EQUILIBRIUM SPECTRA OF SECONDARY COSMIC-RAY POSITRONS IN THE GALAXY AND THE SPECTRUM OF COSMIC GAMMA-RAYS RESULTING FROM THEIR ANNIHILATION

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(Received 12 November, 1968)

Abstract. A general formula is derived for calculating the γ-ray spectrum resulting from the annihilation of cosmic-ray positrons. This formula is used to calculate annihilation-γ-ray spectra from various equilibrium spectra of secondary galactic positrons. These spectra are then compared with the γ-ray spectra produced by other astrophysical processes.

Particular attention is paid to the form of the γ-ray spectrum resulting from the annihilation of positrons having kinetic energies below 5 keV. It is found that for mean leakage times out of the galaxy of less than 400 million years, most of the positrons annihilating near rest come from the β-decay of unstable nuclei produced in cosmic-ray p-C^{12}, p-N^{14}, and p-O^{16} interactions, rather than from pi-meson decay. It is further found that the large majority of these positrons will annihilate from an S state of positronium and that $\frac{3}{4}$ of these will produce a three-photon annihilation continuum rather than the two-photon line spectrum at 0.51 MeV. The results of numerical calculations of the γ-ray fluxes from these processes are given. It is concluded that annihilation γ-rays from the galactic halo may remain forever masked by a metagalactic continuum. However, an 0.51 MeV line from the disk may well be detectable. It is most reasonable to assume that this line is formed predominantly by the annihilation of the CNO β-decay positrons. Under this assumption, the intensity of the line becomes a sensitive measure of the galactic cosmic-ray flux below 1000 MeV/nucleon.

1. Introduction

The annihilation of cosmic-ray positrons has for some time been recognized as a potential source of cosmic γ-rays. γ-ray fluxes from cosmic positron annihilation have been estimated and discussed by various authors (Pollack and Fazio, 1963; Hayakawa *et al.*, 1964; Ginzburg and Syrovatski, 1964a, b). Pollack and Fazio have discussed the possible relationship between the present flux of 0.5 MeV γ-radiation from positron annihilation and the cosmic-ray intensity and galactic gas density 10^9 years ago. Ginzburg and Syrovatski have pointed out that the intensity of the 0.5-MeV line may be a sensitive measure of the leakage rate of cosmic-ray positrons from the galaxy. These authors have also given an approximate formula for the calculation of the γ-ray spectrum from the annihilation of high-energy positrons.

It has therefore become apparent that the cosmic-γ-ray spectrum from cosmic-ray positron annihilation may contain much potential information reflecting various astrophysical conditions in our own galaxy and in possible cosmic-ray sources, both

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galactic and extragalactic. It is for this reason that the author had recently undertaken a more detailed investigation of the various aspects of the cosmic positron annihilation problem (STECKER, 1967b). That work has now led to this further treatment pointing out the potential richness and complexity of the problem.

A typical cosmic-ray positron from secondary pi-meson decay may undergo one of three fates; it may (1) escape from the galaxy, (2) annihilate with an electron while at relativistic energy, or (3) lose almost all its energy before annihilating. One must consider that annihilations may occur either between free electrons and positrons or through the formation of the intermediate bound state of positronium. One must also take into account that in addition to the positrons from pi-meson decay, lower energy positrons are produced in the galaxy by, e.g., cosmic-ray p-C^{12}, p-N^{14}, and p-O^{16} collisions. All of these processes must be considered in calculating the final positron annihilation-γ-ray spectrum. They will be discussed individually in the following sections and their contribution to the total positron annihilation-γ-ray spectrum will be presented. The significance of direct measurement of the positron equilibrium spectrum itself will be discussed as a possible sensitive indicator of the diffusion and leakage of cosmic-ray electrons out of the galaxy.

The point of departure for our discussion will be a derivation of the general formula for calculating the annihilation-γ-ray spectrum (AGS) from cosmic-ray positrons valid at all energies. This formula is then used to calculate AGS from various equilibrium spectra of secondary cosmic-ray positrons. The results of the numerical calculations will be given and discussed.

2. The Annihilation-γ-Ray Spectrum

The cross-section for positron annihilation as a function of energy was first deduced by DIRAC (1930). An excellent presentation of the theory is given by HEITLER (1960). The most important annihilation mode of the free electron-positron system is the annihilation

$$e^+ + e^- \rightarrow \gamma + \gamma . \tag{1}$$

The frequency of this annihilation is 372 times greater than that of the free three-photon annihilation and we may neglect all but the two-photon mode in considering the γ-ray spectrum from free $e^+ - e^-$ annihilation. (However, as we shall see later, the three-photon mode becomes important when we consider the effect of positronium formation by positrons of energies less than 5 keV.)

The differential cross-section for γ-ray production in the collision c.m.s. of a free two-photon annihilation may be written as

$$d\sigma = \frac{\sigma_0}{2\gamma_c^2 \beta_c} \phi(\chi;\gamma)\, d\chi , \tag{2}$$

where $\sigma_0 = \pi r_0^2$, $r_0 = e^2/(Mc^2)$ is the classical radius of the electron, γ is the Lorentz factor of the positron, $\beta = \sqrt{(1-1/\gamma^2)}$, the c.m.s. Lorentz factor and velocity are given by

$$\gamma_c = \sqrt{\frac{\gamma + 1}{2}} \quad \text{and} \quad \beta_c = \sqrt{\frac{\gamma - 1}{\gamma + 1}}, \tag{3}$$

χ is the cosine of the angle between the incoming positron and the outgoing γ-ray in the c.m.s., and the angular distribution function, $\phi(\chi; \gamma)$ is defined as

$$\phi(\chi; \gamma) = \frac{1 + \beta_c^2 (2 - \chi^2)}{1 - \beta_c^2 \chi^2} - \frac{2\beta_c^4 (1 - \chi^2)^2}{(1 - \beta_c^2 \chi^2)^2}. \tag{4}$$

