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The Physics of the Einstein–Podolsky–Rosen Paradox

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The Einstein–Podolsky–Rosen paradox as formulated in their original paper is critically examined. Their argument that quantum mechanics is incomplete is shown to be unsatisfactory on two important grounds. (i) The gedanken experiment proposed by Einstein, Podolsky, and Rosen is physically unrealizable, and consequently their argument is invalid as it stands. (ii) The basic assumptions of their argument are equivalent to the assumption that quantum mechanical systems are in fact describable by unique eigenfunctions of the operators corresponding to physical observables, independent of any observation or measurement. Following an argument due to Furry, it is shown that this interpretation of quantum mechanics must lead to some physical predictions at variance with those of conventional quantum mechanics. A decisive experiment has been performed by Freedman and Clauser, which rules out this interpretation, and imposes severe restrictions on any alternative theory which incorporates the Einstein, Podolsky, and Rosen concept of physical reality.

1. INTRODUCTION

In the more than forty years since the appearance of the celebrated paper by Einstein, Podolsky, and Rosen[1] (referred to as EPR), the debate about the relevance of their argument to the completeness of quantum mechanics and to its "paradoxical" implications has continued without any sign of a final agreed resolution being possible. In recent years a number of experiments have been performed which have a direct bearing on the issues raised by EPR. These experiments have focused attention on the fact that many questions concerning the interpretation of quantum mechanics may be open to experimental resolution. In his detailed examination of the EPR paradox, Hooker[2]

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has recently reemphasized the point made by Furry in 1936[3] that the EPR interpretation of quantum mechanics leads, in certain situations, to predictions different from those of the conventional theory. This point has been taken up in different contexts by Clauser and Horne[4] and d'Espagnat,[5] who investigate in detail the consequences of further, more direct tests of some of the conceptions underlying the rival interpretations of quantum mechanics.

It would seem to be useful to reexamine the EPR argument in some detail to discover how far it is possible to isolate those elements concerned with the physics that can be resolved experimentally, in order that the more philosophical arguments about the general interpretation of quantum mechanics may be firmly placed within the context of established experimental fact. Our argument shall be that if the detailed predictions of conventional quantum mechanics are verified in certain crucial areas, then the interpretation proposed by EPR on which they base their argument for incompleteness is ruled out. The main issues can be objectively decided by careful experimental studies, and the present evidence unambiguously favors the conventional interpretation.

The paradoxical features of the EPR argument are intimately related to the notion of physical reality. The importance of this concept has always been evident, and in his reply to EPR, Bohr[6] stressed that the EPR conception of physical reality contained an essential ambiguity when applied to the actual problems with which we are here concerned. By arguing in terms of the principle of complementarity, Bohr showed that their concept was inconsistent with the concept of physical reality implicit in quantum mechanics. If the experimental results force us to abandon the EPR notion of physical reality, quantum mechanics may appear paradoxical because we lose our intuitive feel for the objective physical situation. Put in this way, it is clear that the paradoxical features of the theory are simply those that conflict with the ways of thinking we have evolved in our dealings with the macroscopic world. The importance of detailed experimental tests is then evident, because the nature of physical reality is our central interest and we should avoid any attempt to force our theories to conform with preconceived notions that are not appropriate for new realms of experience.

In order to get the logic of the debate absolutely clear, it is helpful to set up two theories of quantum mechanics, which may or may not be different. In the first place we consider the existing formal structure and mathematical apparatus of quantum mechanics, in some sense abstracted from particular interpretational notions; in particular from any concept of what constitutes the physical reality for such an abstract scheme. The two theories we wish to consider are made by appending to this mathematical scheme one of two definite concepts of physical reality.

Theory I: In this theory, the concept of reality is that associated with the Copenhagen interpretation of quantum mechanics. Broadly, this states that physical reality can be understood only in terms of the results of possible individual measurements on quantum mechanical systems.

Theory II: This theory contains the concept of reality proposed by EPR, which we broadly associate with the idea that elements of physical reality are objectively real properties of isolated quantum mechanical systems.

We can now discuss the various possibilities.

(a) The two theories are equivalent in that they give the same physical prediction in every case. It would seem that EPR probably held this to be the case. They actually use theory II, and their argument is that quantum mechanics, supplemented by the locality assumption of no instantaneous action at a distance, is not consistent in the obvious sense that one can derive the contradiction that an isolated particle can be simultaneously in eigenstates of noncommuting operators, which is contrary to the mathematical formalism. EPR wish to resolve this contradiction by supplementing the mathematical formalism, and they see the existing theory as simply incomplete.

However, their argument is not valid as it stands because they fail to prove a contradiction within theory II, since, as we shall argue in detail, their example to prove the possibility of a crucial step in their argument is not physically realizable on the basis of theory II. They could, nevertheless, have chosen a different example which did fulfil the necessary quantum mechanical requirements, and we give such an example. Unfortunately for EPR, this example also illustrates that theories I and II are not equivalent.

(b) Theories I and II give different physical predictions in at least some realizable instances. This leads to the possibility of a decisive experimental test of which theory is false, or indeed, if both are false. The experiments decisively reject theory II and agree with theory I. So, although theory II of EPR may be internally inconsistent, it is also false.

(c) A different theory (call it theory III) can be devised which contains the EPR concept of physical reality and agrees with quantum mechanics in every case for which it has so far been tested. Nevertheless, such a theory will necessarily give different predictions in some applications, and the necessity for theory III must eventually become apparent through the breakdown of the present theory I. Since we can have no conceivable guarantee that if theory I does break down somewhere the new theory will be of the form of III, commitment to theory III at present requires an overwhelming belief in the truth of the EPR concept of physical reality. Furthermore, as we shall see, the present evidence indicates that any theory of the third type will either be intrinsically nonlocal, with the ensuing difficulties with the theory of relativity, or pathological in some other way.

The points of the above summary of the argument are discussed in detail in the rest of this paper. We begin with an analysis of Einstein, Podolsky, and Rosen's original argument and show that their position is equivalent to Assumption A of Furry.[3] Furthermore, it is shown that the example they give to demonstrate a vital step in their argument is not physically realizable. We then examine Bohm's proposed experiment,[7] and show that in general it does not exhibit the required paradoxical behavior either. The clearest example which is strictly of the EPR type is that of the polarization correlations in the two-photon decay of a spin-zero system, also considered by Bohm.[8] We discuss this case in detail, and show that the existing experiments provide a convincing refutation of theory II. The final section of the paper is devoted to a discussion of the possibilities for the various interpretations of quantum mechanics.

