A NOTE ON THE ANALOGY BETWEEN MOMENTUM TRANSFER ACROSS A ROUGH SOLID SURFACE AND THE AIR–SEA INTERFACE

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Abstract. The aerodynamic classification of the resistance laws above solid surfaces is based on the use of a so-called Reynolds roughness number $\mathrm{Re}_s = h_s u_*/\nu$, where h_s is the effective roughness height, ν – viscosity, u_* – friction velocity. The recent experimental studies reported by Toba and Ebuchi (1991), demonstrated that the observed "variability" of the sea roughness cannot be explained only on the basis of the classification of aerodynamic conditions of the sea surface proposed by Kitaigorodskii and Volkov (1965) and Kitaigorodskii (1968) even though the latter approach gains some support from recent experimental studies (see for example Geernaert *et al.* 1986). In this paper, an attempt is made to explain some of the recently observed features of the "variability" of surface roughness (Toba and Ebuchi, 1991; Donelan *et al.*, 1993). The "fluctuating" regime of the sea surface roughness is also described. It is shown that the contribution from the dissipation subrange to the variability of the sea surface can be very important and by itself can explain Charnock's (1955) regime.

1. Introduction

In a recent summary of small-scale air-sea interaction studies (Donelan, 1990), the data analysis led to rather interesting conclusions about the "variability" of sea surface roughness. Together with the study by Geernaert *et al.* (1986), this summary indicates the relative success of the analogy between momentum transfer in the vicinity of a solid rough surface and the air–sea interface, when waves are considered as moving roughness elements. Such an analogy was suggested and used first by Kitaigorodskii (1968) in an effort to explain both the success and limitations of Charnock's (1955) prediction of the wind dependence of the surface roughness parameter.

We shall argue in this paper that one of the main reasons for the failure of such an analogy is that all previous approaches (Charnock, 1955; Kitaigorodskii, 1968, 1970) didn't take into account the differences in the conditions of wind wave breaking at different stages of wave development as well as the existence of "aging" waves generated by nonlinear energy transfer. To take all these effects into account, a useful tool can be Kitaigorodskii's (1983, 1992) theory of the dissipation subrange. As a possible feed-back mechanism of momentum transfer from waves to the atmosphere, a reasonable explanation can be based on the inverse energy cascade theory (Kitaigorodskii, 1983; Zakharov and Zaslavskii, 1982; Zakharov, 1992).

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2. Description of the "Variability" of the Sea Surface Roughness

When applying Kitaigorodskii's (1968) aerodynamic classification for the air–sea interface, two questions are of primary importance:

a) What is the limitation of the fluid dynamical analogy between the sea surface and solid rough surfaces? and

b) What is the roughness height for different wind wave generation conditions?

Charnock's (1955) idea that all the stress is supported by the high wave-number part of the spectrum, leading to a strictly wind-speed (or friction velocity) dependent roughness length was attractive and adequate for many ocean cases. According to this idea, the effective height of roughness elements of the sea surface h_s depends only on u_* and g, so that

$$h_s \approx u_*^2/g. \tag{1}$$

This leads to a typical value of $h_s \approx 1$ cm, much larger then a typical value of the thickness of the viscous sublayer $\delta_\nu = \nu/u_* \approx 5 \cdot 10^{-2}$ cm for an average value of $u_* = 30$ cm/s. The Reynolds roughness number Re_s based on these two length scales is equal to

$$\mathrm{Re}_s = h_s/\delta_\nu \approx u_*^3/g\nu \gg 1 \tag{2}$$

and practically for all reasonable values of u_* is larger than 100. Therefore the sea surface is generally aerodynamically rough. A more precise description of the critical values of Re_s (or u_*) can be found from the classical relationships, identifying the aerodynamically smooth and transitional regimes

$$z_0 \approx 0.11\delta_\nu = 0.11\nu/u_* \quad \text{for } \mathrm{Re}_s < \mathrm{Re}_s' \approx 5 \tag{3}$$

$$z_0 = A_s h_s, \quad A_s \approx 1/30 \quad \text{for } \mathrm{Re}_s < \mathrm{Re}_s'' \approx 90. \tag{4}$$

The fact that $\mathrm{Re}_s > 10^2$ was a justification to write simply that

$$z_0 = m u_*^2/g \tag{5}$$

where m was later called the Charnock constant. For the range of wind speed 7.5–20 m/s, the empirically determined average value of m is 0.014 (compare with Nikuradse grain roughness $z_0 = h_s/30$ which corresponds to $m \approx 0.03$). This alone shows that the viscous stresses cannot be very important.

However, the drag law mechanism behind Charnock's formula is not well defined and, consequently, one of the deficiencies of Charnock's approach is related to the non-universality of the numerical value of m. For example, expression (5) with m = 0.014 doesn't yield high enough values of z_0 for situations with accelerating wind fields or storm surges.

The first assessment of Charnock's approach was by Kitaigorodskii (1968) whose aerodynamic classification of sea surface roughness "variability" was checked against available field and laboratory observations in his book (Kitaigorodskii, 1970). Since then, several efforts have been made to improve the results presented in Kitaigorodskii and Volkov (1965) and Kitaigorodskii (1970); among them the most important were the recent works of Geernaert *et al.* (1986), Donelan (1990) and Panin and Krivitskii (1992). We would like to mention also a short note by Jones and Toba (1991) as an addition to Donelan's (1990) analysis on the "variability" of the sea surface roughness parameter. All these works gave enough convincing evidence that the mobility of sea surface roughness elements (progressive waves) definitely plays an important role in the "variability" of the sea surface roughness parameter during wave development.

