}$, then from (A6) we find $p_{\text{scatt}} \sim E_{\text{scatt}} \sim 10^{-67}$ cm $\sim 10^{-1}\hbar/\lambda$. The generation process requires a total energy of order 10^{-8} earth masses.

For detectors we use harmonic oscillators of linear dimensions of order 1 cm. We also make the springs so weak that the period is of order 10^5 sec, i.e., ω_0 is 10^{-15} cm^{-1}. We assume that the mass density of detectors is of order 10 g/cm^3, or 10^{-26} cm^{-2}. From (20) the total mass of detectors is of order 10^{19} cm or about 1000 galactic masses, located at an average distance of order 10^{15} cm. Note that this value for M_{tot} is really an upper limit. If we could focus the scattered wave, for example by bending it in strong gravitational fields, the required value of M_{tot} would be reduced by many orders of magnitude.

Our idealized experiment is fantastically difficult to perform, but nevertheless in principle possible. The thought-experiment dealt with weak fields and classical limits so that we could predict accurately what would happen. In the limit of strong fields and short distances, where present theories are uncertain, the effects of the quantum theory on gravitation should be far greater. It has been often proposed that quantization of gravity may have important consequences for gravitational collapse and the small-scale structure of spacetime. It is possible that quantum fluctuations of spacetime may provide cutoffs for otherwise divergent integrals of field theory and thereby permit a laboratory observation of these effects.

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