The energy of an annihilation γ-ray in the laboratory system is given by the Doppler relation as

$$E_\gamma = (Mc^2)\, \gamma_c^2 (1 + \beta_c \chi). \tag{5}$$

If we now define the dimensionless energy $\eta = E_\gamma/(Mc^2)$, we may use Equations (3) and (5) to determine the normalized distribution function uniquely relating χ to the laboratory Lorentz factor of the positron and the laboratory energy of the γ-ray as follows:

$$f(\chi; \gamma, \eta) = \frac{2}{\sqrt{\gamma^2 - 1}}\, \delta[\chi - \chi_0(\eta, \gamma)],$$

where

$$\chi_0(\eta, \gamma) = \frac{(2\eta - 1) - \gamma}{\sqrt{\gamma^2 - 1}}. \tag{6}$$

The production of γ-rays per second by a positron of energy γMc^2 is

$$Q_{\mathrm{TOT}}(\gamma) = \frac{2\sigma_0 n_e \beta c}{2\gamma_c^2 \beta_c} \int_{-1}^{1} \mathrm{d}\chi \phi(\chi; \gamma)$$

$$= \frac{2\sigma_0 n_e c}{\gamma} \int_{-1}^{1} \mathrm{d}\chi \phi(\chi; \gamma),$$

where n_e is the number density of electrons in the medium.

Therefore, the γ-ray source spectrum from the annihilation of cosmic-ray positrons with a density $n(\gamma)$ cm^{-3} γ^{-1} is given by

$$Q(\eta) \quad 4n_e \sigma_0 c \int_1^\infty \mathrm{d}\gamma \frac{n(\gamma)}{\gamma\sqrt{\gamma^2 - 1}} \int_{-1}^{1} \phi(\chi; \gamma)\, \delta[\chi - \chi_0(\eta, \gamma)]. \tag{7}$$

We now wish to integrate over the delta function in Equation (7). The result of this integration is to replace the last integral in Equation (7) by a function $\Phi(\eta, \gamma)$ such that

$$\Phi(\eta, \gamma) = \begin{cases} \phi[\chi_0(\eta, \gamma), \gamma] & \text{for} \quad |\chi_0| \leqslant 1 \\ 0 & \text{for} \quad |\chi_0| > 1 \end{cases}. \tag{8}$$

We may, therefore, replace the integral over $d\chi$ by the algebraic function $\phi(\chi_0(\eta, \gamma), \gamma)$ provided the limiting condition on χ_0 is transformed into a limiting condition on the integration over $d\gamma$. The limiting conditions $\chi_0 = \pm 1$ correspond to the relations

$$\eta_\pm = \gamma_c^2 (1 \pm \beta_c). \tag{9}$$

It follows from Equation (9) that the product

$$\eta_+ \eta_- = \gamma_c^4 (1 - \beta_c^2) = \gamma_c^2 = \frac{\gamma + 1}{2}, \tag{10}$$

and the sum

$$\eta_+ + \eta_- = 2\gamma_c^2 = \gamma + 1. \tag{11}$$

Therefore, η_+ and η_- are the roots of the quadratic equation

$$\eta^2 - 2\gamma_c^2 \eta + \gamma_c^2 = 0, \tag{12}$$

which can be solved for γ_c in terms of η, yielding

$$\gamma_c^2 = \frac{\eta^2}{2\eta - 1} = \frac{\gamma + 1}{2}, \tag{13}$$

or

$$\gamma = \frac{\eta^2 + (\eta - 1)^2}{2\eta - 1}, \tag{14}$$

which is finite and positive for $\eta > \frac{1}{2}$ and has a minimum of $\gamma = 1$ at $\eta = 1$.

Figure 1 shows the curve defined by Equation (14) split into the branch defined by the physical extremes $\chi = \pm 1$. Also shown are the asymptotes $\eta = \frac{1}{2}$ and $\eta = \gamma + \frac{1}{2}$, the line defining $\chi = 0$ and the shaded region corresponding to the physical values of $|\chi| \leqslant 1$. It can be seen from Figure 1 that the physical range of γ defined by the

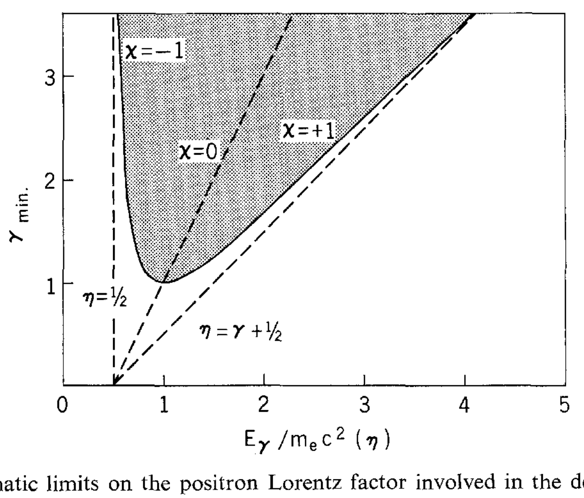

Fig. 1. The kinematic limits on the positron Lorentz factor involved in the determination of the laboratory energies of the annihilation γ-rays produced.

shaded region ($|\chi| \leqslant 1$) is bounded on the bottom by the curve of Equation (14) and is unbounded on the top.

Figure 1 indicates that no γ-rays can be produced from the annihilation process having energies less than or equal to $(Mc^2)/2$. This physical restriction may be seen more clearly as a direct consequence of Equation (9) by noting that

$$\eta_- \leqslant \eta \leqslant \eta_+ , \tag{15}$$

where

$$\eta_- = \gamma_c^2 (1 - \beta_c) = \frac{1 - \beta_c}{1 - \beta_c^2} = \frac{1}{1 + \beta_c} > \frac{1}{2}, \tag{16}$$

so that η_- may approach, but never reach $\frac{1}{2}$ as $\gamma_c \to \infty$. On the other hand

$$\eta_+ = \gamma_c^2 (1 + \beta_c) = \frac{1}{1 - \beta_c}, \tag{17}$$

which increases without bound as $\gamma_c \to \infty$.