However, Figures 5a, 5b from Toba *et al.* (1990) and Figures 11, 12, 13 from Donelan (1990) demonstrate that the geometrical height of the roughness elements $h_s = 30\ z_0$ normalized on $\sigma_\eta = (\xi^2)^{1/2}$ or $H_s = 4(\xi^2)^{1/2}$ ($\xi(x,t)$ is the surface displacement), when plotted against the nondimensional inverse wave age U_a/C_p (U_a – wind speed and C_p – phase speed at the peak of the spectrum) doesn't produce universal relationships. Rather the results lie along two different almost parallel lines. The data presented in Figure 2 of Jones and Toba (1991) are also in agreement with this picture. The same is true also in the observed picture of the "variability" of m. This demonstrates one of the difficulties with the strict analogy between a rough solid and a mobile sea surface with respect to the drag law, and possibly means that the simple descriptions suggested by Kitaigorodskii (1968) and Donelan (1990) are not sufficient.

3. Wind Wave Breaking, Dissipation Subrange in Wind Wave Spectra and Their Influence on the Variability of the Sea Surface Roughness

As was already mentioned above, normalization of experimental data on the sea surface roughness, according to Kitaigorodskii's (1968) classification, doesn't produce the expected collapse of field and laboratory data; rather it produces two different groups of experimental points. We argue below that the reason for this difference is due to the difference in wave breaking conditions. We shall try to relate such difference to the very presence and different location of the dissipation subrange in wind wave spectra for these two regimes.

In laboratory conditions, microscale wave breaking does not reduce sufficiently the h_s values, and practically all wave components contribute to the mobile roughness elements. Furthermore, in this range of U_a/C_p ($U_a/C_p > 2.5$), it is quite likely that the directionality becomes so important that only waves moving into the wind direction contribute to the mobile roughness elements. Thus it is unrealistic to apply an isotropic approximation for this range of U_a/C_p for wavenumbers much higher than the wavenumbers typical for the so-called equilibrium range, where the direct energy cascade toward smaller scales due to nonlinear wave-wave interactions is dominant. As was pointed out by Kitaigorodskii (1992), the very definition and existence of a dissipation subrange can be used effectively in data analysis, only when it can be characterized by more rapid spectral fall-off, compared with one in an equilibrium range. It must not be forgotten that in Kitaigorodskii (1992), the dissipation subrange was defined as a region in the 2-D wavenumber space (k, y), where divergence of nonlinear energy flux is zero,

$$S_{n1}^{(k)} = 0 \quad \text{for} \quad k_{in} < k < \hat{k}_g, \tag{6}$$

and equal to the dissipation of wave energy due to gravitational instability,

$$S_{n1}^{(k)} - S_{\text{diss}}^{(k)} = 0 \quad \text{and} \quad k_\gamma > k > \hat{k}_g. \tag{7}$$

In (6)–(7), k_{in} is the wind input wavenumber, k_γ represents the wavenumber where either viscosity or capillary tension becomes important, and $S_i^{(k)}$ represents the so-called "source" terms in the wave transport equation ($S_{n1}^{(k)}$ – nonlinear interactions, $S_{\text{diss}}^{(k)}$ – dissipation due to breaking). The effective value $\hat{k}_g$ is equal to

$$\hat{k}_g = \int_\vartheta k_g(\vartheta)d\vartheta = Bg/\varepsilon_0^{2/3} \tag{8}$$

where ε_0 – energy flux towards higher wavenumbers in the direct energy cascade region and B is a constant. The expression shows that the "upper" boundary value of the nondissipative part of the wave spectra k_γ depends on the angle ϑ. The value of $\hat{k}_g$ can be related to the transitional frequency w_g in Figure 1 in Kitaigorodskii (1992) through the usual dispersion relationship if Doppler shifting is not taken into account. The values of $\hat{k}_g$ and w_g can be defined empirically (Kitaigorodskii, 1992) as the beginning of rapid spectral fall-off on the rear face of the spectra associated with the asymptotic approach to saturation forms:

$$\psi(k, \vartheta) = Bk^{-4}\sigma(\vartheta) \tag{9}$$

$$S(\omega) = \beta g^2 \omega^{-5} \tag{10}$$

where $B = 2\beta$, $\psi(k, \vartheta)$ is a symmetrical wave number spectrum, $S(\omega)$ – the frequency spectrum. Actually because of the mismatch between the asymptotic saturation forms (9)–(10) and the equilibrium forms of the spectra $\psi(k, \vartheta)$, given as

$$k_{in} < k < \hat{k}_g \quad \psi(k, \vartheta) = A\varepsilon_0^{1/3} g^{1/2} k^{-7/2} F(\vartheta) \tag{11}$$

$$\omega_{in} < \omega < \hat{\omega}_g \quad S(\omega) = 2A\varepsilon_0^{1/3} g\omega^{-4} \tag{12}$$

(ω_{in} – energy input frequency, $F(\vartheta)$ – angular distribution), the spectral fall-off can display a more rapid decrease of energy with k or ω than (9), (10) (Glazman and Weichman, 1989).

