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Complexity out of a Simple Structure: The Intricate Multistable Behaviour of an Optical Resonator Filled with Sodium Atoms*

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Abstract. A Fabry-Perot resonator containing sodium atoms displays a variety of switching phenomena even in the simplified case of homogeneous broadening. The processes are shown to arise from the combined effects of Zeeman pumping and electronic excitation. A simple model can clarify the dazzling complexity of the observed phenomena.

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Nonlinear resonators, i.e. resonators containing a Kerr medium or a medium with saturable dispersion and absorption, have received a great deal of attention during recent years [1]. The combination of the nonlinearity with the built-in feedback mechanism of the resonator can give rise to a wealth of phenomena, ranging from optical bistability to self-oscillations and chaos.

Among the topics studied most intensely in experimental work in this field are resonators filled with sodium atoms [1]. They were used in the first successful experiment on optical bistability [2], and later it was learnt that they can also give rise to polarization switching and tristability [3–8]. It was found that both subcritical and supercritical bifurcations connected with changes in the polarization state of the light field occur under nearly identical experimental conditions [9]. In many cases, however, the detailed interpretation of the observations is complicated by the presence of hyperfine structure and inhomogeneous broadening.

It is the purpose of this paper to shed some light on the relations between the various phenomena occurring in the stationary behaviour of sodium-filled resonators by studying an experimental situation which can be described by a simple model. Even in this case we find that there can be a fairly complicated coupling between bistability and tristability which can lead to quadrustability and which has not yet been described before. We will also briefly discuss the transient behaviour of the device and show that there is a purely *transient* counterpart to polarization-switching.

1. Basic mechanisms and phenomena

In the interpretation of the first experiment on bistability in a sodium-filled resonator [2] the sodium atoms were regarded as two-level systems whose saturation behaviour must then be caused by accumulation of population of the excited state. In real atoms the level structure is more complicated, and it was found that hyperfine pumping in the ground state can play a major role in the experiments on sodium filled resonators [9].

Hyperfine splitting can be expected to be of little impact if it is exceeded by the homogeneous broadening of the optical transition. In the experiment there are indeed indications that with a buffer gas pressure of more than 20 mbar its influence can be neglected [10].

* Dedicated to Prof. Dr. Herbert Welling on the occasion of his 60th birthday

Even then the model of two-level atoms is still oversimplified, because the spin of the sodium atoms is ignored. The ground-state spin orientation of the sodium atoms is hardly affected by collisions with buffer gas atoms, particularly if a noble gas is used. Consequently, two states of different spin orientation have to be distinguished in the sodium ground state even in the case of strong collisional broadening. The interaction of light with sodium atoms of different spin orientation obeys the well-known polarization selection rules. As a consequence the polarization state of the light field necessarily comes into play.

While it is easily understood that the occurrence of ground-state Zeeman pumping in circularly polarized light can reduce the threshold of bistability tremendously [11], one might expect that, for reasons of symmetry, no net spin orientation is created in the case of linearly polarized light input. However, as was first pointed out in [3], at a certain threshold intensity spontaneous symmetry breaking occurs and gives rise to a spin orientation. Simultaneously, the transmitted light becomes strongly elliptically polarized. This phenomenon has been called "polarization switching" [4, 6].

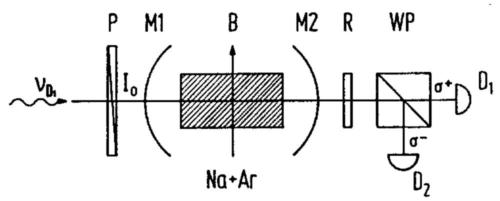

Fig. 1. Simplified geometry of the experimental setup. (P: polarizer, M1, M2: mirrors of the confocal cavity (free spectral range = 1 GHz, beam waist radius about 0.12 mm), B: magnetic field, R: $\lambda/4$-retardation plate, WP: Wollaston prism, D1, D2: photo detectors)

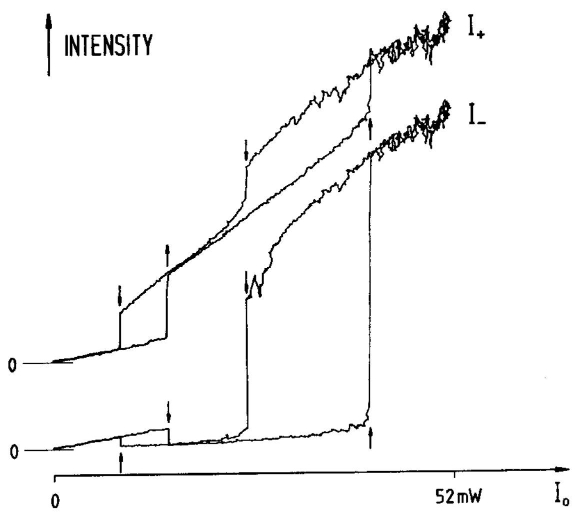

Fig. 2. Dispersive tristability in the case of large laser detuning. Parameters: laser detuning -25 GHz, estimated optical density 0.1 to 0.2, $B_y = 0.5 \times 10^{-4}\,T$