2. THE EPR ARGUMENT

The starting points of EPR are the concepts of completeness and physical reality. For the purposes of their argument they introduce a technical notion of completeness in terms of two basic definitions.

D1. A necessary condition for a complete theory is that "every element of the physical reality must have a counterpart in the physical theory."

D2. An element of physical reality is defined such that, "if, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

These definitions are offered tentatively as a reasonable formulation of what is commonly meant by physical completeness and reality, although EPR are careful to point out that "elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements." In order to interpret D1 and D2 further, we have to look at how EPR apply these concepts in the development of their argument. In passing, we note that in his later writings, Einstein[9] proposes a much more stringent concept of completeness, which amounts to requiring a complete theory to provide a unique prediction for every conceivable sequence of measurements. A complete theory would, therefore, necessarily be deterministic at every level. Quantum mechanics is clearly not complete in this sense, and in this paper we shall concentrate on the implications of D1 and D2 above.

In the next section of their paper, EPR state their explicit interpretation D2 in terms of the formalism of quantum mechanics: A system has an element of physical reality corresponding to a particular observable if, and only if, the system is describable by an eigenfunction of the operator corresponding to that observable. This is clearly the most natural interpretation of D2, and it is of crucial importance, both to the subsequent argument of EPR, and to the detailed criticisms of their general position that will be made in this paper, and it is perfectly clear that this is in fact what EPR do say. The observation is then made that in quantum mechanics, a system cannot simultaneously be in eigenstates of two noncommuting observables. Consequently, for noncommuting operators, the corresponding physical quantities cannot have simultaneous reality. Thus, if we can envisage a physical system for which the results of measurements of two noncommuting observables can both be predicted with certainty, without interfering with that system in any way, the system must have simultaneous elements of physical reality corresponding to these predictions. In other words, according to the above interpretation, the system must, before any measurement of it is made, be simultaneously in an eigenstate for each of the noncommuting operators, which is not allowed in standard quantum mechanics. The construction of such a physical system amounts to a proof of inconsistency of the theory consisting of the formalism of quantum mechanics and the definitions D1 and D2 (i.e., theory II of the introduction). Notice that this argument does not hold unless there is a one-to-one correspondence between elements of physical reality and quantum mechanical eigenstates of the appropriate operator. If a physical system can possess an element of physical reality according to D2 without that system being in a single eigenstate, then there is no necessary contradiction with the formalism in a system possessing simultaneous elements of reality for noncommuting observables.

In order to show that such a physical system might be constructed, EPR consider two particles, called 1 and 2, which initially interact in some way, but eventually separate, so that after a sufficiently long time they can be considered as independent, although of course still correlated by the initial interaction. If this system can be described by two noncommuting observables, A and B say, then the joint wave function for $1 + 2$ can be expanded either in terms of the eigenfunctions of the operator A for particle 1

$$\psi(1, 2) = \sum_{n=1}^{\infty} \xi_n(2)\, u_n(1) \tag{1}$$

where $Au_n(1) = a_n u_n(1)$, or in terms of the eigenfunctions of B for particle 1,

$$\psi(1, 2) = \sum_{s=1}^{\infty} \phi_s(2)\, v_s(1) \qquad (2)$$

where $Bv_s(1) = b_s v_s(1)$. In these equations $\xi_n(2)$ and $\phi_s(2)$ are to be regarded merely as the coefficients of the expansion in terms of the eigenfunctions for particle 1. The possibility of the product form is a consequence of the assumed independence of 1 and 2 after the interaction, and these expansions are equivalent. Now, measurement of observable A on particle 1 gives a value a_k, say, so after the measurement, particle 1 is in the eigenstate $u_k(1)$, and by reduction of the wave packet, particle 2 will be in the state $\xi_k(2)$. Similarly, a measurement of B for particle 1 leaves this particle in the state $v_r(1)$ corresponding to the eigenvalue b_r, say. This measurement reduces the wave packet so that particle 2 will be in the state $\phi_r(2)$. Since we can measure either A or B for particle 1 at will, it follows from D2 that particle 2 must be in the states given by $\xi_k(2)$ and $\phi_r(2)$ simultaneously, and *before any measurement is made on 1*, since the EPR locality assumption rules out the possibility that the physical state of 2 is "created" by the measurement of 1.

In order to prove that there is a contradiction, EPR must now establish that it is possible for $\xi_k(2)$ and $\phi_r(2)$ to be eigenfunctions of some noncommuting operators, P and Q, respectively, which may or may not be A and B. If this is possible, then in such cases particle 2 is simultaneously completely described by a single eigenfunction of *either* of two noncommuting operators, which constitutes the contradiction with the quantum mechanical formalism.

In order to prove this last essential step, EPR consider a particular physical system for which they claim the necessary result is true, but before we consider this part of the argument, some further comment on the situation is in order. The force of the argument is made abundantly clear by the physical idea implicit in D1 and D2. The elements of physical reality of a system, since they necessarily correspond to eigenfunctions of the relevant operators, are "carried along" with the system in a way that is completely independent of the rest of the universe. In other words, isolated physical systems are seen as independent seats of real attributes. This is the position which has been strongly criticized by Bohr,[6] for whom the importance of the individual measuring procedures means that a system cannot be regarded as an independent seat of "real" attributes merely because it has ceased to interact dynamically with other systems. But if it is to be valid, the EPR argument requires an interpretation of quantum mechanics for which the assignment of an element of physical reality to particle 2 after it has separated from particle 1, without in any way disturbing 2, implies that the interaction must have left particle 2 in an eigenstate corresponding to this element of reality. Since we could have measured particle 2 instead of 1, it follows that particle 1 must also have left the interaction region in an eigenstate of the same observable. Thus the position adopted by EPR is exactly that described by Furry[3] as his Assumption and Method A:

> "We assume that during the interaction of the two systems each system made a transition to a definite state, in which it now is, system 1 being in one of the states $u_n(1)$ and system 2 in one of the states $v_s(2)$. These transitions are not causally determined, and there is no way of finding out *which* transitions occurred, except by making a suitable measurement. In the absence of measurements, we know only the probabilities of the different transitions from our knowledge of the initial state, and that if system 1 is in the state $u_r(1)$, system 2 is in the state $v_r(2)$."