Very recently, however, careful data analysis by Hansen *et al.* (1990) and Kitaigorodskii (1991) demonstrated that as a first approximation, (11)–(12) are valid, with the transition to the dissipation subrange ω_g being a monotonically increasing function of U_a/C_p (see Figure 1 in Kitaigorodskii, 1991), which illustrates the shift of ω_g towards lower frequencies with wave growth. The latter fact is in agreement with the assumption about the existence of the dissipation subrange (Kitaigorodskii, 1983), since the transitional frequency in Kitaigorodskii (1991) found in most of the cases can be definitely associated with the existence of an energy cascade pattern in the wind wave spectra at scales larger than the transitional scale $\lambda_g = 2\pi/\hat{k}_g$. Thus as a first result for the description of the shift of the dissipation subrange in frequency and wavenumber domain, we can accept the one given by Figure 1 in Kitaigorodskii (1991), which at least does not contradict Equation (8) because generally it must be that $\varepsilon_0 = \varepsilon_0(\tilde{\omega}_p)$. The existence of the dissipation subrange and its location in wavenumber space must be reflected in the "variability" of the aerodynamic roughness of the sea surface. Indeed the width of the dissipation subrange and its location in wavenumber space indicate the range of scales responsible for creating "sharp crest" waves which, most likely, can be associated with air flow separation behind the crests, the process responsible for creating an effective roughness height h_s in analogy with momentum transfer with a solid surface. The sharply crested waves are the steepest roughness elements among those moving with the same phase velocity, and thus they are most important for flow separation phenomena. Therefore according to the relationship $\tilde{\omega}_g = \tilde{\omega}_g(\tilde{\omega}_p)$, we can expect that with an increase in width of the dissipation subrange (ω_g moves to lower frequencies), the effective roughness must increase, since it must be proportional to the contribution for the mean square surface displacement from sharply crested waves. This is a very important conclusion, which shows that the popular method of scaling air-sea roughness with H_s or σ_η is unable to clarify the reasons for the observed "variability" of z_0, because neither H_s nor σ_η is a fixed portion of the contribution to $\bar{\xi}^2$ from the dissipation subrange. The previously published figures about the dependence of the nondimensional roughness length z_0/H_s on the inverse wave age U_a/C_p indicates a more or less linear increase of z_0/H_s over two to three orders of magnitude when U_a/C_p changes from say 10^{-2} to 10. The roughness parameter itself may behave differently, since such figures can either just reflect the dependence of H_s on U_a/C_p (spurious correlation) or what is more relevant, reflect the decrease of z_0 with a decrease of U_a/C_p. Therefore, it is interesting to plot the Charnock constant m versus U_a/C_p. This was first done and examined by Kitaigorodskii (1970) (Figure 1.20) on the basis of an expedition in the Pacific. These data show an increase of m with $g\sigma_\eta/U_a^2$, which means increasing m with decreasing U_a/C_p or $\omega_p U_a/g$. At that time, however, data were scarce and not very accurate, so that no definite conclusions could be drawn about the dependence of

m on wave age. This problem was considered very recently in an interesting paper by Toba and Ebuchi (1991) and their Figures 8 and 9 illustrate the complex picture of the changes of m, which the authors tried to explain by differences in wind conditions (accelerating or decelerating). However from our viewpoint, the most interesting feature of simultaneous wind and wave time series, presented in their Figures 5 and 6, was the observed correlation between the sea surface roughness and deviations of wind wave spectra from the so-called Toba wind-dependent form, which can be interpreted also in the framework of Kolmogoroff's direct energy cascade theory as in Equations (11) and (12). Toba and Ebuchi (1991) introduced what they call the degree of undersaturation $\tilde{\alpha}_s$ defined as

$$\tilde{\alpha}_s = 1 - \alpha_s/\bar{\alpha}_s \tag{13}$$

where $\bar{\alpha}_s$ is the average value of the coefficient of energy level in the equilibrium spectra given by

$$S(\omega) = \bar{\alpha}_s g u_* \omega^{-4}. \tag{14}$$

According to Toba and Jones (1991), the value of $\bar{\alpha}_s$ is 0.062. The range of frequencies used to determine α_s was from 0.5 to 3 Hz. The Toba spectrum can be interpreted in the framework of equilibrium direct cascade theory (11), (12) (Kitaigorodskii, 1983). In Hansen *et al.* (1990), it was shown that $\bar{\alpha}_s = 0.052$ $(g/\omega_p u_*)^{1/3}$ exists. This relation gives for example in the vicinity of $\tilde{\omega}_p = 2$, the value of $\alpha_s = 0.041$ which is not far from the value reported by Toba and Jones (1991) $\alpha_s = 0.062$. According to Toba and Ebuchi (1991), fluctuations in $\tilde{\alpha}_s$ are caused by the delay for the adjustment of the energy level of the equilibrium range to the wind variation. We reproduce here Figure 5 and 6 from Ebuchi and Toba (1991) as Figure 1a, b which both demonstrate that time series of $\tilde{\alpha}_s$ and z_0 have some common broad features. The most important and intriguing question now is what are the reasons for the observed correlation between z_0 and $\tilde{\alpha}_s$? Are they related to changes in the level in the equilibrium part of the spectrum or to changes in the width of the dissipation subrange, or to both? We'll address this question in the next section.

4. Examples of Data Analysis

Toba and Ebuchi (1991) suggested that in order to explain the observed "variability" of z_0 the correlation between $\tilde{\alpha}_s$ and z_0 must be studied on time scales smaller than usual. Although the wind speed is usually averaged over 15 min periods, it is also possible by using 3 min moving average wind data to investigate the mechanism of the variations of wind stress in relation to the wind waves. The physical interpretation by Toba and Ebuchi (1991) was as follows: variations of U_a on time scales of several minutes to one hour cause variations of $\omega_p = 2\pi f_p$ and

$\tilde{\alpha}_s$. Large negative values of $\tilde{\alpha}_s$ correspond to the situation where the level of the wave equilibrium spectra "increases rapidly". Such a condition must be associated with larger z_0, since the wave components at the very high frequency part of the wave spectra outside the equilibrium range are steeper in this situation, and small-scale processes related to viscosity and surface tension can be more important in maintaining momentum flux. The latter notion disagrees with the idea proposed by us about the relationship between the width of the dissipation subrange and values of the surface roughness, because the dissipation subrange in Kitaigorodskii's (1991) definition is outside the region where surface tension or viscosity are important. According to observations by Toba and Ebuchi (1991), when $\tilde{\alpha}_s$ is changing, the shape of the spectrum remains still close to ω^{-4}, which must reflect the important role of wave-wave interactions in maintaining the equilibrium form (14). Consequently, air momentum entering the waves at the high frequency part of the spectrum is quickly transferred to the whole equilibrium range "up" to near ω_p. Toba and Ebuchi (1991) also mentioned the subtle variation with $\tilde{\alpha}_s$ of the wave spectral shape near ω_p. Their conclusions, though interesting and intuitively probably correct, still don't give a clear answer to the main question: what are the reasons for the correlation between variations of $\tilde{\alpha}_s$ and the surface roughness z_0 for different situations with $\tilde{\alpha}_s < 0$ or $\tilde{\alpha}_s > 0$?