The general restriction on the range of η may be designated by the introduction of the Heaviside step function, $H_+(\eta_0)$ which is defined by the relation

$$H_+(\eta_0) = \begin{cases} 1 & \text{for} \quad \eta > \eta_0 \\ 0 & \text{for} \quad \eta \leqslant \eta_0 \end{cases} \tag{18}$$

We may therefore rewrite Equation (7) in the form

$$Q(\eta) = 4H_+\left(\frac{1}{2}\right) n_{e^-} \sigma_0 c \int_{G(\eta)}^{\infty} \mathrm{d}\gamma \, \frac{n(\gamma)}{\gamma\sqrt{\gamma^2 - 1}} \, \phi(\chi_0(\eta, \gamma), \gamma), \tag{19}$$

where

$$G(\eta) = \frac{\eta^2 + (\eta - 1)^2}{2\eta - 1}.$$

From the general results which we have obtained, we may immediately arrive at formulas for asymptotic spectra which may be used as guides in examining the results of numerical calculations. These are obtained as follows:

(a) For two-photon annihilations at rest, it follows from Equation (5) that the AGS is simply a line at energy $\eta = 1$ (0.51 MeV). This is, of course, a familiar and expected result.

(b) The AGS from the two-photon annihilation of an ultrarelativistic positron may be obtained from a consideration of the angular distribution function, $\phi(\chi; \gamma)$, given by Equation (4). At ultrarelativistic energies, the angular distribution function peaks sharply at $\chi = \pm 1$ so that the γ-rays resulting from the annihilation lie close to the asymptotes of Figure 1, viz. $\eta = \frac{1}{2}$ and $\eta = \gamma + \frac{1}{2}$. This result, as pointed out by Heitler (op. cit.), may be discussed physically as follows. In an ultrarelativistic annihilation, the resulting photons are emitted in a sharply backward and sharply forward direction in the collision c.m.s. respectively. In the laboratory system, the forward photon carries off almost all the energy of the collision while the backward photon carries off an energy between about 0.25 and 0.5 MeV. Therefore, the AGS for $\eta \gg 1$ may be obtained by using the approximate production cross-section

$$\sigma_A(\eta;\gamma) \simeq \sigma_A(\gamma)\,\delta(\eta-\gamma) \quad \text{for} \quad \eta \gg 1. \tag{20}$$

The two-photon annihilation cross-section at ultrarelativistic energies has the asymptotic form

$$\sigma_A(\gamma) \simeq \frac{\sigma_0}{\gamma}\left[\ln(2\gamma)-1\right], \quad \gamma \gg 1. \tag{21}$$

Therefore, the ultrarelativistic asymptotic form of the AGS is given by the expression

$$Q(\eta) \simeq n_{e^-}\,\sigma_0 c\,\frac{n(\eta)\left[\ln(2\eta)-1\right]}{\eta}, \tag{22}$$

as has been previously noted by GINZBURG and SYROVATSKIJ (1964b, c). These asymptotic formulas should be kept in mind when examining the numerical evaluations of the AGS obtained from the exact formula given by Equation (19).

3. The Equilibrium Spectrum of Secondary Galactic Positrons from Pi-Meson Decay

In order to calculate plausible annihilation-γ-ray spectra from our own galaxy, it will be assumed that the only source of galactic cosmic-ray positrons above a few MeV energy is the result of primary cosmic rays colliding with atoms of interstellar hydrogen and helium gas in the galaxy. These high-energy collisions are known to produce positive pi-mesons which rapidly decay into positrons and neutrinos. Much accelerator data is available on the production of pi-mesons of interactions up to 30 GeV/c and many cosmic-ray studies have been made of higher energy interactions. Various calculations have been made using this data to calculate positron source spectra in the galaxy (POLLACK and FAZIO, 1963; GINZBURG and SYROVATSKI, 1964b; HAYAKAWA *et al.*, 1964; JONES, 1963; RAMATY and LINGENFELTER, 1966a). In these calculations, it is usually assumed that the primary cosmic-ray spectrum is essentially the same as that observed at the earth. Ramaty and Lingenfelter have shown that in the energy range where most of the pions are produced (above 500 MeV), the primary galactic cosmic-ray spectrum observed at the earth is little affected by local conditions in the solar system and that local effects induce very little error in the positron flux calculation. We therefore take for our source spectrum of cosmic-ray positrons, the one derived by Ramaty and Lingenfelter. (This spectrum is shown in Figure 2.)

We further assume that these positrons have reached a quasi-equilibrium condition in the galaxy determined by leakage out of the halo, annihilation, and energy loss by ionization and coulomb interactions, bremsstrahlung, Compton collisions and synchrotron radiation. The equilibrium density spectrum, $n(\gamma)$, which will be used in Equation (19), is then taken to be determined by the continuity equation*

$$\frac{\partial}{\partial \gamma}\left[n(\gamma)\, r(\gamma)\right] = q_+(\gamma) - \frac{n(\gamma)}{\tau_s(\gamma)}, \tag{23}$$

where $q_+(\gamma)$ is the positron source spectrum from pi-meson decay, $r(\gamma)$ is the total energy loss rate per unit time and $\tau_s(\gamma)$ is the effective positron survival time given by

$$\frac{1}{\tau_s(\gamma)} = \frac{1}{\tau_A(\gamma)} + \frac{1}{\tau_l(\gamma)}, \tag{24}$$

with $\tau_A(\gamma)$ and $\tau_l(\gamma)$ being the annihilation time and mean leakage time (or diffusion

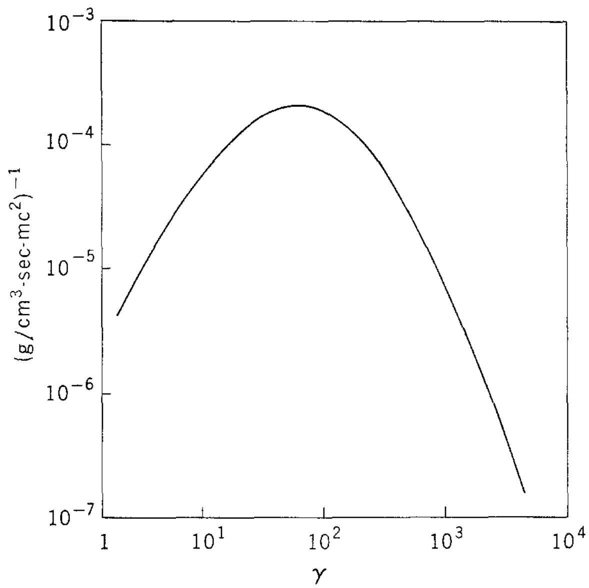

Fig. 2. The source spectrum of positrons produced from the decay of positive pi-mesons formed in cosmic-ray collisions (Ramaty and Lingenfelter, 1967).