The $u_n(1)$ and $v_s(2)$ are simply the eigenfunctions of the operators A and B above, and, as we shall see, it is usually the case that in order to know this much about the system, A and B must be in fact the same observable, but this is not necessary. If more than one observable is involved (e.g., a pair of noncommuting observables), Furry makes it clear that a separate Assumption A is made for each observable.

It does not appear to have been always recognized that this Assumption A of Furry was intended as a clear restatement of the interpretation of quantum mechanics implied by the use EPR make of their definitions D1 and D2. Because of the argument they make, EPR necessarily interpret quantum mechanics as containing Assumption A, which is not to say that EPR believe this assumption to be true, since clearly if one believes quantum mechanics to be incomplete, one would not make Assumption A. Furry does not propose A as a resolution of the EPR paradox, and he expressly rejects A because, as he clearly proves, it is inconsistent with quantum mechanics in just those cases that EPR consider. Furry rejects as untenable the idea of the independent existence of two entities, the state of system 2 (given by the eigenstate in which it "actually" is), and one's knowledge of its state, only the latter being affected by measurements made on system 1. It is made quite clear by EPR, however, that in their interpretation, the reality of the system is something inherent in it as it stands, since they admit that if it is insisted that two physical quantities can be simultaneous elements of reality *only when they can be simultaneously measured or predicted,* then the contradiction does not arise. On the other hand, they feel that the consequence of taking such a position is that the reality of the observables for particle 2 depends on the measurement made on 1, which they reject as contrary to a reasonable definition of reality for noninteracting systems.[3]

[3] This interpretation of *EPR* in terms of Furry's Assumption A is lent further weight by Einstein's unambiguous restatement of his position in his autobiographical notes.[9]

The final stage of the EPR argument is to claim that an isolated system of the type considered is simultaneously in single eigenstates of noncommuting operators, contrary to the mathematical formalism. If a system can have an element of physical reality according to D2 without being in a single eigenstate, then the conflict does not arise. Thus EPR must have interpreted quantum mechanics as implying that unique eigenstates are the only possible counterparts of elements of physical reality. If locality is assumed, this interpretation is equivalent to Furry's Assumption A for the separated but correlated two-particle systems considered by EPR. Since, as Furry points out, quantum mechanics rules out Assumption A, the theory criticized by EPR (theory II) is not conventional quantum mechanics (theory I).

Returning to the original argument, we consider the gedanken experiment proposed by EPR to show that $\xi_k(2)$ and $\phi_r(2)$ could be eigenfunctions of noncommuting operators. They consider the two-particle system described by the wave function

$$\Psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp[i(x_1 - x_2 + x_0)\, p/\hbar]\, dp \tag{3}$$

where x_0 is some constant. Carrying through the above argument, they claim that if particle 1 is measured to have momentum p, then particle 2 has momentum $-p$, and if particle 1 is measured to be in the position $x_1 = x$, then particle 2 is at $x_2 = x + x_0$. Now, according to their assumptions, Eq. (3) is the wave function of the system for all times after the interaction has ceased to be effective. But it is clear that such a system is physically impossible, because from Eq. (3) we have immediately

$$\Psi(x_1, x_2) = h\, \delta(x_1 - x_2 + x_0) \tag{4}$$

which is just the configuration space eigenfunction for two particles a fixed distance x_0 apart. Now the momentum conjugate to the relative position $x_1 - x_2$ is $p = \frac{1}{2}(p_1 - p_2)$, and by the uncertainty principle, if $x_1 - x_2$ is known exactly $(=x_0)$, then p can take on all values between $+\infty$ and $-\infty$. The particles, therefore, have a completely unspecifiable relative momentum, and can never remain a fixed distance apart. At best, the wave function (3) can describe the system at only one instant.[4]

However, it is true that the total momentum $P = p_1 + p_2$ commutes with $x_1 - x_2$, so that the total momentum can be exactly specified for $\Psi(x_1, x_2)$. Then momentum conservation assures us that this momentum is the same at all subsequent times, so a measurement of p_1 does indeed lead to a

[4] This argument is equivalent to the unpublished argument of Epstein, quoted by Jammer,[34] in which neglecting the time dependence of the wave function (3) is criticized. We remark with Jammer that this argument has never been satisfactorily answered.

unique prediction for p_2. On the other hand, since the particles do not remain a fixed distance apart, measurement of x_1 leaves x_2 completely undetermined,[5] even at the time for which Eq. (3) is the true wave function, because the measurement of x_1 is necessarily at some subsequent time, after particle 1 has moved an indeterminable distance.

Predictions for this system can be made only on the basis of the principle of momentum conservation. The impossibility of predicting x_2 from measurements on 1 alone is a direct consequence of the fact that there is no independent conservation law for position. It was the attempt to impose such a conservation law in the form of assuming Eq. (4) to be valid for all times that makes the example of EPR physically unrealizable.

The general argument of EPR breaks down because the coefficients of the expansion in terms of the eigenfunctions for particle 1 are not in general eigenfunctions for particle 2. The eigenvalues for the two separated systems need not be correlated, so the general expansion is

$$\psi(1, 2) = \sum_{i,j=1}^{\infty} w_{ij} u_i(1)\, u_j(2) \tag{5}$$

(or the corresponding double integral for continuous variables). This is reducible to the form required by EPR,

$$\psi(1, 2) = \sum_{i=1}^{\infty} f_i u_i(1)\, u_i(2) \tag{6}$$

only if the eigenvalues a_1 and a_2 of $u_i(a_1)$ and $u_1(a_2)$, respectively, are correlated by an exact conservation law for all i, so that we can write

$$u_i(1)\, u_j(2) = u_i(a_1)\, u_i(a_2)\, \delta_{ij}$$

with a_2 equal to some function of a_1. In the absence of a conservation law, measurement of A on particle 1 gives $u_i(1) = u_n(a_n)\, \delta_{in}$, and the wave function (5) is reduced to

$$\psi(1, 2) = \sum_{j=1}^{\infty} w_{nj} u_n(a_n)\, u_j(2) \tag{7}$$

[5] In his realization of the EPR wave function in terms of a two-slit experiment, Bohr[6] is careful to note that the width of the slits must be much greater than the characteristic wavelength of the particle. It is surprising, therefore, that in his footnote pointing out that EPR take the limiting case of infinitely narrow slits, he does not comment on the ensuing diffraction effects, which rule out subsequent position predictions.

for which there is still a distribution over the eigenfunctions $u_j(2)$ for particle 2. The two "different" wave functions obtained by EPR are, consequently, exactly the same quantum mechanical state simply expanded in terms of different complete bases, which is not inconsistent or incomplete.