If the intensity of the input beam is further increased, the asymmetric state of elliptically polarized output persists in a broad intensity interval, until at some second threshold the output becomes linearly polarized again [6, 7]. This recovery of symmetry through an inverse polarization switching was attributed to increased population of the excited state which reduces the possible population difference between the Zeeman levels of the ground state. Both the polarization switching at the first threshold and the recovery of symmetry at the second threshold show hysteresis.

2. Experimental Observations

The experiments under consideration here are performed in a setup as used in [4–8]; it is schematically shown in Fig. 1 and was described in detail in [8]. A Fabry-Perot resonator contains sodium vapor. A laser beam tuned to the D_1 sodium resonance transition is irradiated into the resonator with controlled polarization. In contrast to a conventional bistability experiment, the transmitted light is sent through polarizing optics in order to separate the right- and left-hand circularly polarized components.

We had also the possibility to apply a transverse magnetic field and compensated for the earth magnetic field. In contrast to other experiments we used a fairly high buffer gas pressure (Ar at 200 mbar) in order to approach the model of spin-$\frac{1}{2}$ atoms.

2.1. Stationary Behaviour

A typical plot of the intensities of the right- and left-hand circularly polarized components I_+ and I_- of the transmitted light is shown in Fig. 2. There is a sizeable regime of input intensities for which the polarization components I_+, I_- of the output intensity are completely different. As the input intensity is increased, symmetry is broken through "polarization switching", and through "inverse polarization switching" it is recovered. The downward scan of input intensity reveals that all switching occurs with hysteresis. It has been found that the right-hand and left-hand circularly polarized components can interchange their roles. In effect, there is *tristability* within the range of either hysteresis loop, because there are three stable states coexisting. One we call the "symmetrical" state; it corresponds to linearly polarized output and no spin orientation. The other two are "asymmetrical", with a spin orientation of one or the other sign.

The physical origin of the polarization switching is indeed the creation of ground-state orientation. This can be demonstrated experimentally with the help of a weak probe beam, which is sent off-axis through uncoated spots of the resonator mirrors. By means of polarizers the probe beam was made sensitive to the birefringence of the sample, which is determined by the ground-state orientation. A typical birefringence measurement is shown in Fig. 3a; it has to be compared to the behaviour of the resonator output I_+ depicted in Fig. 3b. As expected, normal and inverse polarization switching in Fig. 3b correspond directly to the appearence and disappearence of orientation in Fig. 3a, respectively.

The width and height of the hysteresis loops depend critically on the cavity detuning. Actually, the left hysteresis loop can be made to shrink to zero width. Correspondingly, the size of the jumps of I_+ and I_- also shrinks and vanishes. In other words, the bifurcation can become supercritical [9].

The threshold for polarization switching is increased by a transverse magnetic field; at the same time the width of the hysteresis loop is increased. On the other hand, for the inverse polarization switching we do not see any noticeable effect of the magnetic field. Let us remark that there is a threshold value for the transverse magnetic field above which one has magnetically induced self-oscillations [12, 13]; the present discussion is restricted to fields below that threshold.

The behaviour of Figs. 2 and 3 is the most typical behaviour we observe in our experiments. It occurs in a wide range of laser frequencies on either side of the resonance line. There is a difference, however, to the observations reported in the literature [6]. These look more like the behaviour shown in Fig. 4, i.e. the intensity of the highly transmitted polarization component switches to a *lower* value at the second threshold. In spite of this qualitative difference, the behaviour of the birefringence is as shown in Fig. 3a in either case. We expect that there is no fundamental physical difference between the two cases. Systematic studies demonstrate that the behaviour of Figs. 2 and 3b occurs in the wide range of laser frequencies where the behaviour of the device is governed by dispersion, while the behaviour of Fig. 4 occurs in the fairly narrow frequency range where the role of absorption is dominant.

There is another prominent change if the sodium density is slightly increased over the values used for Figs. 2–4. In the dispersive regime the behaviour of Fig. 2 changes to that of Fig. 5. Here obviously the two hysteresis cycles have "collided" and interlaced.