time) for positrons of energy γMc^2. These quantities are given by

$$\frac{1}{\tau_A(\gamma)} = \frac{n_{e-}\,\sigma_0 c}{\gamma}\sqrt{\frac{\gamma-1}{\gamma+1}}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right] \tag{25}$$

and

$$\frac{1}{\tau_l(\gamma)} = \frac{\sqrt{\gamma^2-1}}{\gamma T_l}, \quad T_l = \text{const}, \tag{26}$$

* Shen (1967) has suggested that the inclusion in the continuity equation of a term which takes account of intragalactic spatial diffusion, may affect the index of the spectrum above 10 GeV ($\gamma > 2 \times 10^4$). This term need not be considered here since $\gamma > 2 \times 10^4$ is well above the region of primary interest for consideration of the positron AGS and at lower energies the correction is negligible (see also, Ramaty and Lingenfelter (1966b)).

where we have assumed a mean leakage time inversely proportional to velocity.

Most of the interstellar gas in our galaxy is unionized with the exception of the so-called H II regions near the very hot O and B stars which are powerful sources of ionizing ultraviolet radiation. ALLEN (1963) gives the proportion of space near the galactic plane occupied by clouds of interstellar gas and dust as 7% and that occupied by ionized clouds (H II regions) as 0.4%. We will therefore assume that the galactic gas is entirely neutral for the purposes of these calculations and take for the energy loss rate from ionization the expression

$$r_{\mathrm{I}}(\gamma) = \frac{8}{3}\,\sigma_0 n_{e-}\, c\,\frac{\gamma}{\sqrt{\gamma^2-1}} \times \left[22 + \ln\{\gamma(\gamma-1)(\gamma^2-1)\} - 1.695\left(\frac{\gamma^2-1}{\gamma^2}\right) - \frac{1.39}{\gamma}\right]; \tag{27}$$

see HEITLER (1960); MORRISON (1961).

The energy loss rate from bremsstrahlung may be taken as

$$r_{\mathrm{B}}(\gamma) = 7.3\times 10^{-16}\, n_e \gamma\,, \tag{28}$$

based on radiation lengths for hydrogen and helium given by DOVZHENKO and POMANSKII (1964).

The loss rate from synchrotron radiation and Compton collisions may be taken as

$$r_{s+c}(\gamma) = 1.3\times 10^{-9}\,(H^2 + 3\times 10^{-11}\rho_\gamma)\,\gamma^2\,, \tag{29}$$

where H is given in gauss and the radiation density, ρ_γ, is given in ev/cm^3 (RAMATY and LINGENFELTER, 1966a).

The total positron energy-loss rate is taken to be the sum of Equations (27)–(29) and is simply denoted by $r(\gamma)$. Equation (23) may then be solved to yield the equilibrium positron density spectrum in the form

$$n(\gamma) = \frac{1}{|r(\gamma)|}\int_\gamma^\infty \mathrm{d}z\, q_+(z)\exp\left[-\int_\gamma^\infty \frac{\mathrm{d}w}{|r(w)|\tau_s(w)}\right] \tag{30}$$

(STECKER, 1967b).

Equation (30) will be used to obtain numerical solutions for $n(\gamma)$, the positron equilibrium flux

$$I_+(\gamma) = \frac{c}{4\pi}\,\frac{n(\gamma)\sqrt{\gamma^2-1}}{\gamma} \tag{31}$$

and the resulting AGS of

$$\left.\begin{aligned} Q(\eta) = 4H_+\left(\frac{1}{2}\right) n_{e-}\,\sigma_0 c \int_{G(\eta)}^\infty \mathrm{d}\gamma\,\frac{\phi[\chi_0(\eta,\gamma),\gamma]}{\gamma\sqrt{\gamma^2-1}\,|r(\gamma)|} \\ \times \int_\gamma^\infty \mathrm{d}z\, q_y(z)\exp\left[-\int_\gamma^z \frac{\mathrm{d}w}{|r(w)|\tau_s(w)}\right]. \end{aligned}\right\} \tag{32}$$

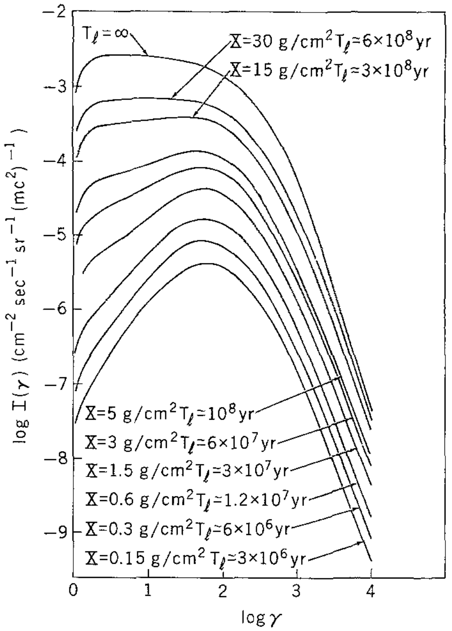

Fig. 3. Various positron equilibrium fluxes for the halo model of the galaxy given for various approximate mean path lengths and mean leakage times.