3. BOHM'S PROPOSAL

An alternative gedanken experiment has been suggested by Bohm.[7] He considers the decay of a spin-zero composite system into two (distinguishable) spin-$\frac{1}{2}$ particles. Since angular momentum is conserved, he argues that if we measure the spin projection of particle 1 in any direction, the corresponding spin projection of particle 2 must have the opposite sign, because the spin projections are correlated by the additive conservation of angular momentum. All the commentators[2,10–15] on this example of the EPR paradox agree that this is the correct quantum mechanical description of this particular system. If this is correct, then clearly Bohm's experiment is of the type required by EPR. According to the standard argument, the total system has spin zero, so the wave function can be expanded in terms of the eigenfunctions of the spin operator $S_{\hat{a}}$ in the direction specified by the arbitrary unit vector $\hat{\mathbf{a}}$. Namely, if $S_{\hat{a}} u_{\pm}^{\hat{a}}(1, 2) = \pm\hbar u_{\pm}^{\hat{a}}(1, 2)$, then

$$\psi(1, 2) = (1/\sqrt{2})[u_+^{\hat{a}}(1)u_-^{\hat{a}}(2) - u_-^{\hat{a}}(1)\, u_+^{\hat{a}}(2)] \tag{8}$$

Then, if the decay axis in the center-of-mass frame for the system is taken to be the z direction, measurement of S_x for particle 1, giving a result σ_{1x}, implies that particle 2 is in the state $u^x_{-\sigma_{1x}}(2)$. Similarly a measurement of S_y for 1 implies that particle 2 is in the state $u^y_{-\sigma_{1y}}(2)$. Since either of these measurements can be made without disturbing particle 2 in any way, the conclusion that particle 2 must be in a state which can be simultaneously described by eigenfunctions of both of the noncommuting operators S_x and S_y appears unavoidable on the basis of the definitions D_1 and D_2 of EPR.

This standard analysis of the system appears unsatisfactory on several counts, however. In the first place, the total angular momentum operators $J_{\hat{a}} = L_{\hat{a}} + S_{\hat{a}}$, where $L_{\hat{a}}$ is the orbital angular momentum operator for the direction $\hat{\mathbf{a}}$, are clearly not all conserved. This is because the two-body decay in itself defines a direction. In order that we can say that a decay has occurred, we need to perform some measurement on at least one of the particles (in fact a coincidence measurement on both particles is the only certain test for the decay) and any measurement capable of recognizing a decay product and of giving more information than the bare fact that a decay has occurred must necessarily involve some, even if an inexact, position measurement.

Thus for measurements of the type under consideration we know, at least approximately, the decay axis, so we know that the original spherical symmetry of the spin-zero state has been reduced to an axial symmetry. If we take the z direction as along this decay axis, the symmetry assures us that the Hamiltonian must commute with L_z, but it certainly does not commute with L_x and L_y. This is because the original interaction potential is dependent on the separation of the particles, so once this direction is externally defined, $[L_x, H] \neq 0$, and $[L_y, H] \neq 0$. Thus, although the angular momentum $\mathbf{L}^2$ and the z component L_z are conserved, L_x and L_y are not conserved. However, at present we have no reason to suppose that S_x and S_y do not commute with H, so possibly they are separately conserved. It clearly depends on the detailed spin dependence of the potential, and not simply on the fact that the initial state has spin zero, as seems to have been the usual assumption.

A more serious objection to the usual analysis, however, is the fact that its conclusions are manifestly false for several well-known decays of the type considered, such as charged pion decay and K_{l2} decay. In π^- decay, for instance, the spin-zero π^- decays mainly into a muon and an antineutrino, both of which have spin $\frac{1}{2}$. Since the neutrino is massless, its velocity is always that of light, and it can exist only in definite helicity states, i.e., in states of definite spin projection in the direction of motion. In general, a neutrino and its associated antineutrino must have opposite helicities, and the universal $1 - \gamma_5$ form of the weak interaction Hamiltonian responsible for the decay projects out just that case for which the neutrinos are left-handed (negative helicity) and antineutrinos are right-handed. For π^- decay, therefore, this two-component theory of the neutrino, together with the conservation of angular momentum, predicts that the muon will be completely polarized with positive helicity, and this has been verified experimentally.[16,17] The two-component theory further specifies that the antineutrino cannot have a definite spin in the x or y directions, but it is clear that the accompanying muon can be measured to have a definite spin in either of these transverse directions, so the spin projections in any direction perpendicular to the direction of propagation in the center-of-mass frame are not correlated for this decay, and only $\mathbf{J}^2$ and $J_z = L_z + S_z$ are conserved. It seems unreasonable to dismiss this and similar decays as a testing ground for Bohm's analysis (as Peres and Singer[12] do) on the grounds that the spins are oriented, and we know that the answer is inconsistent with the general argument. A more careful analysis is called for in which the special features (if any) of these decays can be seen to fit into the general pattern.