We shall try to present a consistent picture of the "variability" of z_0 with wave parameters which, from our viewpoint, can explain the observations summarized by Toba and Ebuchi (1991) in Figure 7 where the Charnock constant m was plotted versus U_a/C_p. First to be mentioned in this context must be that, from the very definition of $\tilde{\alpha}_s$, it follows that negative values $\tilde{\alpha}_s$ correspond to local values of α_s exceeding the average value α_s. This can be interpreted (see Figure 2) as the case when the dissipation subrange given by its saturation form (9–10) prevails over the equilibrium range since, for given ω, the value of $S(\omega)$ in the dissipation subrange is then larger than the equilibrium value (14). This is possible in the situation when the dissipation subrange moves towards lower frequencies. Such situations correspond to wind-wave growth (either with fetch or duration) and probably also to an accelerating wind field because, in the latter case, many wave components are moving more slowly than the wind and the role of wave-wave interactions producing the direct energy cascade increases.

Thus we come to the very important conclusion that the observed correlation between the increase in absolute negative values of $\tilde{\alpha}_s$ and the increase in roughness parameter z_0 (see some dashed parts of Figure 1b based on Toba and Ebuchi Figure 6) can be associated with the existence of the dissipation subrange and its different widths in the frequency domain. In such cases ($\tilde{\alpha}_s < 0$), we can expect the roughness to increase with the shift of the dissipation subrange boundary ω_g towards lower frequencies (decrease of U_a/C_p according to Figure 1 in Kitaigorodskii, 1992). Such a shift means that there is an increase of width of the dissipation subrange. This in turn can be understood as an increase of a number of sharp-crested (breaking) waves responsible for air flow separation that must lead to an

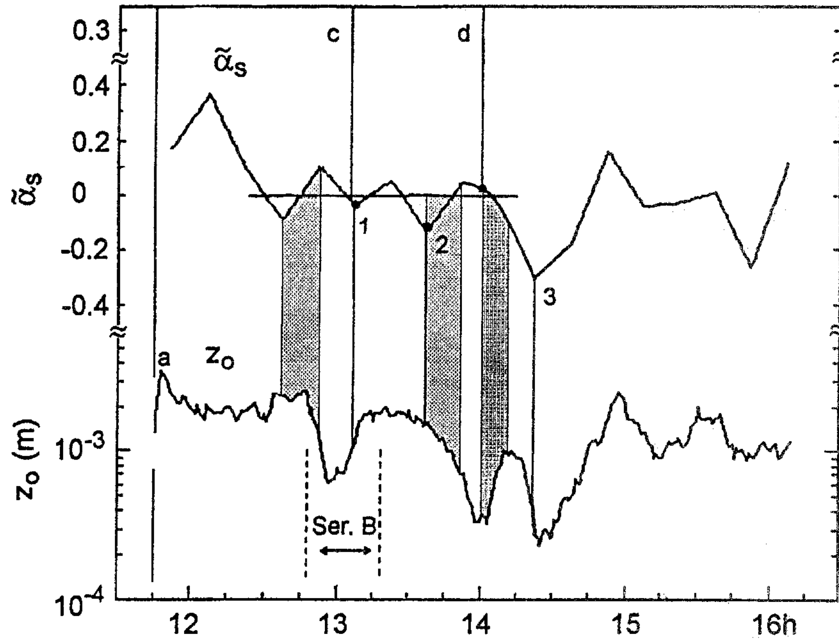

Fig. 1a. The 4 hr time series (Series A) of the degree of undersaturation $\tilde{\alpha}_s$ defined by Equation (13) and the roughness length z_0. The straight lines 1, 2, 3 identify the points chosen for the case $\tilde{\alpha}_s \leq 0$ in Table I.

increase in z_0. This can be seen only in a few sections of the time series of $\hat{\alpha}_s$ and z_0 in Figure 1a, b which by itself demonstrates that the contribution from the dissipation subrange is not necessarily the only factor influencing sea-surface roughness values. Figure 1a gives another example of the influence of the width of the dissipation subrange on the roughness: the increase in absolute positive values of $\hat{\alpha}_s$ coincides with the decreases in z_0.

To give some examples of the relationships between z_0 and wave age, we present in Table I some simple calculations of the values of the nondimensional roughness z_0 and nondimensional peak frequency f_p for some points already identified on Figure 1a, b.

From Figure 1a we choose three basic points as extremes in the region $\tilde{\alpha}_s < 0$. The two other similar points in Figure 1a (at 12.30 h and 16.00 h) were not used since Table I was constructed for illustration purposes only. As can be seen from Table I, these points from Figure 1a associated with an increase of $\omega_p U_a / g$, though in a very narrow range (a decrease of width of the dissipation subrange), correspond to a decrease of nondimensional roughness (in terms of $g z_0 / u_*^2$ or z_0 / σ_η). Figure 1a illustrates the usually accepted 15 min mean wind and corresponding z_0, whereas Figure 1b illustrates 28 time series where u_* and z_0 were derived as 3 min averages and f_p and $\hat{\alpha}_s$ as 1 min average quantities.