The conventional type of optical bistability, i.e. intensity switching of a linearly polarized light without any polarization switching, is *not* observed in our

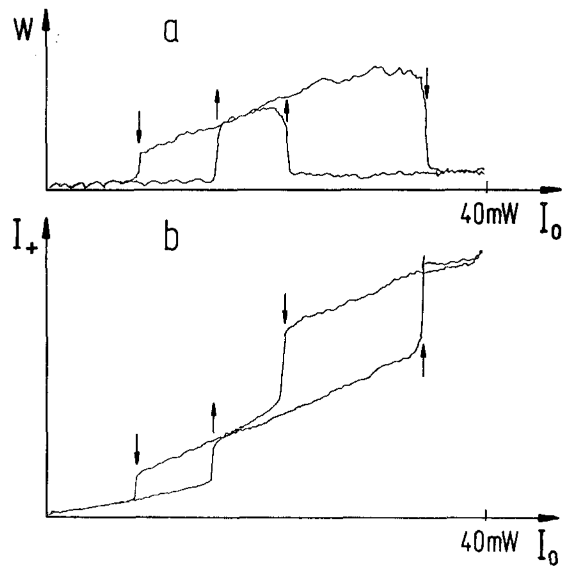

Fig. 3. **a** Orientation of the sample, **b** transmitted intensity I_+. Parameters: laser detuning 12 GHz, optical density similar as in Fig. 2, $B_y = 0.1 \times 10^{-4}$ T (from [23])

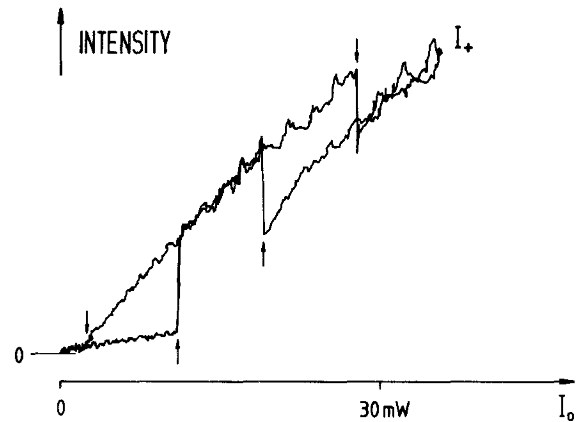

Fig. 4. Normal and inverse polarization switching of the σ^+-light in the case of small laser detuning. Parameters: laser detuning −1 GHz, optical density similar as in Fig. 2, $B_y = 0.2 \times 10^{-4}$ T

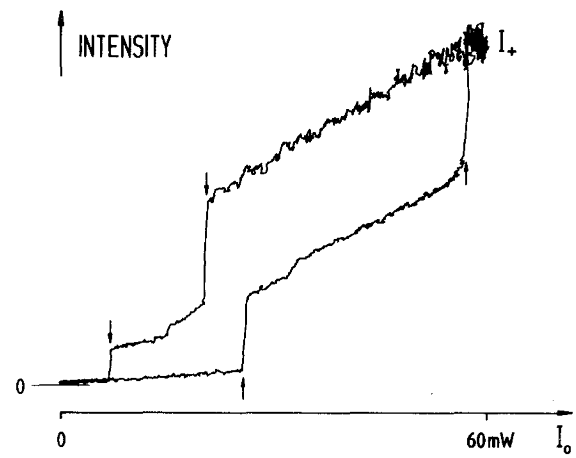

Fig. 5. Interwoven hysteresis cycles in the case of increased optical density (about a factor 2 with respect to Fig. 2). Parameters: laser detuning 15 GHz, $B_y = 0.4 \times 10^{-4}$ T

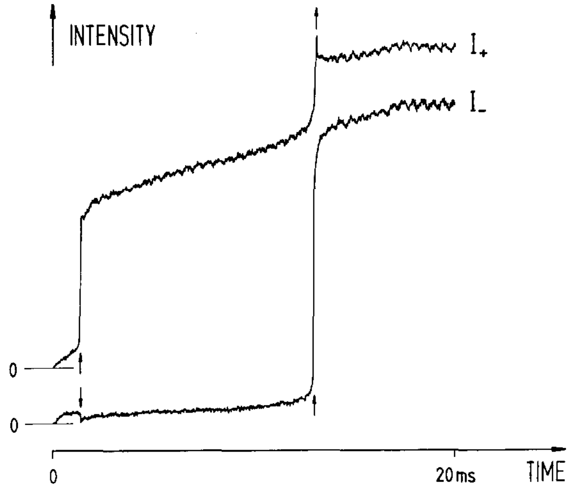

Fig. 6. System response to a sudden switch of input intensity. Parameters: $I_0 = 30\,\mathrm{mW}$, laser detuning 5 GHz, optical density similar as in Fig. 2, $B_y < 0.1 \times 10^{-4}\,T$

experiment. Polarization switching always comes first. Even if the output intensity were not discriminated with respect to its polarization components, polarization switching would be almost impossible to overlook, because it leaves its fingerprints also in the total intensity. We caution, however, that this statement may not hold when inadvertently there is a polarization-selective or birefringent element in the resonator, or when there is a longitudinal magnetic field.