We shall in fact argue that π^- decay is completely typical, and that the reasons for nonconservation of S_x and S_y in this decay apply generally to all spin-zero decays into spin-$\frac{1}{2}$ particles. Weak interactions are described by the phenomenologically successful Fermi V–A theory in which the interaction

Hamiltonian is of current x current form with a pointlike interaction. The interaction Hamiltonian for π^- decay in the rest frame from standard weak interaction theory is proportional to

$$H_{\text{int}} \sim \bar{u}_\mu(p_\mu)\, \gamma_0(1 - \gamma_5)\, v_{\bar{\nu}}(p_{\bar{\nu}})$$

and the directional information is contained in the Dirac spinors $u_\mu(p_\mu)$ and $v_{\bar{\nu}}(p_{\bar{\nu}})$ associated with the outgoing particles. Because the antineutrino is massless, $v_{\bar{\nu}}(p_{\bar{\nu}})$ is simply a Pauli spinor for a state of positive helicity, invariant under Lorentz boosts, but the muon spinor is a boosted Pauli spinor of the form

$$u_\mu(p_z) = \left(\frac{E_\mu + m_\mu}{2m_\mu}\right)^{1/2}\left(1 + \frac{\alpha_z p_z}{E_\mu + m_\mu}\right)\binom{u_\pm{}^z}{0}$$

where E_μ is the muon energy, $u_+{}^z$ and $u_-{}^z$ are the eigenfunctions of $S_z = \frac{1}{2}\hbar\sigma_z$, and $\alpha_z = \left(\begin{smallmatrix}0 & \sigma_z\\ \sigma_z & 0\end{smallmatrix}\right)$. It is this Lorentz boost factor that provides the essential directional and spin information controlling the commutation properties of the Hamiltonian with the angular momentum operators.

We can see from this discussion why π^- decay with its obvious nonconservation of S_x and S_y is typical of all decays into spin-$\frac{1}{2}$ particles, even when the basic potential is spin independent. Since the decay products are necessarily moving relative to each other, at least one such boost factor is an essential ingredient of the total Hamiltonian. If we look at the Hamiltonian nonrelativistically as an operator on Pauli spinors, this boost must be incorporated in the effective potential, and thus, with complete generality, only S_z and L_z can be conserved in a decay with two spin-$\frac{1}{2}$ particles, contrary to the conclusion reached above in terms of a purely nonrelativistic theory.

The essential point is that the rotation through 90° about the y axis to give Eq. (8) in terms of

$$u_\pm{}^x(p_z) = (1/\sqrt{2})[u_+{}^z(p_z) \pm u_-{}^z(p_z)] \tag{9}$$

from the definite helicity states $u_\pm{}^z(p_z)$ has a different significance for moving spinors. Since S_x does not commute with the boost in the z direction, $u_\pm{}^x(p_z)$ are not eigenfunctions of the spin operator in the x direction. The general expansion (8) is true for all directions only if the various $u^{\hat{a}}$ are related by the simple two-dimensional rotations, as in (9), so Eq. (8) is an expansion in spin eigenfunctions only for the z axis. For $u_+{}^x(p_z)$ defined as in Eq. (9), the expectation value of σ_x is

$$\langle\sigma_x\rangle = u_+^{x\dagger}(p_z)\, \sigma_x u_+{}^x(p_z)/u_+^{x\dagger}(p_z)\, u_+{}^x(p_z) = (1 - \beta^2)^{1/2} \tag{10}$$

where β is the ratio of the center-of-mass frame velocity to the velocity of light, $\beta = v/c$. Equation (10) is strictly less than unity, and vanishes in the extreme relativistic limit. Thus a measurement of S_x for particle 1 does not necessarily imply that particle 2 has the opposite spin projection in the x direction. By expanding the wave function (for the case of equal-mass decay products)

$$\psi(1, 2) = (1/\sqrt{2})[u_+^x(p_z)\, u_-^x(-p_z) - u_-^x(p_z)\, u_+^x(-p)] \tag{11}$$

in terms of true eigenfunctions of S_x, we obtain the following probabilities for the various possible outcomes of simultaneous measurement of S_x for both particles:

$$\begin{aligned} P(\uparrow, \downarrow) &= P(\downarrow, \uparrow) = \tfrac{1}{2} - \tfrac{1}{4}\beta^2 \\ P(\uparrow, \uparrow) &= P(\downarrow, \downarrow) = \tfrac{1}{4}\beta^2 \end{aligned} \tag{12}$$

where $\uparrow$ and $\downarrow$ represent a measurement of spin up or down, respectively, in the x direction. Clearly, it is only for $v = 0$ that a measurement on particle 1 allows a unique prediction for particle 2, and in the extreme relativistic limit, $\beta \to 1$, all four possible outcomes are equally likely.

In general, therefore, Bohm's gedanken experiment is not suitable for proving the EPR argument, since it is only a helicity measurement on 1 that leads to a unique prediction for 2, and the argument does not go through.

This system is interesting, however, because a specific (center-of-mass-frame velocity dependent) correlation between the transverse spin projections is predicted, which would not be predicted by a theory incorporating Furry's Assumption A. In the latter theory, for consistency each particle must be emitted in a specific helicity state, and thus all results for transverse spin measurements are equally probable [i.e., Eq. (12) with $v = c$]. It would be interesting to perform an experiment to check predictions (12) for the equal mass decay in detail. The unsuitability of this experiment for EPR is a consequence of the fact that the rotated helicity states (9) are not directly measurable, since the Wigner rotation associated with the Lorentz boost of a spin-$\frac{1}{2}$ particle is more than a simple spatial rotation of the spin axis in that it mixes positive and negative energy solutions of the Dirac equation. Thus a z-boosted eigenfunction of S_x is not an eigenfunction of spin in any other direction. The spin projection in the direction of motion has a special importance, and the conservation of only S_z in these decays is a consequence of covariance.

4. TWO-PHOTON CORRELATIONS

The conservation of L_z and S_z is a general feature of two-body decays, but fortunately there is one case in which the limitations discussed in connection with spin-$\frac{1}{2}$ particles do not obtain. This is for two-photon decays, for which the combinations similar to (9) of definite helicity states are directly observable as a plane-polarized photon. The significance of photon polarization correlations has been noted by several authors (for example, see Bohm and Aharanov[8]), and the relevant experiments are easier to perform than those for spin-$\frac{1}{2}$ systems. The decay of the ${}^1S_0$ ground state of positronium has been studied by Snyder *et al.*,[18] Bleuler and Bradt,[19] and Wu and Shaknov,[20] and a very detailed experiment has been performed on the correlated photons in the $6\ {}^1S_0 \rightarrow 4\ {}^1P_1 \rightarrow 4\ {}^1S_0$ atomic cascade in calcium by Kocher and Commins,[21] Clauser *et al.*,[22] and Freedman and Clauser.[23] Once again, however, there appears to be some confusion in the literature over the interpretation of these experiments. We shall analyze the experiment on calcium in some detail in order to make the significance of the observed polarization correlations clear.