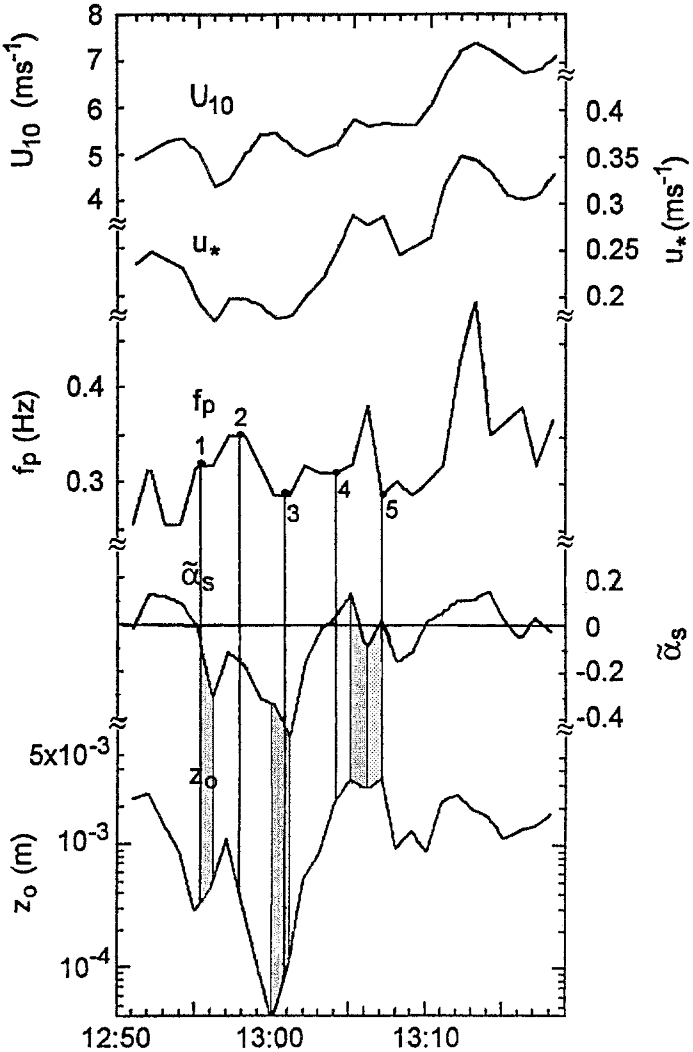

Fig. 1b. The 28-min time series (series B) which is a part of series A. The data of $f_p = \omega_p/2\pi$ and $\tilde{\alpha}_s$ are here 1 min values. The straight lines 1, 2, 3, 4, 5 identify the points chosen for case $\tilde{\alpha}_s \leq 0$ in Table I.

From Figure 1b for $\tilde{\alpha}_s < 0$, we choose another five points, shown by numbers 1–5 and corresponding lines. The points 4, 5 roughly corresponds to $\tilde{\alpha}_s \approx 0$, i.e., to the average position of the dissipation subrange (Figure 2). Since it is clear from Figures 5, 6 of Toba and Ebuchi (1991) that the behaviour of u_* is similar to U_a, we prefer in Table I to choose $U_a = U_{10}$ as the wind parameter (the results will not be affected if we replace U_{10} by u_*). It is evident from Table I that in the very narrow range of $\omega_p U_a/g$ (0.93–1.12), there is no systematic change in nondimensional roughness (if we use the points from Figure 1b). The latter points

TABLE I

The main characteristics for three basic points in Figure 1a and five points in Figure 1b for $\tilde{\alpha}_s$

N points	f_p (Hz)	$\omega_p = 2\pi f$	U_a (m/sec)	z_0 (m) 10^{-3}	gz_0/U^2 10^{-5}	$\omega_p U_a/g = U_a/C_p$
Figure 1a						
1	0.30	1.88	5.2	1.5	55	0.97
2	0.33	2.07	7.0	1.5	30	1.45
3	0.32	2.01	9.0	0.3	6	1.89
Figure 1b						
1	0.32	2.01	4.5	0.07	2.8	0.9
2	0.35	2.20	5.0	0.3	15	0.92
3	0.28	1.76	5.0	3.0	94	1.01
4	0.32	2.01	5.5	0.52	20(?)	1.12
5	0.28	1.76	5.6–5.7	2.5	100	1.13

in Table I rather represent what Toba and Ebuchi (1991) called the "fluctuating" regime of the sea-surface roughness which can be used to define the mean value of the Charnock constant. One of the possible reasons why data from Figure 1b in Table I cannot be interpreted in terms of a variable width of the dissipation subrange (as data from Figure 1a) is that 1 min average values of $\tilde{\alpha}_s$ and $\tilde{f}_p$ cannot characterize the process of adjustment of the width of the dissipation subrange to wind-wave growth (changes in U_a/C_p).

The data in Figure 1a, b cover a rather narrow range of wave age or nondimensional peak frequency ($\omega_p U_a/g \approx 0.9$–$1.9$), corresponding to well-developed wind waves. If we simply take $u_* = U_a/30$, the range $\omega_p u_*/g$ would be 0.03–0.063. The corresponding range of variation of the Charnock constant $m = gz_0/u_*^2$, covered by the data in Figures 1a, b is then 0.025–0.9. We reproduce here Figure 8 from Toba *et al.* (1990) as Figure 3. It can be clearly seen that this is essentially the range of values of m and $\omega_p u_*/g$ which represent that part of the data set circled by us in Figure 3, cited in Toba and Ebuchi (1991).

The remaining points in Figure 3 can be used only for determination of the average value of $m = gz_0/u_*^2$ in the region $\omega_p u_*/g \approx 0.06$–$0.12$. On the basis of Figure 3, we suggest a hypothesis that there are several patterns of behaviour of the nondimensional roughness with wave age U_a/C_p. For waves with $u_*/C_p < 0.05$, the roughness diminishes with increasing U_a/C_p, which can be related to the decrease of the width of the dissipation subrange with wave growth (Figure 1 in Kitaigorodskii, 1991, which we reproduce here as Figure 4). For wave ages typical of well developed seas (up to the Pierson-Moscovitz condition), say $u_*/C_p < 0.05$ in Figure 3, the tendency is opposite. The latter can be explained by the occurrence of long fast gravity waves generated due to the inverse energy cascade which can