2.2. Transient Behaviour

Given the rich dynamics of bistable systems an interest in transient phenomena is well warranted. "Critical slowing down" [1], "noncritical slowing down" [14–16] and "transient noise-induced bimodality" [17–19] have been observed and analyzed. In most experiments there was just one dynamic variable like the population of the excited state or the ground state spin orientation. In the present experiment obviously both these quantities play their role simultaneously and we wonder whether the transition from a one-dimensional to a two-dimensional system gives rise to new dynamical effects. Up to now, however, we have not yet performed a detailed study of the transient behaviour. Nevertheless we already observe a new phenomenon, which might by called "transient symmetry breaking".

It occurs when we switch the input intensity from (nearly) zero to a value slightly above the *second* threshold. Thus both the initial and the final state are symmetrical states, i.e. they have no ground-state orientation. One might therefore expect that there is no orientation during the transient. The experiment, however, shows a different behaviour (Fig. 6). I_+ switches rapidly to high transmission and then changes slowly, until after about 13 ms it switches to an even higher value. I_- stays very low, until I_+ switches to its final value, at which moment I_- switches to the same high-transmission value as I_+. This means that finally the output is linearly polarized as expected. For an interval of intermediate times, however, the output is strongly elliptically polarized, i.e. the symmetry between I_+ and I_- is broken. So there is symmetry breaking, but as a *transient* phenomenon only.

3. Interpretation of the Results

3.1. Theoretical Model

In [8] a theoretical model describing the behaviour of spin-$\frac{1}{2}$ atoms in a resonator under conditions of strong homogeneous broadening was presented in detail. The model starts from the density matrix formalism, but by adiabatic elimination techniques a model is obtained which finally resembles the result of a rate equation approach. The population of the excited state was neglected, but its inclusion in the model is straightforward. For the sake of simplicity, we assume, however, that the Zeeman sublevels of the excited state are always equally populated. This is regarded to be an excellent approximation, since the orientation of the excited state is very sensitive to collisions due to its nonzero orbital angular momentum.

In the analysis the direction of propagation of the laser beam is used as quantization axis (z-axis). We allow for the effect of a static magnetic field $\mathbf{B}$ of arbitrary direction. Its transverse component defines the y-direction. The inclusion of a z-component of $\mathbf{B}$ ($\mathbf{B} = (0, B_y, B_z)$) is a straightforward generalization of [8] and provides the means of discussing the effect of spurious static magnetic fields.

In the extended model to be used here, we describe the atomic system in terms of the quantities u, v, and w, as defined in [8], which are proportional to the expectation values of the x, y, and z components of the spin in the sample, and an additional quantity s. s is defined as the sum of the diagonal elements of the density matrix over the Zeeman sublevels of the excited state and therefore represents the (total) population of the excited state.

Within certain approximations detailed in [8] the behaviour of the atomic system is determined by the equations of motion

$$\dot{u} = -(\gamma + P_+ + P_-)u + (\Delta(P_- - P_+) + \Omega_z)v\,, \tag{1a}$$

$$\dot{v} = -(\gamma + P_+ + P_-)v - (\Delta(P_- - P_+) + \Omega_z)u - \Omega_y w\,, \tag{1b}$$

$$\dot{w} = -(\gamma + P_+ + P_-)w + \Omega_y v + (P_- - P_+)(1 - 2s)\,, \tag{1c}$$

$$\dot{s} = -(\Gamma + 2P_+ + 2P_-)s + P_+ + P_- - (P_- - P_+)w\,. \tag{1d}$$

Here Δ is the laser detuning from the resonance frequency of the D_1 transition, P_+ and P_- are the pump rates introduced by the right-hand (σ^+) and left-hand (σ^-) circularly polarized components of the light field in the resonator, whose spatial variation is neglected. γ is the relaxation constant of the ground-state orientation, and Γ is the relaxation constant of s. Ω_z and Ω_y are the Larmor frequencies belonging to B_z and B_y, respectively.