Since the cascade is from spin zero to spin zero, the overall parity of the system is positive (whereas it is negative for positronium) and the wave function can be written in the form (Jauch and Rohrlich,[24] p. 282)

$$\psi(1, 2) = (1/\sqrt{2})[\omega_+(1)\,\omega_+(2) + \omega_-(1)\,\omega_-(2)] \tag{13}$$

where $\omega_\pm$ are states of positive or negative helicity (right and left circularly polarized photons). In terms of the plane polarization states e_x and e_y, these are

$$\begin{aligned} \omega_\pm(1) &= (1/\sqrt{2})[e_x(1) \pm ie_y(1)] \\ \omega_\pm(2) &= (1/\sqrt{2})[e_x(2) \mp ie_y(2)] \end{aligned} \tag{14}$$

since the particles move in opposite directions. Equation (13) can consequently be written as

$$\psi(1, 2) = (1/\sqrt{2})[e_x(1)\,e_x(2) + e_y(1)\,e_y(2)] \tag{15}$$

and this form is invariant under rotations about the z axis that change the direction of the plane polarization basis

$$e_{x'} = e_x \cos\alpha + e_y \sin\alpha, \qquad e_{y'} = -e_x \sin\alpha + e_y \cos\alpha$$

Now the plane polarization states are just the rotations (9) (apart from a trivial overall phase) of the definite helicity states (14):

$$\begin{aligned} e_x(1) &= (1/\sqrt{2})[\omega_+(1) + \omega_-(1)] \\ e_y(1) &= -(i/\sqrt{2})[\omega_+(1) - \omega_-(1)] \end{aligned} \tag{16}$$

and similarly for $e_x(2)$ and $e_y(2)$. The difference is the important one that plane polarization states are directly observable. This means that the photon polarization correlations implied by Eqs. (13) and (15) provide a realizable experiment of the form required by EPR.

Before we discuss this aspect of the system, however, it is useful to make the interpretation of the various polarizations absolutely clear. The photon has spin one, but only two independent polarization states. The third polarization state of a normal spin-one object is absent because the photon is massless, and its interactions are necessarily gauge invariant. The fact of only two spin states has led to the idea that the spin correlations of photons are the same as those of massive spin-$\frac{1}{2}$ particles, but this is not the case. Gauge invariance implies that the massless photon, in common with all other massless particles,[25] can have a well-defined spin projection *only* in the direction of motion. Definite spin states in any transverse direction are forbidden, since for a spin projection of $+1$ in the x direction, for instance, rotational invariance about the z axis necessarily requires all three spin projections, $+1$, 0, and -1, in any other transverse direction, which is ruled out by gauge invariance. Notice that the absence of a *helicity* zero state is consistent with rotational and gauge invariance, since for a massless particle, necessarily moving with the velocity of light, there is no axis of symmetry other than that of the direction of motion. Consequently, linear polarization states *do not* correspond to definite spin projections in some transverse direction. Linear polarizations are coherent superpositions of circular polarization (definite helicity) states, and not in themselves spin eigenstates. The existence of correlations for planes of polarization in the decay is therefore a consequence of the conservation of S_z alone.

The noncommuting operators to be considered in applying the EPR argument are not therefore the usual spin operators. The relevant operators for photon polarizations are the noncommuting Stokes operators (Jauch and Rohrlich,[24] pp. 40–47). The EPR argument appears to be complete, because, from Eq. (15), the result of a measurement on photon 1 with a polarization filter oriented in the x direction (denoted F_{1x}) completely determines the state of plane polarization of photon 2. If photon 1 passes F_{1x}, then Eq. (15) tells us that photon 2 will invariably pass a similar filter F_{2x} and be absorbed in F_{2y}. If photon 1 is absorbed in F_{1x}, then 2 will also invariably be absorbed in F_{2x}, but pass through F_{2y}. According to EPR, this means that photon 2 is in a state of definite plane polarization, since, depending on the result of the measurement in F_{1x}, we can predict the result for photon 2 with certainty. But we could equally well have chosen a different orientation (relative to the laboratory) for filter 1, such as $F_{1x'}$. A similar argument would lead to the conclusion that photon 2 must be in a definite state of plane polarization with respect to this new direction x'. But the Stokes operators for the x and x' directions do not commute, and we have a quantum mechanical system simultaneously in eigenstates of two noncommuting operators.

But the oddity of this conclusion is immediately apparent. The EPR argument appears to lead to the conclusion that the photons must be emitted simultaneously plane polarized in all directions, which is logically impossible given what we normally mean by a state of plane polarization. It is not merely quantum mechanics that rules out this interpretation of the system, but, as Peres and Singer[12] have suggested, the contradiction is more fundamental than that sought by EPR on the basis of their theory II. In order to get a consistent description, EPR would presumably have to argue that the photons are emitted in some definite polarization state, and the physical reality corresponding to the polarization in any other direction could only be the projection of the polarization axis on the new direction (or something similar for initially circularly polarized states). The necessity for the conflict with experiment at some point proved by Furry[3] is obvious. If plane polarized photons behave according to quantum mechanics, then photons emitted in definite polarization states have a different polarization correlation than that predicted by Eqs. (13) and (15). On the other hand, if we preserve the predictions of (13) and (15) by assuming some additional rule, such as that photons pass or fail to pass a polarization filter according to whether the angle between the filter axis and the polarization plane is less than or greater than 45°, then there is a conflict with the experimentally known behavior of plane polarized photons.

Thus the experiment that fulfils all the criteria of EPR as a test case for their argument for the incompleteness of quantum mechanics also provides convincing evidence that their general interpretation of quantum mechanics, specified by theory II of the introduction, fails to account for the observations. The experimental results of Freedom and Clauser[23] are very clear.[6] In the correlated calcium transitions analyzed above, there is an absolute correlation between the behavior of the two photons: If photon 1 passes a polarizing filter in any direction, then photon 2 invariably passes a filter with the same orientation. If photon 1 fails to pass, then photon 2 invariably also fails to pass. The behavior of the photons for filters at some relative orientation α follows the $\cos^2\alpha$ distribution expected from conventional quantum mechanics. The only way in which this behavior can be understood is on the basis that the photons are not emitted in a definite polarization state, but that they

[6] It should be mentioned that there is a similar experiment by Holt[26] in mercury which does not agree with the quantum mechanical predictions. These measurements were made at only two relative orientations of the polarizing filters, and because any experimental error will tend to give a lower degree of correlation, we shall not consider this experiment further. The experiment on mercury has recently been repeated by Clauser[27] with results in complete agreement with quantum mechanics, so the evidence strongly favors the conventional theory, although further independent high-precision experiments would be desirable.

are correlated. Observation of the effect of a particular filter on photon 1 enables us to predict with certainty the way in which photon 2 will behave when confronted with a similar filter with a relative orientation of 0° or 90°. We know that photon 2 will behave *as if it were plane polarized* in a direction at 0° or 90° depending only on whether photon 1 passed or failed to pass filter 1. Quantum mechanics as given by theory I above does not allow one to infer that photon 2 was in this state of plane polarization *before* the measurement on photon 1. All that theory I does is to allow one to make certain predictions about the results of future possible observations of the behavior of particle 2.