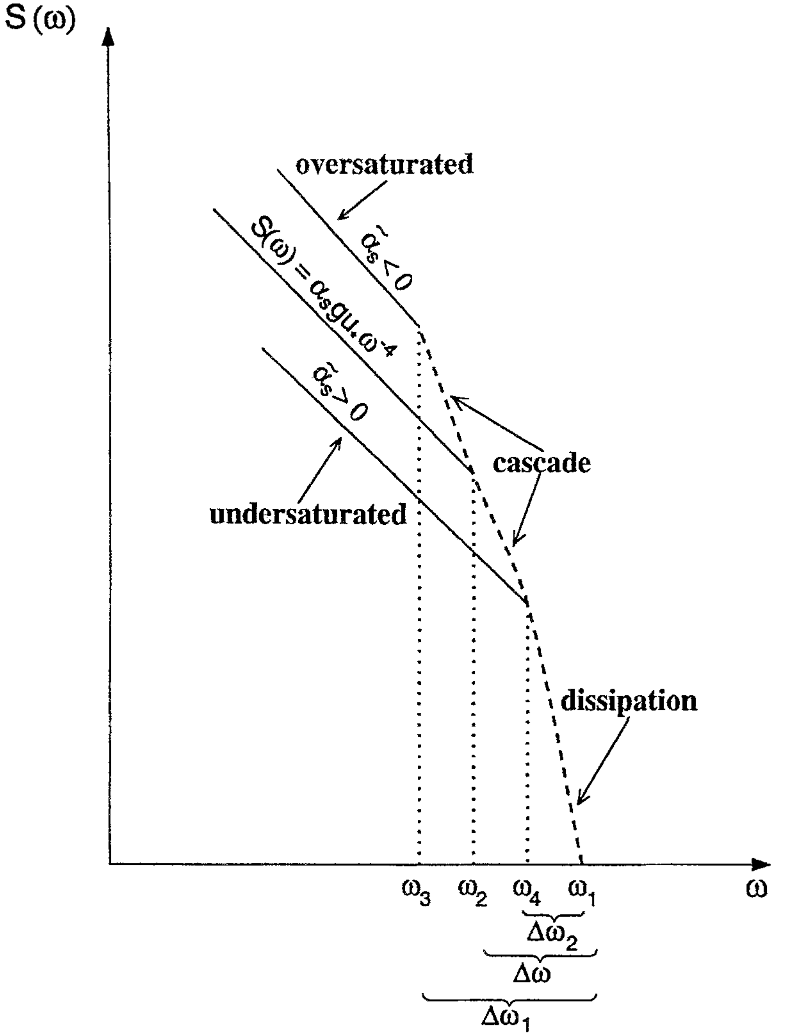

Fig. 2. Sketch illustrating the shift of the low frequency boundary of the dissipation subrange, and the variations of width of the dissipation subrange with $\tilde{\alpha}_s \leq 0$ and $\tilde{\alpha}_s \geq 0$. $\Delta\omega = \omega_2 - \omega_1$: average width of dissipation subrange. $\Delta\omega_1 = \omega_3 - \omega_1$: increased width of dissipation subrange, corresponding to $\tilde{\alpha}_s < 0$ in (13). $\Delta\omega_2 = \omega_4 - \omega_1$: decreased width of dissipation subrange, corresponding to $\tilde{\alpha}_s > 0$ in (13).

be responsible for feedback mechanisms transferring momentum from waves to wind in this range of scales. This decrease of roughness at U_a/C_p smaller than 1 ($u_*/C_p < 0.05$), i.e. under conditions of a predominant inverse energy cascade in wind-wave spectra (Kitaigorodskii, 1983) can explain the behaviour of some of the old data (Kitaigorodskii, 1970) where direct measurements of momentum flux in the presence of "aging" waves (swell) show very small, even sometimes negative, values of $\tau = \rho\langle u'w'\rangle$. The remaining feature of the data points in Figure 3, i.e. the

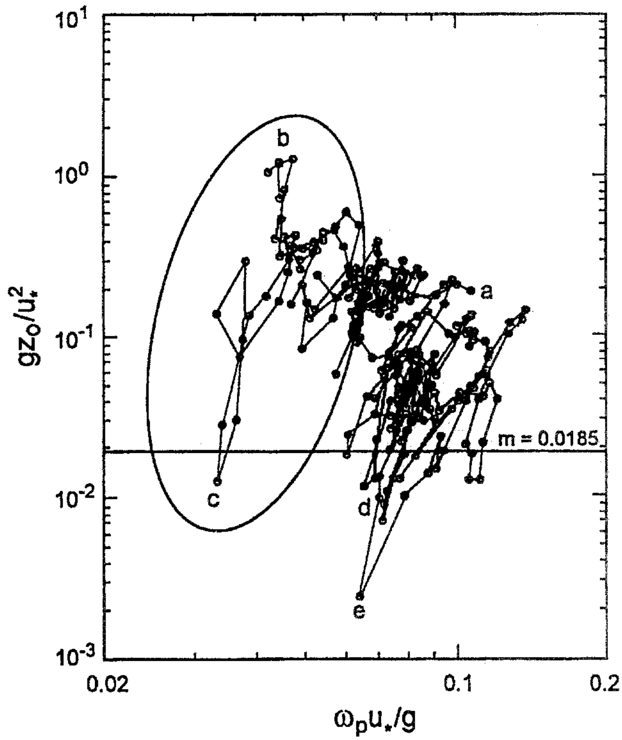

Fig. 3. The nondimensional roughness gz_0/u_*^2 vs nondimensional peak frequency $\omega_p u_*/g$.

"fluctuation" of m with U_a/C_p (for $\omega_p U_*/g > 0.06$) permits estimation of only the average value of m in this range.

5. Examples of Calculations of the Sea Surface Roughness Parameter and its Wind Dependence

In the framework suggested by Kitaigorodskii (1970) that

$$z_0 = A_s h_s = A_s \left\{ 2 \int_0^\infty S(\omega) \exp[-2\varkappa C(\omega)/u_*] d\omega \right\}^{1/2}, \tag{15}$$

it is now possible to estimate the validity of the hypothesis about the governing role of the dissipation subrange in the determination of the values of z_0. To do this, we need first of all reliable experimental data about the contribution from the dissipation subrange to the mean square surface displacement. We used here the data, recently published by Volkov *et al.* (1992). Their measurements permit one

