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Comparison of NH_3 Laser Dynamics with the Extended Lorenz Model*

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Abstract. We present a comparison of the intensity and the electric field measured from an optically pumped FIR NH_3 laser showing instabilities with the respective results obtained from numerical integration of the Lorenz model extended to allow for detuning of the laser cavity with respect to emission line center. Good agreement between experimental and numerical results for high gas pressure suggests that this laser is appropriately described by the Lorenz model for higher operating pressure, while in the low pressure domain qualitative differences are found.

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The correspondence between the dynamics observed on NH_3 laser transitions [1, 2] and the laser-Lorenz model has recently been widely disputed; see e.g. [3]. In order to provide additional experimental evidence, more detailed comparisons between the model and measured intensity and field dynamics are presented.

Without detuning the solutions of the Lorenz Model display the well-known spiralling on a double sided attractor with irregular jumps from one side to the other. Figure 1a shows the projection of this attractor along the electric field axis of the phase space while Fig. 1b gives a plot of the corresponding electric field vs. time for that case. In contrast, plots of the intensity vs. time would not show the change of sign associated with the switching between the two leaves and therefore would not allow one to determine whether the underlying attractor is single sided or double sided. In an attempt to determine whether or not a given experimental situation may be described by the Lorenz model it is therefore important to consider not only the intensity but also the electric field including its phase.

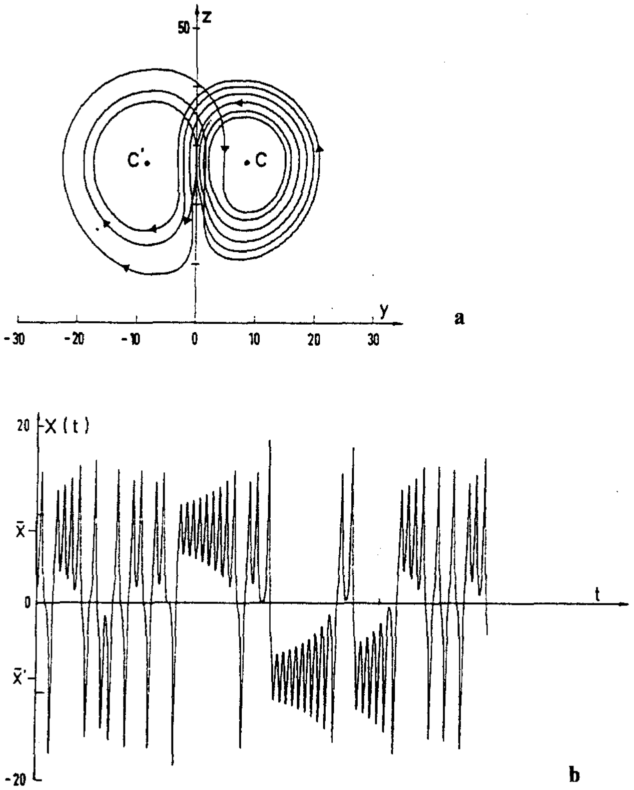

Fig. 1. a Projection of the Lorenz attractor along the electric field axis of the phase space. **b** Plot of electric field versus time from a numerical integration of the Lorenz equations. From [4]

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* Dedicated to Prof. Dr. Herbert Welling on the occasion of his 60th birthday

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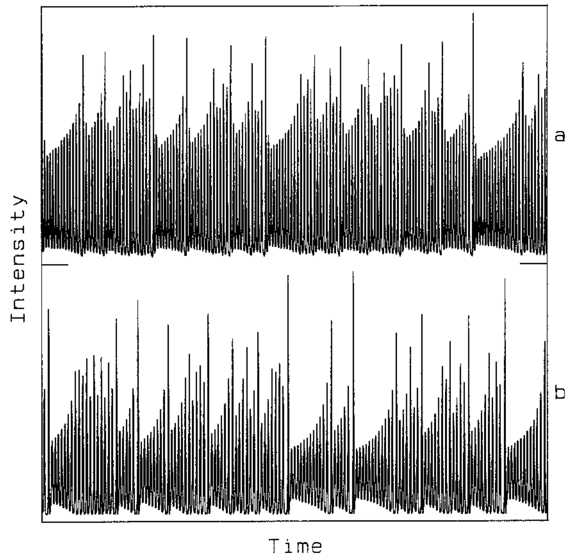