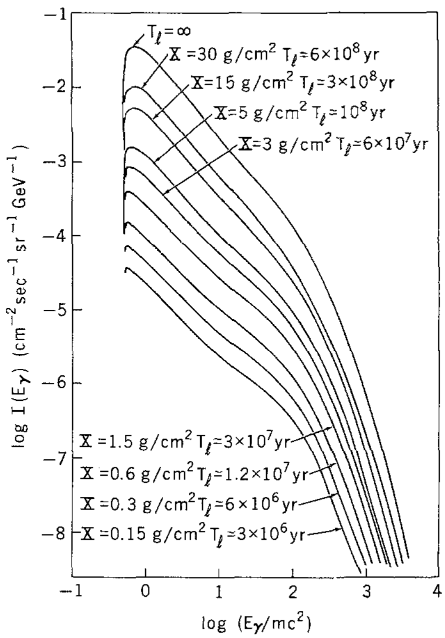

Fig. 4. The annihilation-γ-ray flux spectra for the positron equilibrium fluxes given in Figure 3 resulting from annihilations-in-flight of positrons having energies greater than 5 keV.

In order to utilize Equations (30)–(32) to determine the positron equilibrium flux and AGS in the galaxy, we chose typical average values for the quantities n_e, H, ρ_γ and T_l for both the galactic disc and the galactic halo. These values are given in Table I. The radiation density, ρ_γ, includes the contribution of 0.25 eV/cm^3 from the 2.7 K universal microwave field (STOKES *et al.*, 1967).

Using the values given in Table I for the galactic halo to numerically integrate

TABLE I

Average astrophysical parameters for the galactic disk and halo

	n_e (cm^{-3})	H (μg)	ρ_γ (eV/cm^3)	T_l (10^6 yrs.)	L_{eff} (cm)
Disk	0.5–1	6	0.45	2–4	3×10^{22}
Halo	$1\text{–}3 \times 10^{-2}$	3	0.65	100–300	5×10^{22}

Equations (30)–(32), typical galactic spectra are obtained for $I_+(\gamma)$ and $Q(\eta)$. The results are given in Figures 3 and 4 for various values of mean path length X(g/cm^2) and corresponding T_l.* The flux of annihilation-γ-radiation observed at the earth, $I_A(\eta)$, is given by

$$I_A(\eta) = \frac{L_{eff}}{4\pi} Q(\eta), \tag{33}$$

where L_{eff} is the effective path length for γ-ray production.

For leakage times less than 10 million years, the equilibrium positron flux has roughly the same characteristics of the source spectrum of Figure 2 and its magnitude is proportional to the leakage time. In the case of longer leakage times, the positrons are trapped in the galaxy for a sufficient time for the energy loss processes, particularly ionization loss, to affect the spectrum by progressively flattening it below 30 MeV. Studies of spallating cosmic-ray nuclei yield a mean path length, $X = 4 \pm 1$ g/cm^2 for cosmic-ray nuclei, corresponding to a mean leakage time of the order of 100 million years. The curve in Figure 3 corresponding to $X = 5$ g/cm^2 is in agreement with the cosmic-ray positron measurements of HARTMAN (1967). It may be noted that measurements of the galactic positron flux below 30 MeV would yield a more sensitive check on the galactic mean leakage time.

* The relation between mean path length, X, and mean leakage time, T_l, is given by the equation $X = \rho c T_l$, where ρ is the mean density of the gas in the medium (RAMATY and LINGENFELTER, 1966a). Assuming the galactic medium is 90% hydrogen and 10% helium and with X in g/cm^2, n_H in atoms/cm^3 and T_l in millions of years, this relation becomes $X \simeq 5 n_H T_l$. Thus, for a halo model with $X = 5$ g/cm^2 and $n_H = 10^{-2}$ cm^{-3}, $T_l = 10^8$ yr. whereas, for a disk model with $X = 5$ g/cm^2 and $n_H = 2$ cm^{-3}, $T_l = 10^6$ yr. We chose here to discuss the background γ-ray spectrum from a halo model of the galaxy since it has been shown by RAMATY and LINGENFELTER (1966b) that such a model gives a positron equilibrium spectrum, even in the disk, which is almost identical with that obtained for a disk-plus-halo model. The inclusion of a spatial diffusion term of the form $D_0 \nabla^2 N$ in Equation (23) will have little effect for positrons with $\gamma < 2 \times 10^4$ (see footnote to Equation (23)). It may also be noted that a recent study by ANAND *et al.* (1968) indicates that there is no large gradient of the cosmic-ray electron intensity between the disk and the halo for energies less than 10 GeV.

Figure 4 shows the annihilation-γ-ray spectra obtained using the positron equilibrium fluxes of Figure 3. The spectra shown are from annihilations-in-flight of positrons having energies greater than 5 keV. AGS from positrons annihilating with energies below 5 keV will be discussed later. It can be seen that the peaks of these spectra lie in the region $\frac{1}{2}<\eta<1$. This effect is due to a pile-up of those γ-rays from the annihilation of relativistic positrons which are emitted in the backward direction in the c.m.s.

In Figure 8, this flux is compared with γ-ray spectra from neutral pi-meson production in galactic cosmic-ray collisions (STECKER, 1967) and with the positron's own bremsstrahlung spectrum ($I_\pi(E_\gamma)$ and $I_B(E_\gamma)$ respectively). It should be noted that these spectra are rigidly related since the ultimate source of $I_\pi(E_\gamma)$ is the same cosmic-ray collision process generating the positrons and $I_B(E_\gamma)$ is completely determined by $I_+(\gamma)$ and n_e. The total galactic bremsstrahlung spectrum is of course determined by the sum of the cosmic-ray positron and electron fluxes in the galaxy and may be expected to be at least twice as large as the $I_B(\eta)$ flux shown in Figure 8. We have also shown the expected bremsstrahlung γ-ray flux from the observed cosmic-ray electron spectrum.

Recent measurements have indicated that for $\eta\leqslant 2$ the observed isotropic γ-ray spectrum follows a power law of the form $I_{obs}(\eta)\sim\eta^{-2.3}$ (see review paper by GOULD, 1967). An extrapolation of the observed isotropic spectrum is also plotted in Figure 8 along with the conjectured extrapolation of SHEN and BERKEY (1968). Various authors have suggested that this flux may be due to Compton collisions between intergalactic positrons and electrons and the universal thermal microwave radiation (HOYLE, 1965; GOULD, 1965; FELTEN, 1965; FAZIO *et al.*, 1966; FELTEN and MORRISON, 1966) or background from external galaxies (GOULD and BURBIDGE, 1963; SILK, 1968). If this is indeed the case, it may place severe restrictions on observations of the galactic AGS.