It would be very interesting to check the quantum mechanical predictions of Eq. (13) for circular polarization measurements. With the calcium atomic transitions this should be possible. Circular polarization, or definite helicity states, can be measured for optical photons by inserting a quarter-wave plate before the polarizing filter. A circularly polarized beam is changed by a quarter-wave plate into a beam plane polarized at an angle of 45° to the optical axis of the plate, the relative orientation depending on whether the initial photon is right or left circularly polarized. From Eq. (13) we see that both photons are emitted in the same state of circular polarization, and since they are going in opposite directions, insertion of quarter-wave plates before the filters in the experiment of Freedom and Clauser should reverse the pattern of coincidence versus the relative orientation of the filters. Equation (13) predicts that if circular polarizations are observed in this way, the maximum coincidence rate occurs for filters at a relative orientation of 90°, with no coincidences when the filters are aligned. This change would be spectacular, because, as we have stressed, the photons are not emitted in states of definite helicity any more than in states of definite plane polarization, and unpolarized light is unaffected by a quarter-wave plate according to standard optical theory. (Note that this change of orientation of the filters for coincidence when quarter-wave plates are inserted does not occur for negative parity states such as positronium, because the coincidences are expected at a relative filter orientation of 90° for both plane polarized and circularly polarized photons in this case.)

To summarize, the experiments of Kocher and Commins[(21)] and Freedman and Clauser[(23)] and the positronium experiments of Wu and Shaknov[(20)] and Bleuler and Bradt[(19)] are decisive for the choice between theory I of conventional quantum mechanics and theory II of EPR. The experiments unambiguously reject theory II, and are completely consistent with theory I. It is clear from the way in which the contradiction arises in the application of theory II to this process that something more serious is wrong than a simple incompleteness of the quantum mechanical description of a state described by two or more noncommuting operators. In this case the existence of simultaneous eigenfunctions of the noncommuting Stokes operators for plane polarizations in different directions is logically impossible, and it is clearly the EPR definition of physical reality that is at fault. If EPR wish to retain their concept of physical reality, they must accept the consequences of Furry's result, and acknowledge that theory II gives different physical predictions, which are, furthermore, in conflict with the experimental evidence.

5. DISCUSSION

The argument of Einstein, Podolsky, and Rosen can be summarized in the following way. The formal mathematical structure of quantum mechanics is supplemented by the following interpretational assumptions.

1. *Physical reality.* Definition D2 of Section 2 is a sufficient condition for a system to have an element of physical reality. A corollary of this assumption as interpreted by EPR is that elements of physical reality are in one-to-one correspondence with the eigenstates of the mathematical formalism.

2. *Locality.* There can be no instantaneous action at a distance. The real physical state of an object is not influenced by other objects spatially separated from it, except by physical interactions propagating at no greater than the speed of light.

3. *Completeness.* Quantum mechanics is a complete theory in the sense of definition D1 of Section 2.

By considering correlation effects between spatially separated systems in this scheme, EPR derive the formal contradiction that single systems can simultaneously be in eigenstates of noncommuting operators, which is not allowed by the mathematical formalism. Thus, at least one of the assumptions is suspect, and EPR suggest that the third assumption of completeness is at fault.

Although the physical example that EPR consider in order to establish the latter part of the argument is unrealizable, we have seen that the two-photon correlation experiments do satisfy the necessary conditions. Nevertheless, the argument as it stands does not prove that quantum mechanics is incomplete, since the simple contradiction obtained does not uniquely indicate assumption 3 as being at fault. The difficulty is resolved more directly by abandoning either assumption 1 or 2. However, neither alternative possibility is acceptable to Einstein,[9] who insists that an individual system has definite values for all observables before measurement, and hence that the quantum mechanical ψ-function is incomplete.

This argument is essentially irrelevant to the issue of the completeness of quantum mechanics because the theory outlined above (theory II of the introduction) is not conventional quantum mechanics. It leads to different predictions for those correlation experiments that are central to the argument, and, what is more, Furry's analysis[3] of the EPR theory shows the specific origin of the disagreement to be assumption 1 above. Conventional quantum mechanics is inconsistent with such a conception of physical reality. The decision between theories I and II is then an experimental question, and not a matter of preference. The results of Freedman and Clauser,[23,27] if confirmed, are conclusive, and theory II must be rejected as objectively false.

Let us assume that the quantum mechanical predictions are borne out in future experiments, and consider the position of the EPR argument. The question of locality then becomes the central issue. The position we have reached for the two-photon correlation experiment is that before any measurement, neither photon is in a polarization eigenstate. Before photon 1 is measured we cannot make any detailed predictions other than that there will be a correlation. What this correlation implies for photon 2 in a filter is unknown until we measure photon 1. In this sense the measurement on 1 creates the reality for 2, because it is only after the measurement that we can make the unique prediction required by the EPR definition of reality. This is seen in the fact that the conventional description of the combined system is essentially different after the measurement on photon 1. Before any measurement, we cannot assign separate states to particles 1 and 2—they form a single system even if they are not interacting, and can only be described together. In particular, neither can be described by a single eigenfunction of any operator. After the measurement on 1, we can predict with certainty the result of a similar measurement on 2, and it is only in this situation that the state of system 2 can be described by a single eigenfunction of the appropriate operator. The basic assumption of locality means that this measurement on 1 does not change the *real* physical state of 2. Thus, since 2 was not in the eigenstate before the measurement, it is not in this eigenstate after such a measurement. The alternative point of view for a strictly local theory, namely that since 2 is in an eigenstate after the measurement, it must have been in this eigenstate before the measurement, is ruled out by the experimental verification of the standard correlation predictions.