Fig. 2. **a** Intensity versus time measured from a chaotically emitting NH_3 laser. **b** Result of numerical integration of the Lorenz equations for parameters approximately describing the experimental situation in **a**

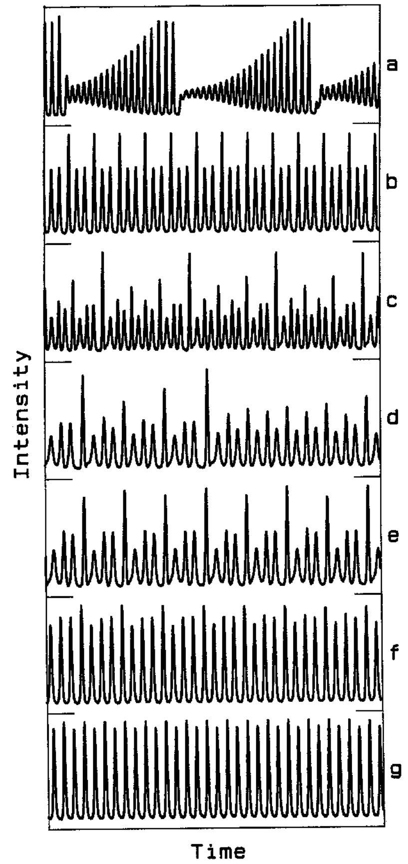

Fig. 3a–g. Intensity signals measured for increasing detuning of the laser cavity with respect to emission line center: **a** resonant case: Lorenz-like chaos, **b** small detuning: stable period-3 oscillation, **c** period-12 window within the period doubling chaos domain, **d** period doubling chaos, **e** stable period-8 oscillation, **f** stable period-4 oscillation, **g** stable period-2 oscillation

1. Intensity Measurements

Figure 2a shows a plot of the intensity vs. time measured from a chaotically emitting $^{14}NH_3$ laser tuned to line center. The experimental set up was the same as the one previously described [1]. For comparison, Fig. 2b shows the result of a numerical integration of the Lorenz equations with parameters chosen to approximately describe the experimental situation. Good qualitative agreement is found suggesting a close similarity between the experimental system and the Lorenz system. Minor differences may indicate that the parameters chosen for the numerical solution do not exactly match the conditions given in the experiment.

Under the same operating conditions, the detuned laser goes through a series of instabilities as it is tuned towards line center, as presented in Fig. 3. Starting with large detuning a period doubling sequence is observed first (Fig. 3g–e) leading to period doubling chaos (Fig. 3d). Within the chaotic regime, various windows are found. Figure 3c shows a period-12 window as an example. On further reduction of the detuning the chaos disappears and the system enters a period-3 state as given in Fig. 3b. Finally, for very small detuning the Lorenz chaos is found, characterized by sequences of exponentially growing pulses (Fig. 3a).

This bifurcation scheme agrees well with the numerical results obtained by Mandel and Zeghlache for the extended Lorenz model [5]. In Fig. 4 we give their bifurcation diagram which shows the type of pulsations expected for various detunings as a function of the pump parameter.

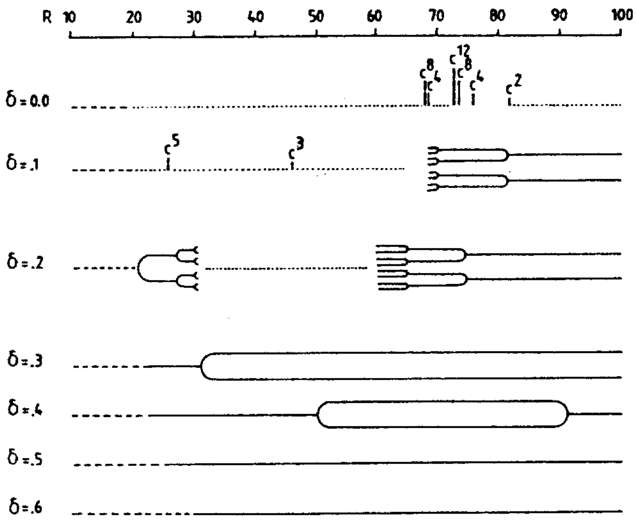

Fig. 4. Bifurcation diagram from [5]. Dashed lines: stable steady states, dotted lines: chaotic domains, solid lines: periodic solutions

Since in our measurements the pump parameter was held fixed, our results correspond to a vertical scan from bottom to top through that diagram. In addition, for the centrally tuned laser we observed the abrupt onset of chaos with increasing pump parameter corresponding to a scan from left to right along the uppermost line of their diagram.