4. The Annihilation-γ-Ray Spectrum from Cosmic-Ray Positrons Annihilating near Rest

We now come to the problem of determining the AGS from cosmic-ray positrons annihilating near rest. Because some aspects of this determination seem deceptively simple, there has been a tendency to oversimplify this problem in the literature. For this reason, I will first list various aspects of the problem essential to an accurate analysis before proceeding to treat them.

(a) The most important source of the cosmic-ray positrons having energies greater than a few MeV is the source we have been considering, the decay of secondary charged pi-mesons. There are, however, sources of relatively low energy positrons (less than a few MeV) which, as we shall see, may have a greater probability of being trapped inside the galaxy until they annihilate near rest. These sources are the β-emitters, which are produced predominantly by low energy cosmic-ray interactions involving carbon, nitrogen and oxygen. Therefore, any observable 0.5 MeV line radiation from the galaxy may be primarily due to these β-emiters and an observation of the intensity of the 0.5 MeV line could supply information on the intensity of low-energy cosmic radiation in the galaxy.

(b) In considering annihilations near rest, one must consider the possibility of the intermediate formation of the bound electron-positron system, i.e., the positronium atom. At low energies, the cross-section for positronium formation becomes much greater than the cross-section for free annihilation.

(c) Once formed in interstellar space, a positronium atom will annihilate 75% of the time into three photons. This situation contrasts sharply with the case of free annihilations where three-photon annihilations occur with a probability of less than $\frac{1}{2}$%. Therefore, the three-photon annihilation process, which produces a continuous spectrum from 0–0.5 MeV, must be considered along with the two-photon line annihilations. Their relative importance depends directly on the fraction of positrons which ultimately form positronium.

5. The Number of Cosmic-Ray Positrons annihilating near Rest

The positrons which annihilate near rest most likely come from two sources:

(a) Positrons from the decay of secondary charged pi-mesons which were created at low enough energies to be trapped for a sufficiently long time in the galaxy to lose essentially all their energy before either annihilating in flight or escaping from the galaxy.

(b) Positrons from the decay of β-emitting nuclei formed in collisions of low-energy cosmic rays involving nuclei of carbon, nitrogen and oxygen.

The fraction of the original positron flux from the decay of secondary pi-mesons which annihilate near rest, f_+, is given by

$$f_+ = \frac{\int_1^\infty \mathrm{d}z q_+(z) \exp\left[-\int_1^\infty \frac{\mathrm{d}w}{|r(w)|\tau_s(w)}\right]}{\int_1^\infty \mathrm{d}z q_+(z)}. \tag{34}$$

This fraction was calculated numerically using Equations (24)–(29) and is given in Table II and Figure 5 for various possible mean leakage times, T_l. Table IV gives the corresponding values of $Q_{\text{rest}, \pi}$, the total number of positrons from pi-meson decay per cm^3 per second annihilating below 5 keV for the halo model of Table I ($Q_{\text{rest}, \pi}$ being defined by the numerator of Equation (34) and the resulting γ-ray fluxes).

In the case of an infinite leakage time (all positrons being trapped and annihilating in the galaxy) we find that 80% of the positrons produced annihilate near rest, a figure which is in perfect agreement with that given by Heitler (1960) as an asymptotic value for the annihilation of ultrarelativistic positrons when the dominant energy loss comes from ionization. However, for the leakage time usually considered as plausible for the galactic halo, 10^8 years (corresponding to $\chi \simeq 5$ gm/cm^2), only 1–2% of these positrons annihilate near rest.

TABLE II

Fraction of positrons from π^+-decay which survive to annihilate near rest ($T_+ \leqslant 5$ keV)

T_l (10^6 yrs.)	f_+ (%)
∞	80
600	20
300	9.3
100	1.6
60	0.61
30	0.14
10	0.022
6	0.0065
3	0.0025

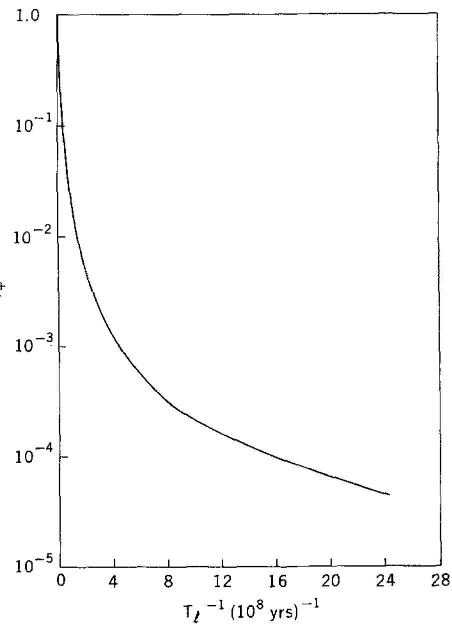

Fig. 5. The fraction of pi-meson produced positrons which annihilate at energies less than 5 keV as a function of mean leakage time.

We next consider the positrons produced in the decay of β-emitting nuclei produced in low-energy cosmic-ray collisions. The important reactions to be considered are listed in Table III (Audouze *et al.*, 1967). Whereas the majority of pi-mesons are produced by cosmic rays having energies over 500 MeV, β-emitting nuclei can be produced by cosmic rays with energies down to below 20 MeV. The positrons produced have energies less than 5 MeV with the singular exception of those produced from the decay of N^{12}. Using the cross-section data given in Table III, and the quiet-sun observations of the cosmic-ray spectrum from 20–1000 MeV/nucleon (Comstock *et al.*, 1966), we obtain the value for the fluxes given in Table IV. This value probably

TABLE III

Principal reactions leading to production of β-emitting nuclei. (See AUDOUZE *et al.* (1967).)