The optical properties of the medium are determined by w and s only. They enter the absorption coefficients α_+ and α_- and the indices of refraction n_+ and n_- (for σ^+- and σ^--light) via

$$\alpha_\pm(w,s)=\alpha_0(1\pm w-2s)\,, \tag{2a}$$

$$n_\pm(w,s)=1+(n_0-1)(1\pm w-2s)\,. \tag{2b}$$

As described in [8], we also have to write an equation of motion for the electric field in the resonator; it is coupled to the properties of the medium via $\alpha_\pm$ and $n_\pm$. Provided that the time constant of the resonator is small compared to the time constants of the medium, the electric field can adiabatically be eliminated, and the pump rates can be described by

$$P_\pm(w,s,P_0,\kappa,\delta)=A(\alpha_\pm(w,s),n_\pm(w,s),\delta)^*(1\pm\kappa)^*P_0\,. \tag{3}$$

Here A is the Airy function pertinant to Fabry-Perot resonators. For a given resonator it depends, besides of the cavity detuning δ, on the absorbtion coefficient $\alpha_\pm$ and the index of refraction $n_\pm$. These, in turn, are coupled to the dynamic variables w and s via (2a) and (2b). κ characterizes the polarization of the incident light ($\kappa=\pm 1$ for circular polarization, $\kappa=0$ for linear polarization), and $(1\pm\kappa)^*P_0$ are the pump rates which would be introduced without a resonator. (For details, see [8].)

In the case $\Gamma \gg P_0$ the solution of (1d) is $s=0$ and (1a–c) reduce to the ones given in [8]. In this case the adiabatic elimination of the electric field is well justified, since the time-constant of the orientation is of the order of 10^{-5} s, while the time constant of the resonator is in the 10^{-8} s range. In the case treated here the procedure is problematic, since the lifetime of the population of the excited state is also about 10^{-8} s. It can be assumed, however, that the long-time behaviour of the system is slaved by the slow evolution of the orientation.

3.2. Discussion

In correspondence to the role of I_0 in the experiments we will treat P_0 as the only control parameter. We start from the situation of the simplest experiment on tristability, i.e. we assume linearly polarized light

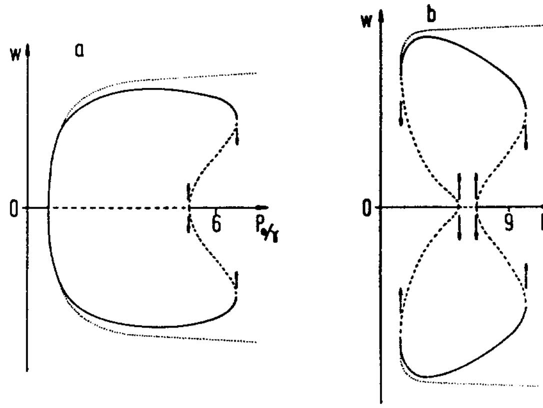

Fig. 7a, b. Typical bifurcation diagrams of the system. Dashed lines mark unstable, solid lines stable states. The dotted line is calculated under the assumption $s=0$. Parameters: **a** $\alpha_0 L=0.25$, $\Delta=10$, $\Omega_y/\gamma=0$, $\Gamma/\gamma=40$, $\delta=0.032$, **b** $\alpha_0 L=0.1$, $\Delta=6.5$, $\Omega_y/\gamma=30$, $\Gamma/\gamma=100$, $\delta=0.08$ (δ is measured in units of the free spectral range). Finesse of the resonator is $F=17.5$ in Figs. 7–12

($\kappa=0$) and vanishing magnetic field ($\Omega_y=\Omega_z=0$). In this case the stationary solution of (1a and b) is $u=v=0$, and any initial deviations from these values decay rapidly. By examination of the remaining equations (1c and d) it is immediately clear that $w=0$ is a stationary solution for all P_0, since $P_+(w=0)=P_-(w=0)$ as a consequence of (2a and b). Linear stability analysis reveals that the "symmetric" solution $w=0$ becomes unstable with increasing values of P_0, and it bifurcates to an asymmetric branch $w\neq 0$. As a consequence of the symmetry $P_+(w,s)=P_-(-w,s)$, which follows from (2), every point w_0, s_0 on the asymmetric branch has its counterpart $-w_0, s_0$. This conjugate point has the same stability properties, but describes a state of the system with opposite orientation and interchanged roles of P_+ and P_-.