2. Field Measurements

Intensity measurements do not contain the full information needed to describe the symmetry of an attractor, as the phase of the electric field is lost in taking its absolute square. Therefore, heterodyne measurements were carried out to recover the amplitude and the phase of the electric field emitted by the laser. It is only from such data that a double-sided attractor can be distinguished from a single-sided attractor. To facilitate such measurements, a $^{15}NH_3$ laser emitting at 153 μm was used instead of the $^{14}NH_3$ laser operating at 81 μm. The experimental set up was described in [6].

Figure 5 shows the in-phase and quadrature components of the electric field in the upper traces and the corresponding intensity signal in the lowest trace for a centrally tuned laser emitting chaotically. Parameter values were the same as for Fig. 2a. Note that the phase of the electric field (except for some slow drift due to a frequency change of the local oscillator) shows only minor variations within one sequence of intensity pulses and does not change sign. However, at the end of such a sequence the electric field shows a phase jump of π, corresponding to the transition from one leaf of the Lorenz attractor to the other. Close inspection of Fig. 5 reveals that at certain times there is only a single revolution around the fixed point before the system switches back to the former leaf again, as can also be observed in numerical simulations.

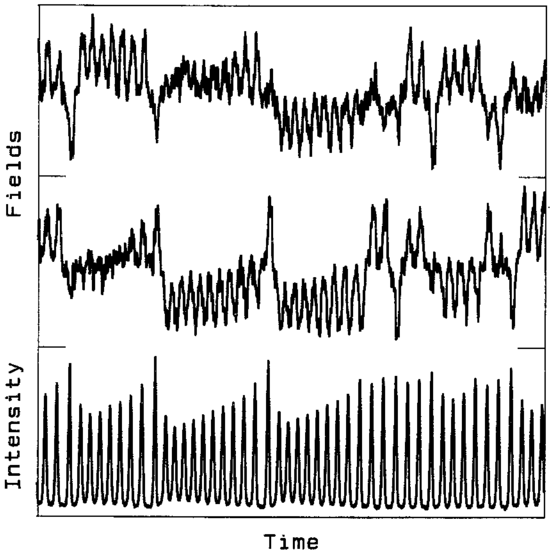

Fig. 5. Upper traces: in-phase and quadrature components of the field emitted by the laser without detuning. Lower trace: intensity emitted by the laser. Phase jumps of π indicate a symmetric attractor: Lorenz-like chaos

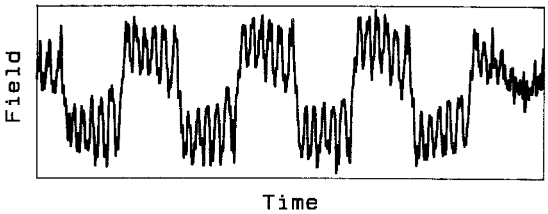

Fig. 6. One of the electric field components for a symmetric window within the chaotic domain for centrally tuned cavity: the laser performs regular motion on a symmetric attractor with six loops on each side

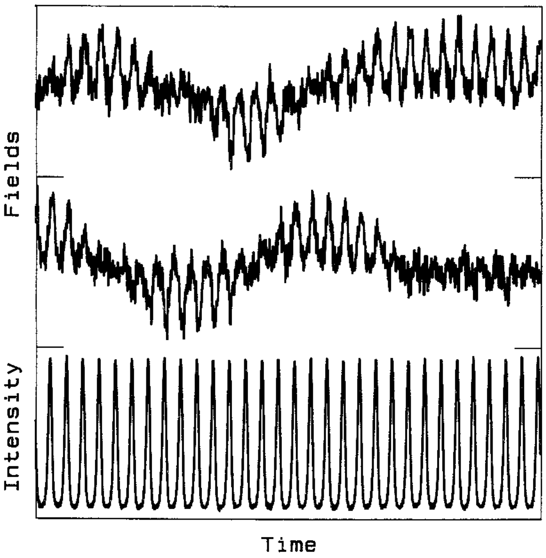

Fig. 7. Electric field components and intensity of a laser with detuned resonator: Absence of field sign changes indicates the detuned period-1 state to be described by a single-sided attractor