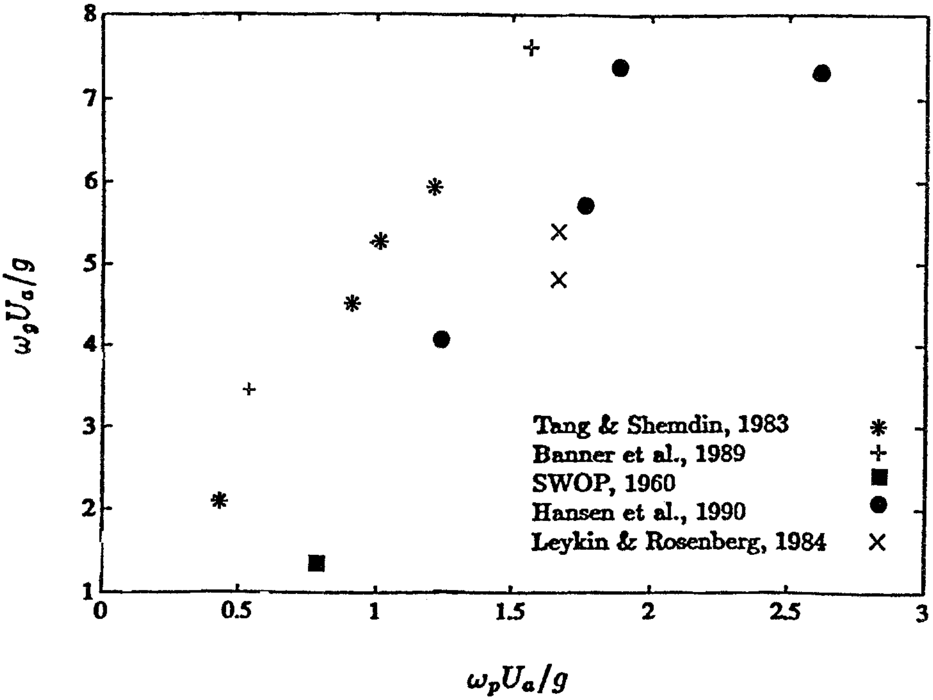

Fig. 4. Changes of the dissipation subrange low frequency boundary with nondimensional peak frequency $\omega_p U_a/g$ (from Kitaigorodskii, 1992).

to determine the effective height of the sea-surface elements h_s from the following formula

$$h_s = \left[2\int_{\infty}^{\omega_g} S(\omega)d\omega\right]^{1/2}. \tag{16}$$

In Volkov *et al.* (1992), ω_g and $S(\omega)$ in the range $\omega > \omega_g$ were determined experimentally from the observed high frequency part of the wind-wave spectra (both for light and moderate wind speeds). The experimental values of h_s in Volkov *et al.* (1992) for different wind speeds are shown in Figure 5. We can also calculate the value h_s from the parameterized form of the wave spectra in the dissipation subrange (Kitaigorodskii, 1992) using empirically found values of β and $\tilde{\omega}_g = f(\tilde{\omega}_p)$ (Kitaigorodskii, 1992). The result is

$$h_s = (2\beta/2)^{1/2}(U_a^2/g)f^{-2}(\omega_p U_a/g). \tag{17}$$

For well-defined empirical values of $\beta = 0.025$ and $f = 4$, this leads to

$$h_s = 7 \cdot 10^{-3}(U_a^2/g). \tag{18}$$

The line on Figure 5 calculated according to (18) shows that the expression (18) is a reasonably good approximation for directly measured values of h_s (though

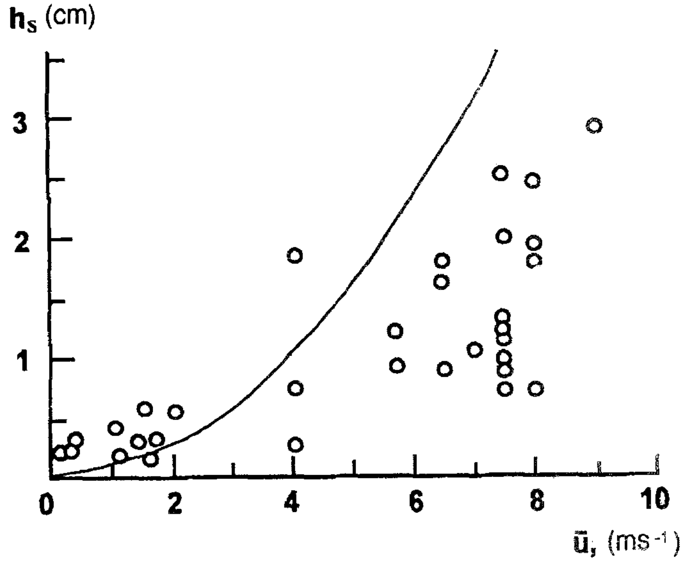

Fig. 5. The effective roughness height h_s of the sea surface for different wind speeds (from Volkov *et al.*, 1992). Line is the h_s from (18).

high winds speeds $\geq$ 8 m/s, the calculated values exceed the observed ones). The data presented in Volkov *et al.* (1992) don't give enough information about $\tilde{\omega}_p$ thus in our calculations $f(\tilde{\omega}_p)$ was taken as a constant. If we accept the values of h_s given by (18), we can estimate at least approximately the value of m corresponding to (18). For example, for values of the drag coefficient $C_d = u_*^2/U_a^2$ = (1.0–1.5) 10^{-3}, formula (18) gives

$$z_0 = h_s/30 = (0.15 - 0.23)u_*^2/g \tag{19}$$

which is about ten times larger than the usually accepted average values of $m \approx 0.0140$–0.0185. The relatively high value of $m \approx gz_0/u_*^2$ in (19) most likely corresponds to the second group of points in the circle in Figure 3 with $u_*\omega_p/g <$ 0.06, where the roughness increases with decreasing U_a/C_p due to increasing width of the dissipation subrange. Thus values presented by Volkov *et al.* (1992) most likely correspond to situations in the wind-wave field, when contributions to the roughness come mainly from the dissipation subrange, where waves can be considered as sharp-crested practically non-moving roughness elements. These values of m can be compared with the contribution to roughness from longer faster waves outside of the dissipation subrange by using the Kitaigorodskii expression (15). To estimate the contribution to roughness from an equilibrium range, the simple model for wave spectra where

$$\begin{aligned} S(\omega) &= 0 && \text{for} \quad \omega < \omega_m \\ S(\omega) &= \alpha_s u_* g \omega^{-4} && \text{for} \quad \omega > \omega_g \end{aligned} \tag{20}$$