A plot of $w_0(P_0)$ is shown in Fig. 7a and b; it can be seen that both supercritical and subcritical behaviour is obtained; in the latter case there is hysteresis and we find the phenomenon of tristability. (In the calculations we assumed parameters typical for the sodium system.) It is found that in the vicinity of the left bifurcation there is close similarity with the plot $w_0(P_0)$ obtained under the assumption $s=0$ also shown in the figures. In this regime of low P_0 values there can exist a pronounced orientation $w\neq 0$ of the atoms as a consequence of the slow relaxation of w (small values of γ), while the population s of the excited state with its fast relaxation ($\Gamma/\gamma \gg 1$) is still negligible.

In the intensity range well beyond the first bifurcation point the population in the excited state s can no longer be neglected, since the rates Γ, P_+, and P_- become compareable. The effect of a nonzero s is to reduce the available population in the ground state,

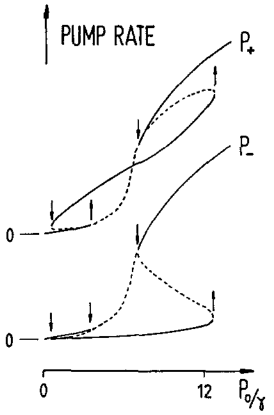

Fig. 8. Dependence of the pump rates P_+ and P_- on the light input in the dispersive case. Parameters: $\alpha_0 L = 0.1$, $\Delta = 7.5$, $\Omega_y/\gamma = 10$, $\Gamma/\gamma = 100$, $\delta = 0.08$

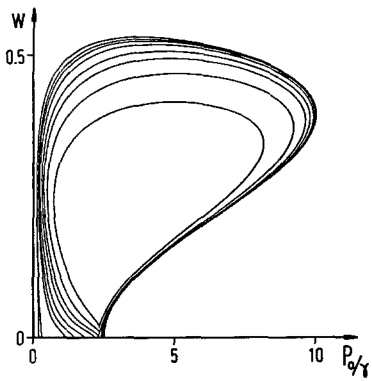

Fig. 9. Asymmetric branch of the state curve for different Larmor frequencies Ω_y. (For convenience only shown for $w > 0$.) Parameters: $\Gamma/\gamma = 75$, Ω_y/γ variing from 0 up to 50 from outer to inner curves, $\alpha_0 L = 0.25$, $\Delta = 15$, $\delta = 0.08$

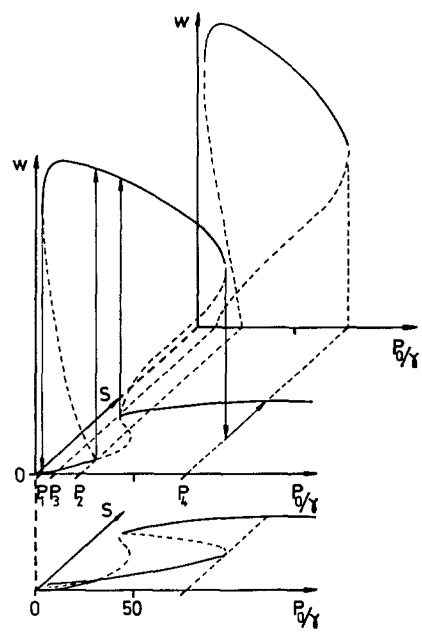

Fig. 10. 3*D*-plot of a state curve displaying interwoven hysteresis cycles. P_1 to P_4 mark the positions and the arrows the directions of switching. Parameters: $\Gamma/\gamma = 100$, $\alpha_0 L = 0.25$, $\Delta = 10$, $\delta = 0.26$, $\Omega_y/\gamma = 20$

$(1-s)$, and therefore w. This is manifest in a bending of the $w \neq 0$ states towards smaller values of w. That goes so far as to produce turning points of the asymmetric branches and a second bifurcation point, where the asymmetric branches meet the symmetric one. In other words, for very high input intensities the symmetric is recovered, and only the symmetric solution $w = 0$ exists, as described in the literature [4, 6, 7].

It turns out that the dependence of w on P_0 is very similar in the absorptive and in the dispersive case. The quantities observed in the experiment, I_+ and I_-, are proportional to P_+ and P_-, and these can be calculated from w and s by means of (3). Typical results for the dispersive case are shown in Fig. 8; they have to be compared to the experimental result of Fig. 2. In the absorptive case we obtain the type of behaviour shown in Fig. 4. In both cases all qualitative features are well reproduced. We conclude that the marked differences between Figs. 3 and 4 just reflect the different dependence of the Airy function on $\alpha_\pm$ and $n_\pm$, while the underlying physical mechanisms are very similar.