Figure 6 shows one of the electric field traces for conditions under which the laser regularly performs 6 revolutions about one of the fixed points before it switches to the other side of the attractor. As for the previous set, these data were sampled for zero detuning of the laser cavity, but for a slightly different pump parameter. Analysis of the phase jumps demonstrates this symmetric state to be a periodic window in the Lorenz chaos with 6 revolutions around each fixed point.

In contrast to Fig. 5, the field components and the intensity for a regularly pulsing detuned laser are shown in Fig. 7. Under those conditions, no jumps are found in the electric field phase. Thus we conclude that the attractor is single sided in this case, in accordance

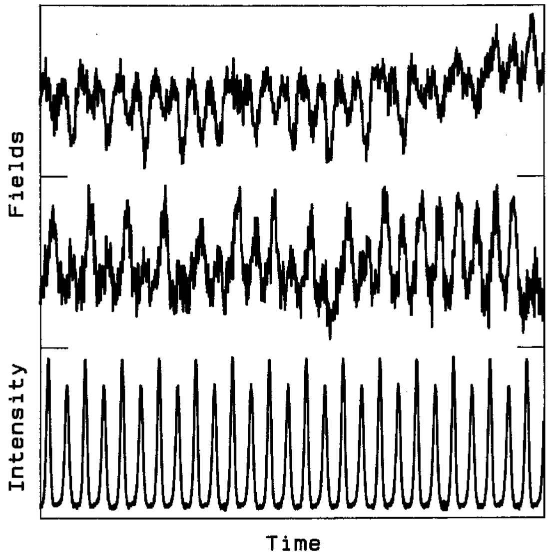

Fig. 8. Similar to Fig. 7, but for less detuning, showing the single sided detuned period-2 state

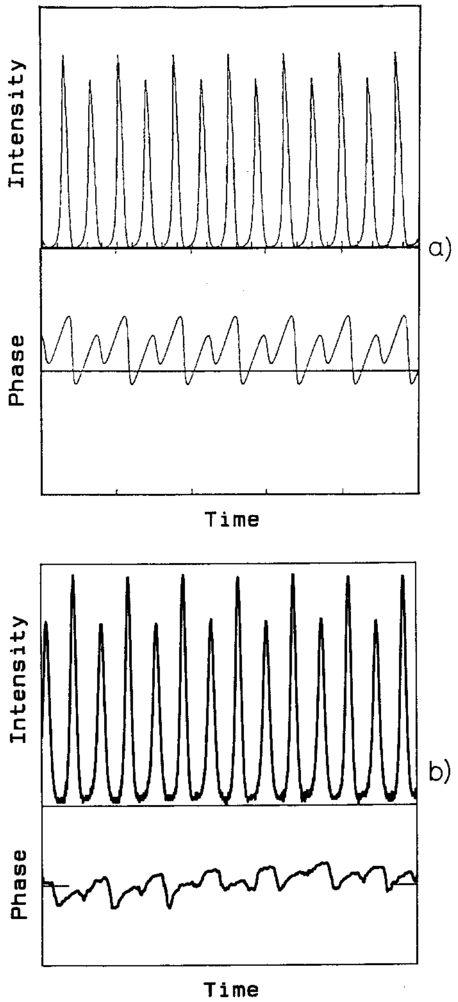

Fig. 9a, b. Intensity (upper trace) and phase (lower trace) for the detuned period-2 state: **a** numerically generated data, **b** measured from a NH_3 laser

with the numerical results for a detuned laser. Figure 8 gives corresponding results for slightly less detuning, resulting in a period doubled intensity output from the laser as a first step along the period doubling sequence quoted above. Again no phase jumps are found in the electric field traces, confirming that the underlying attractor is single sided period doubled and not double sided asymmetric. This attractor configuration is expected from the extended Lorenz model, as is demonstrated in Fig. 9, which gives numerically generated intensity and phase plots vs. time for a detuned Lorenz system in (a) and, for comparison, intensity measured from a detuned laser and the phase reconstructed from the electric field data in (b). The scale for both phase traces is π/div.