The conclusion must be that even though a system may be describable by a single eigenfunction, the independent physically real state of the system belongs to a different category of description. This would agree with the interpretation of quantum mechanics for which the wave function is simply a summary of the possible outcomes of future measurements. In this interpretation no commitment is made to whether this state is "objectively real" or to what the system is "in itself." This is not to say, for example, that the concept of an electron "in itself" is necessarily meaningless, or that individual electrons do not really exist objectively. Such questions are merely not relevant to the ψ-function, which does not correspond to a physical field at each spacetime point. This is a perfectly conventional view, since no procedure exists by which we can measure $\psi(x)$ directly (as with infinitesimal test charges, which give physical meaning to the **E** and **B** fields of electromagnetic theory, for example). The completeness claim of quantum mechanics consists in saying that the information contained in ψ is *all* that is available for the prediction of any future measurement.

The "paradoxical" nature of this result for correlated systems would seem to originate in the loss of a direct, simple, physical explanation of the quantum mechanical predictions in terms of the properties of each of the separated systems of the correlated pair in itself. The problem is that of the quantum theory of measurement in general, and concerns the physical interpretation of the "reduction of the wave packet." This is the difficulty that Hooker[2] sees as the heart of the EPR argument, and is the basis of his plea for a plausible physical explanation of correlations between widely separated systems. What is being asked for is that the theory should provide some mechanism for this correlation. How is it in fact that particle 2 "knows" the result of the measurement on particle 1, and acts in the measuring apparatus accordingly? The failure of quantum mechanics to provide such a mechanism is interpreted as an incompleteness of the theory.

What form would such a mechanistic account of correlation phenomena take? The localized description of the separated systems in terms of single eigenfunctions has been ruled out by Furry's general result. The only alternative is that the measurement of one system does in fact influence the second system. In other words, we must abandon the assumption of locality and the physical independence of the (widely) separated subsystems. It is clear that for the predictions of quantum mechanics to be maintained in all situations, such a physical interaction between the subsystems would have to be instantaneous, as in the original hidden variable theory of Bohm.[28,29] Such a solution was emphatically rejected by Einstein, and is no more acceptable today, because it is in direct conflict with the special theory of relativity. The necessary interaction can be instantaneous in only one Lorentz frame, so the principle of equivalence of all inertial frames would have to be abandoned. Moreover, we know of no possible mechanism for the instantaneous propagation of physical information over arbitrarily large distances, so such an explanation becomes less physically plausible than the alternatives.

Another possibility is that there is an interchange of physical information propagated at the speed of light between the separated systems. Such a theory is still local in the quantum field theoretical sense, but it leads to violations of the standard results for spacelike separations of the two measurements. If system 2 is measured before there is time for information of the result of the measurement on system 1 to be transmitted to 2, the modified theory predicts that there would be no correlation. In the experiments to date the distance between the subsystems is not large, so the measurements are unlikely to have been made at purely spacelike separations, and do not test the proposed modified causal theory. If it is found in future experiments that the standard quantum mechanical correlation does not obtain for spacelike separations of the measurements on the subsystems, then the standard theory is clearly incomplete. The difference in results between spacelike and timelike separations would be unambiguous evidence for a physical interaction between the systems, and present quantum mechanics does not provide any account of such an interaction.

However, if such a situation were to be found experimentally, the repercussions would be far-reaching. The basis of the quantum mechanical result for spin or polarization correlations is very simple. We require only the conservation and quantization of angular momentum. The phenomenon of space quantization, which leads to the notion of quantization of angular momentum, is very well established, and the "all-or-nothing" character of polarized photon transitions in linear polarizing filters is beyond dispute. Thus the absence of the predicted correlation for spacelike separations would be clear evidence for the nonconservation of angular momentum. Apart from undermining the whole of modern physics, this would imply the existence of an anisotropy of space, which would be very hard to reconcile with our current understanding of the nature of space and time.[7]

Furthermore, the type of physical interaction required is rather unusual. The interaction conveying the causal correlation information cannot be a standard quantum field because it is completely specific to one particle and to one interaction. It carries no energy or momentum, and must propagate over arbitrarily large distances without diminution. Since momentum and angular momentum conserving collisions always involve such correlations, and are the most common events in the universe, this zero-energy field must be all-pervasive, but without any effect on other than the specific partner of a particular interaction.[8]

It would appear that no resolution of the EPR paradox is possible by abandoning the locality assumption. In view of Bell's theorem,[(10)] this seems

[7] This difficulty arises also for strictly local hidden variable theories, and indeed for any theory that predicts violations of the strict correlations for individual pairs of systems. Universal conservation laws apply to single systems, and not just in the average over ensembles of similar systems.

[8] The possibility of "filter enhancement" suggested by Clauser and Horne[(4)] is even more bizarre. It provides no solution to the conceptual problems involved, and no plausible reason has been given for such an ad hoc conspiratorial effect.

to rule out hidden variable theories in general, since local hidden variable theories are already unable to reproduce all the results of standard quantum mechanics—a fact which is at the heart of all the "proofs" of the impossibility of such theories.[10,30–33] The possibility of a completely different theory that incorporates the EPR concept of physical reality and reproduces all the quantum mechanical results known to date, including the two-photon correlation experiments (theory III of the introduction), cannot be ruled out a priori. But from the foregoing discussion it is clear that such a theory is very tightly constrained if it is not to be less physically plausible than the existing theory I. Whatever the form of such a theory, however, its acceptance or rejection is an experimental question. If it is a different theory, it must have different predictions for some characteristic situations, and the outcome of crucial experiments will objectively decide the issue.

The position we have reached is perfectly clear. If quantum mechanics is experimentally correct, and if locality is maintained, then no more detailed account of the phenomenon of correlation can plausibly be given. The photon polarization experiments of Freedman and Clauser, when taken with Furry's point, provide a complete answer to the EPR argument by showing that the classically based concept of physical reality is inadequate for quantum mechanics. The onus is on the opponents of the conventional interpretation of quantum mechanics to show that a theory both consistent with observation and more complete in the required sense is indeed possible.

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