The shape of $w(P_0)$ is not qualitatively changed if a weak transverse magnetic field is taken into account. The most pronounced result is the shift of the first bifurcation point to higher values of P_0, accompanied by an increase of the width of the hysteresis loop (Fig. 9). This agrees well with the experimental observations (see above). The second bifurcation point does not strongly depend on the magnetic field, and therefore at some strong field the bifurcations can collide. In this case they are annihilated, and the asymmetric branches form closed loops isolated from the symmetric branch ("isolas"), as shown in Fig. 9.

We restrict the discussion again to the special case $\Omega_y = \Omega_z = 0$. In principle, a proper representation of the state curves requires the three dimensional $\{P_0, w, s\}$-space. Up to now projections to the $\{P_0, w\}$-plane capture all essential features of the state curves. We now encounter the situation, however, in which s displays the same S-shaped dependence on P_0 that is familiar for conventional two-level bistability. The condition for this to happen is that the "cooperativity parameter" C is sufficiently large [20]. For a given resonator C increases with the particle density. Therefore we should anticipate a more complicated behaviour of the state curve for large optical absorptions.

This is indeed confirmed by numerical results. Figure 10 shows the state curve in three-dimensional space and also its projections on the $\{P_0, s\}$- and $\{P_0, w\}$-planes. The projection to the $\{P_0, s\}$-plane is in fact S-shaped in the symmetric branch $w=0$; in addition there is a feature representing the projection of the asymmetric branch. In the projection on the $\{P_0, w\}$-plane we can have a crossing, since the second bifurcation point of the state curve can lie at lower values of P_0 than the first one due to the s-shaped bending seen in the $\{P_0, s\}$-plane. In this example there exist up to nine fixed points for the same value of P_0. This occurs in the interval between P_3 and P_2 (Fig. 10). Here the stable fixed points can be characterized by

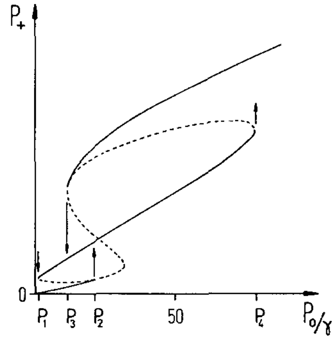

Fig. 11. Pump rate P_+ derived from the state curve of Fig. 10

$w=0$, s low
("symmetric low transmission state"),

$w=0$, s high
("symmetric high transmission state"),

$w=\pm w_0 \neq 0$ (two asymmetric states).

In this situation the system displays *quadrustability*. The corresponding behaviour of P_+ is illustrated in Fig. 11. Obviously it can give a satisfactory qualitative explanation of the observation shown in Fig. 5.

In the calculations there is no indication of the existence of conventional two-level bistability. For large values of Γ ($\Gamma/\gamma \gg 1$) the branch $w=0$ of the state curve *always* becomes unstable before its turning point is reached. In any transition to the asymmetric branches, however, not only P_+ and P_- change, but also their sum. Such transitions should therefore show up in an experiment even if that does not discriminate the different circular polarization components.

Again we caution that this statement only holds under idealized conditions. In the calculations we do not only assume that the incident light is linearly polarized and that there are no polarization-selective elements in the resonator, but also that there is no longitudinal component of the magnetic field present. Violation of such condition would change the issue considerably, because bifurcations like Fig. 2 are structurally unstable [21].

It suffices that the incident light is only slightly elliptically polarized, and $w=0$ is not any longer a solution of (1c). With the symmetry broken by the polarization of the light, there is no *spontaneous* symmetry breaking, and bistable behaviour takes the place of tristability (see also [22]). The bistability would also be observed in the total output intensity, $(P_+ + P_-)$. However, this bistability is not related to a saturation of the optical transition, but to Zeeman pumping.

In the case of perfectly linear polarization but with both a longitudinal and a transverse component of the magnetic field ($\Omega_y \neq 0$, $\Omega_z \neq 0$) the symmetric branch $w=0$ still exists. However, it becomes unstable through a *transcritical* rather than a pitchfork bifurcation. That implies that the asymmetric branches lose their symmetry with respect to each other and bifurcate from the $w=0$ branch under an oblique angle. Again, the symmetry is broken already so that no spontaneous symmetry breaking occurs. An experiment would again display bistability which might easily be mistaken for two-level bistability, particularly since a component of the earth magnetic field would already be sufficient to break the symmetry.