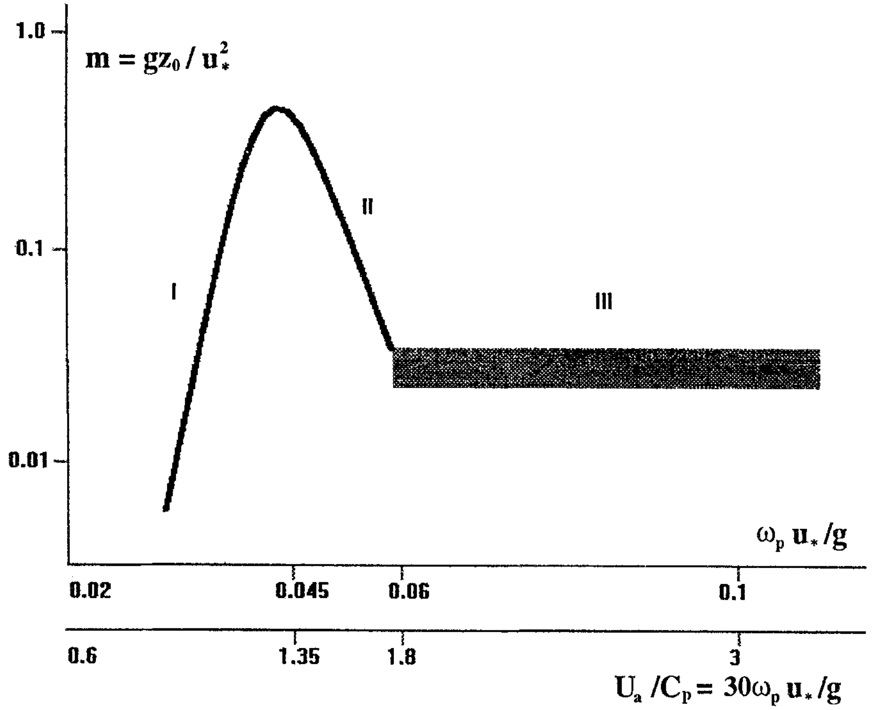

Fig. 6. Sketch illustrating the main features of the variability of the nondimensional sea roughness m with wave age.

is appropriate. The expression for roughness derived from (15, 20) can be written as

$$\begin{aligned} z_0 &= A_s u_*^2 g^{-1} f_0(\omega_p u_*/g) \\ f_0(\omega_p u_*/g) &= f_0(\tilde{\omega}_p) = [2\alpha_s/(2\varkappa)^3]^{1/2} g(\omega_p) \end{aligned} \tag{21}$$

with $g(\omega_p)$ given as

$$g(\tilde{\omega}_p) = [1 - (\varkappa/2\tilde{\omega}_p)^2 \exp(-2\varkappa/2\omega_p) + \varkappa/\tilde{\omega}_p + 1/2]^{1/2}. \tag{22}$$

It can be seen from (21, 22) that $f_0(\omega_p)$ effectively reaches the asymptotic value $(2\alpha_s/2\varkappa^3)^{1/2}$ when $\tilde{\omega}_p < 0.1$ $(U_a/C_p \approx U_a\omega_p/g < 3)$. Such small asymptotic values of m correspond approximately to the point C in Figure 3 and so represent the very small contribution to the entire area circled in Figure 3. This fact by itself immediately shows that the contribution from the dissipation subrange is decisive for the whole area in Figure 3 for values $\omega_p u_*/g < 0.06$ and thus for the basic range of variation in the numerical values of the Charnock constant. The calculation performed by K. Kahma (unpublished) in the framework of the Kitaigorodskii theory (15) using the most realistic model for the whole spectrum $S(\omega)$ also shows that the greatest contribution to z_0 and the drag coefficient comes from the ω^{-5}

high frequency ($\omega > \omega_p$) part of the spectrum. Thus the basic features of Figure 3 in our interpretation can be explained by assuming that there are two patterns of behaviour of the nondimensional roughness with wave age $U_a/C_p = \omega_p U_a/g$. For waves with $U_a/C_p < 1$, the roughness diminishes with increasing U_a/C_p, which can be related to the decrease in width of the dissipation subrange with wave growth (Figure 4). For wave ages typical of well developed seas (up to the Pierson-Moscovitz condition), the tendency is reversed, which can be explained by the appearance of long fast waves generated by the inverse energy cascade being responsible to feedback mechanism of transferring momentum from waves to wind in this range of scales (Kitaigorodskii, 1970). The important conclusion from our analysis is that the main part of the observed "variability" of m with wave age U_a/C_p in Figure 3 is not described by simple formulae (like Toba-Koga, Tikey or Charnock). Finally, the remaining feature of Figure 3, the "fluctuating" behaviour of the sea-surface roughness in the region $\omega_p U_*/g > 0.06$ can be explained by introducing an "effective" value of the Charnock number m defined through the concept of mobile roughness elements (Kitaigorodskii, 1968, 1973) in the region where most of the contribution to z_0 comes from the equilibrium rear face of the spectra. To summarize our explanation of the variability of z_0, we present a schematic Figure 6 based on the data of Toba and Ebuchi (Figure 3) and Donelan *et al.* (1993). Figure 6 demonstrates that the variation of m with U_a/C_p is more complex than can be expected from the simple Charnock approximation. The basis of the theory underlying the features presented in Figure 6 was discussed in previous sections. However, an explanation of the decreasing roughness after an extremum point with decreasing U_a/C_p remains an unsolved theoretical problem, since quantitative estimates of the feedback mechanism for transferring momentum from aging waves to the air in the presence of a whole spectrum of wind generated waves was never done. It is clear only that in this range of U_a/C_p not all wave components represent traditional roughness properties of the sea-surface.

6. Conclusions

We have demonstrated that the variability of the sea-surface roughness parameter can be related to the process of wave breaking and its characteristics. To explain and parameterize such "variability", it is useful to take into account the existence and "variability" of the width of the so-called dissipation subrange (Kitaigorodskii, 1983, 1991) as well as the presence of aging "waves", produced by an inverse energy cascade (Zakharov and Zaslavskii, 1992; Kitaigorodskii, 1983).

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