Reaction	σ (mb)	Decay	Half-life	Positron energy (MeV)
C^{12}(p, 3p2n) B^8	?	$B^8(\beta^+)$ Be^8	0.78 sec	1.4
C^{12}(p, p2n) C^{10}	~ 3			
N^{14}(p, 2p3n) C^{10}	?	$C^{10}(\beta^+)$ B^{10}	19 sec	1.9
O^{16}(p, 3p,4n) C^{10}	< 10			
C^{12}(p, pn) C^{11}	~ 70			
N^{14}(p, 2p2n) C^{11}	~ 30	$C^{11}(\beta^+)$ B^{11}	20.5 min	0.97
O^{16}(p, 3p3n) C^{11}	~ 10			
C^{12}(p, n) N^{12}	?			
N^{14}(p, p2n) N^{12}	?	$N^{12}(\beta^+)$ C^{12}	0.011 sec	16.4
O^{16}(p, 2p3n) N^{12}	?			
N^{14}(p, pn) N^{13}	~ 15			
		$N^{13}(\beta^+)$ C^{13}	10.0 min	1.19
O^{16}(p, 2p2n) N^{13}	~ 10			
N^{14}(p, n) O^{14}	~ 50[a]			
		$O^{14}(\beta^+)$ N^{14}	71 sec	1.18, 4.14
O^{16}(p, p2n) O^{14}	< 10			
O^{16}(p, pn) O^{15}	~ 50	$O^{15}(\beta^+)$ N^{15}	2.06 min	1.74

? – Unknown and not included in estimate of positron production.
[a] This cross-section has a value of about 100 mb at energies below 12 MeV but is negligible above 150 MeV.

TABLE IV

Positron annihilation rate and γ-ray flux from positrons annihilating near rest*

Source	$Q_{T,\text{rest}}$ (cm^{-3} sec^{-1})	Flux* (2γ-line) (cm^{-2} sec^{-1} sr^{-1})	Flux* (3γ-continuum) (cm^{-2} sec^{-1} sr^{-1})
π^+: T_1 (10^6 yrs.)			
∞	1.4×10^{-27}	2×10^{-5}	1×10^{-4}
600	3.5×10^{-28}	5×10^{-6}	2.5×10^{-5}
300	1.6×10^{-28}	2.5×10^{-6}	1×10^{-5}
100	3×10^{-29}	5×10^{-7}	2×10^{-6}
60	1×10^{-29}	1.5×10^{-7}	7×10^{-7}
30	2.5×10^{-30}	4×10^{-8}	2×10^{-7}
10	3.8×10^{-31}	6×10^{-9}	2.5×10^{-8}
6	1.1×10^{-31}	1.5×10^{-9}	7.5×10^{-9}
3	4.4×10^{-32}	7×10^{-10}	3×10^{-9}
p-C, N, O	2.5×10^{-28}	4×10^{-6}	1.6×10^{-5}

* Assuming $\langle nL_{\text{eff}} \rangle = 3 \times 10^{21}$ cm^{-2}.

represents a lower limit to the true astrophysical value since over 95% of these β-ray positrons will remain trapped in the galaxy and annihilate near rest.* For mean leakage times less than about 400 million years most of the positrons annihilating near rest come from the β-emitting unstable nuclei produced in cosmic-ray p-C, N, O interactions.

6. Galactic Positronium Formation

The cross-section for positronium formation by fast positrons in atomic hydrogen has been calculated by CHESHIRE (1964) and will be used as an approximation for the interstellar medium. The ratio of positronium formation to free annihilation is only significant at non-relativistic energies and may be approximated by

$$\frac{\sigma_{\text{pos}}(\gamma)}{\sigma_{\text{A}}(\gamma)} \equiv S(\gamma) = \left\{ \begin{array}{lll} 0 & \text{for} & (\gamma - 1) > 10^{-2} \\ 10^{-6}(\gamma - 1)^{-3} & & 10^{-4} < (\gamma - 1) < 10^{-2} \\ 10^{-2}(\gamma - 1)^{-2} & & 2.5 \times 10^{-5} < (\gamma - 1) < 10^{-4} \\ 6.4 \times 10^{11}(\gamma - 1) & & 10^{-5} < (\gamma - 1) < 2.5 \times 10^{-5} \\ 0 & & (\gamma - 1) < 10^{-5} \end{array} \right\}. \quad (35)$$

At energies of the order of the hydrogen binding energy, the probability for positronium formation in the ground state has been estimated to be between 0.25 and 0.50 and the probability for positronium formation in excited states is small (DEUTSCH, 1953).

Equation (35) was used to determine the amount of positronium being formed by positrons combining with electrons as a function of energy. The result of this calculation is shown in Figure 6. This figure shows the percentage of positrons which, after having survived to reach a 10 keV kinetic energy, survive to reach lower energies. The dashed line indicates the survival fraction for free annihilation only; the solid line indicates the survival fraction when positronium formation is taken into account. Figure 6 shows that almost all of the positrons annihilating near rest do so through intermediate positronium formation with an average energy of about 35 eV.

7. γ-Ray Spectra from Two- and Three-Photon Annihilations at Rest

The cross-section for free annihilation into ζ photons, i.e., for the process

$$e^{+} + e^{-} \rightarrow \zeta\gamma, \quad (36)$$

is of the order

$$\sigma_{\text{A}, \zeta\gamma} \sim \alpha^{(\zeta - 2)} \sigma_{\text{A}, 2\gamma}, \quad (37)$$

* The range of a 1.5 MeV positron is about 3 g/cm² (BERGER and SELTZER, 1964) so that almost all of the positrons formed will annihilate near rest. Therefore, for a simple halo model, the equilibrium AGS is directly proportional to the positron production rate along the line of sight (cf. Equation 33). It should be noted that the quiet-sun observations represent a modulated cosmic-ray spectrum and that the true galactic cosmic-ray spectrum (and therefore AGS line flux) could be much greater.