A particularly transparent description of the behaviour of bistable and tristable systems can often be given by means of the analogy to the overdamped motion of a particle in a potential. In the present case the description can be used, if we exclude the presence of a magnetic field. In the following we assume $\mathbf{B}=0$, $\kappa=0$. If we further assume that at any instant of time $u=v=0$, then for all times $u=v=0$, *and the system is fully described* by (1c) and (1d). They can formally be written

$$\dot{\mathbf{x}} = -\operatorname{grad} V(\mathbf{x}). \tag{4}$$

The components of $\mathbf{x}$ are defined by $x_1=w$ and $x_2=\sqrt{2}s$. The potential V is easily obtained by numeric integrations; it depends on P_0 as a parameter. As an example Fig. 12 shows contour lines of V for a set of parameters corresponding to the situation $P_3<P_0<P_2$ in Fig. 10. It can be seen that there are four stable fixed points, i.e. the system is quadrustable. In addition, there are five unstable fixed points whose nature can be deduced from the figure. By regarding the variation of the potential with P_0 an improved understanding of Fig. 10 can be obtained.

It is obvious that in two- or even more dimensional potential surfaces the trajectories describing the transient behaviour of the system are much more complicated than in the one-dimensional case predominantly discussed so far in the literature on optical bistability. For example, we find that in the situation of

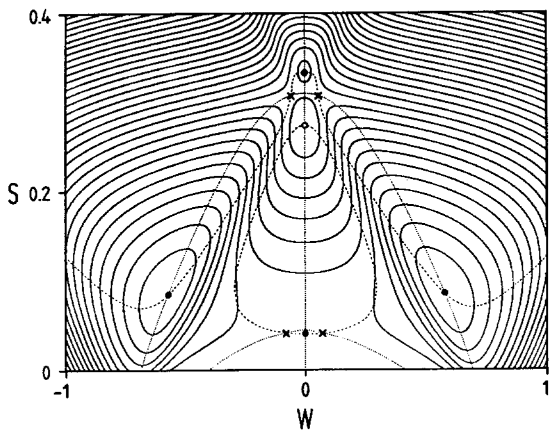

Fig. 12. Contour lines of the potential surface in a situation of quadrustability. Full circles: stable nodes, crosses: saddle, open circle: unstable node. The dashed and dotted lines define the sites of $\partial V/\partial s=0$ and $\partial V/\partial w=0$, respectively. Parameters: $\Delta=10$, $\delta=0.4$, $\alpha_0 L=0.4$, $\Gamma/\gamma=40$, $P_0/\gamma=9$

the transient experiment discussed above in connection with Fig. 6 any small deviation from the initial condition $w=0$ can yield tremenduous asymmetries occuring for intermediate times, before the system finally approaches a symmetric steady state. Since the trajectories can pass very flat regions of the potential there can be generalizations of the "critical slowing down"-phenomena discussed in one-dimensional systems. In computer simulations we clearly find the phenomenon of "transient symmetry breaking" observed in the experiment, if not yet in convincing quantitative agreement with Fig. 6. It is obvious, however, that the sensitivity to the initial conditions mentioned above implies a strong sensitivity to small perturbations and to imperfections of the experiment.

In this context we would like to point out that the existence of a potential facilitates the treatment of the *stochastic* behaviour of the system. It may be rewarding to generalize the discussions of phenomena like "transient noise-induced bimodality" [17–19], which are described by a one-dimensional Fokker-Planck equation, to the present case.

4. Conclusion

In this paper we presented observations on the behaviour of sodium-filled resonators under conditions of high buffer gas pressure. We explained them qualitatively by means of a simple model which assumes spin-$\frac{1}{2}$-atoms and takes into account the ground state orientation and the population of the excited state. It turns out that in experiments involving linearly polarized light input and the absence of magnetic fields and polarizing optics, the conventional two-level bistability due to population of the excited state always mingles with spontaneous symmetry breaking due to ground-state Zeeman pumping. The effects can be interwoven in a fairly complicated way and can then give rise to new phenomena like a type of quadrustability which involves just one transmission peak of the optical resonator.

In experiments on the transient behaviour we also observed "transient symmetry breaking". This new phenomenon occurs in the evolution of the system from a symmetric initial to a symmetric final state. While the sensitivity of the calculated trajectories to the initial conditions, to stochastic forces and to minute deviations of parameters from idealized assumptions render it difficult to perform a comparison between theory and experiment, the new phenomenon probably can be explained by the analogy of the overdamped motion of a particle on a two-dimensional potential surface.

We demonstrated that a conceptionally simple system – spin-$\frac{1}{2}$ atoms inside a cavity, interacting with a *cw* light field – can display considerable complexity in its steady state behaviour. Complexity arising from a simple structure seems to be the "leitmotif" of nonlinear feedback systems, yet one cannot help being surprised time and again.

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