3. Deviations from the Lorenz Model

Figure 10 shows another apparently period-doubled signal. However, these data were taken from a laser tuned to line center where the attractor is expected to be symmetric. In fact, the electric field traces show that every second intensity pulse is accompanied by a change in sign of the field while the other intensity pulse results in only a small change of the field phase. Thus the system regularly performs two revolutions around the fixed point of one leaf of the double-sided attractor before it switches to the other leaf. Note that this state is not part of a period doubling sequence because it is governed by a symmetric attractor, even though it could not be distinguished from such a state on the basis of intensity data alone.

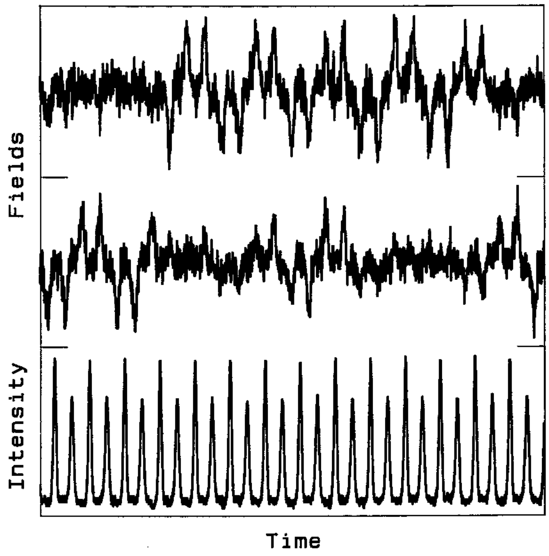

Fig. 10. Electric field components and intensity for central tuning of the cavity (lower pressure: coherence effects from pump are present). The laser is shown to be in a symmetric state performing two loops on each side of the attractor

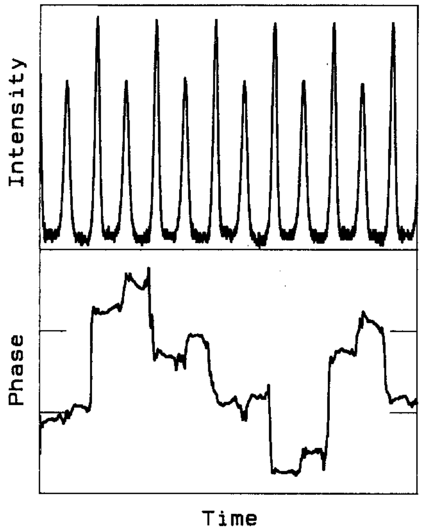

Fig. 11. Intensity and phase data for the state given in Fig. 10

Figure 11 gives the intensity and the phase for that state to demonstrate the switching behaviour more clearly. While there is some drift of the local oscillator frequency superimposed on the phase data, it is clearly seen that each big intensity pulse is preceeded by a jump of π in the phase of the electric field. This differs significantly from what would be expected for a Lorenz-like attractor, where the zero crossing follows the biggest intensity pulse. In addition, there are not only phase jumps of π, but also smaller phase excursions associated with the smaller pulses which would not occur in a tuned Lorenz system, where all variables may be treated as real numbers, thereby allowing only for phase changes of zero or π.

The lower gas pressure used when measuring Fig. 11 is assumed to be a reason for this deviation from Lorenz-like behaviour. We believe that pressure broadening is too small here to counteract the coherence effects of the optical pumping. Therefore, a proper description would need at least a three-level model. However, no attempts have yet been made to confirm this assumption.

In conclusion, we have found that the bifurcation diagram obtained from numerical solutions of the extended Lorenz equations for a scan of the detuning is reproduced in experiments using a FIR NH_3 laser. In addition, for small homogeneous broadening conditions deviations from the Lorenz-like behaviour are found, which are attributed to the coherence effects of the optical pump, making the use of a three level model necessary in the low pressure domain.

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