where

$$\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137}^{*} . \tag{38}$$

Since we have seen that the positronium formation process should also be considered, we must also consider reactions of the type

$$e^+ + e^- \to \Pi \quad , \qquad \Pi \to \zeta\gamma \tag{39}$$

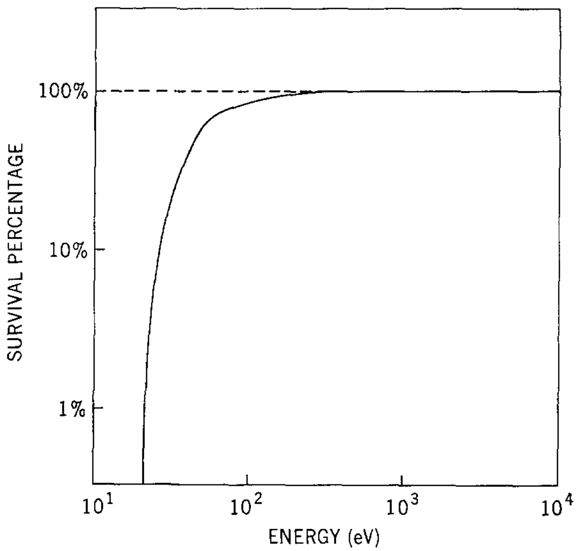

Fig. 6. The percentage of positrons which, after having survived to reach an energy of 10 keV, survive to reach lower energies. The dashed line indicates the survival fraction found by taking into account free annihilation only; the solid line indicates the survival fraction when positronium formation is also taken into account.

where the capital pi symbol will stand for positronium. Here again, we can neglect processes involving $\zeta > 3$.

The positronium annihilations, unlike the free annihilations, obey selection rules since positronium is in an eigenstate of charge conjugation C, and C is conserved in electromagnetic interactions.

If ζ photons are produced in the final state, then

$$C = (-1)^{\zeta} . \tag{40}$$

It can be shown that under particle exchange, positronium obeys a kind of generalized Pauli principle and changes sign. The exchange of particles involves the

* Positrons may annihilate with bound electrons producing a single photon, with momentum being conserved by the binding nucleus. However, the cross-section for this process is always less than $\pi r_0^2 Z^5/(137)^4$, where the atomic number Z is almost always 1 or 2 under astrophysical conditions. Therefore, the single-photon annihilation can be beglected in the following discussion (Heitler, 1960).

exchange of the product of the space, spin, and charge-conjugation parts of the wave function. Therefore,

$$(-1)^l (-1)^{S+1} C = -1\,, \tag{41}$$

where l is the orbital angular-momentum quantum number and S is the spin quantum number. By combining Equations (40) and (41), we obtain the selection rule

$$l + S = \zeta\,. \tag{42}$$

We may therefore specify the processes given by Equation (39) as

$$e^+ + e^- \rightarrow \Pi\,(^1S_0), \quad \hookrightarrow 2\gamma \tag{43}$$

and

$$e^+ + e^- \rightarrow \Pi\,(^3S_1). \quad \hookrightarrow 3\gamma \tag{44}$$

Processes involving $l \neq 0$ can be neglected (Deutsch, 1953).

The lifetime for positronium annihilation into 2 γ-rays is given by

$$\tau_{^1S_0 \rightarrow 2\gamma} = 1.25 \times 10^{-10}\, n^3 \text{ sec}\,,$$

where n is the principal quantum number of the positronium state.

The lifetime for positronium annihilation into 3 γ-rays is given by

$$\tau_{^3S_1 \rightarrow 3\gamma} = 1.4 \times 10^{-7}\, n^3 \text{ sec}\,. \tag{45}$$

(Ore and Powell, 1949).

The lifetimes for both the two- and three-photon annihilations of positronium are therefore so short that for considerations of galactic γ-ray production, we may consider the annihilation to take place effectively at the time that positronium is formed. Since the probability for positronium formation in the triplet state is 3 times the probability for positronium formation in the singlet state, it follows from Equations (43) and (44) that $\frac{3}{4}$ of the positronium formed in the galaxy annihilates into 3 photons.

The energy spectrum of the two-photon annihilation in the center-of-momentum system (c.m.s.) of the electron-positron pair is single line at $E_\gamma \simeq 0.51$ MeV ($\eta = 1$), as can easily be seen from considerations of conservation of energy and momentum. The natural width of this line due to the uncertainty principle is small, being on the order of

$$\Delta E = \frac{\hbar}{\tau} \simeq 5.3 \times 10^{-12}\,\text{MeV}\,. \tag{46}$$

Dominant broadening can be expected to be due to the Doppler effect, and is of the order

$$\Delta E/E = \Delta E/Mc^2 = \beta\,. \tag{47}$$

The effect of astrophysical conditions on the broadening of the 0.51-MeV line from two-photon annihilation can be determined as follows:

In free $e^+ - e^-$ annihilations, we may consider a gas or plasma at temperature T. Then the distribution of the component of particle velocity along the line of sight of the observed γ-ray is of the form

$$f(\beta_{\parallel})\, d\beta_{\parallel} = (b/\pi)^{1/2}\, e^{-b\beta_{\parallel}^2}\, d\beta_{\parallel}, \tag{48}$$

where

$$b = Mc^2/2kT. \tag{49}$$

It then follows from Equations (47) and (48) that the spectral-line shape has the Gaussian form

$$f(\eta)\, d\eta = \sqrt{\frac{b}{\pi}}\, e^{-b[(\eta - 1)/\eta]^2}\, d\eta. \tag{50}$$

Equation (50) has the same form as Equation (58), since the number of collisions involving velocity v is proportional to $\sigma v \simeq \sigma_0 c$ independent of v.

The order of magnitude of the broadening is of the order of

$$\Delta\eta = (\Delta E_\gamma/M) \simeq b^{-1/2} \simeq 1.8 \times 10^{-5}\, T^{1/2}, \tag{51}$$

so that for $T = 100\,\mathrm{K}$, $\Delta E \simeq 10^{-1}$ keV. However, we have shown in the last section that most of the annihilations near rest occur following the formation of positronium by positrons having an average energy of about 35 eV. It therefore follows from the results of the previous section and Equation (47) that 75% of the resultant annihilations occur via the three-photon channel and 25% produce a two-photon line with a Doppler width of about 5 keV.

The energy spectrum from the three-photon annihilation is continuous, as allowed by conservation of momentum. It has been calculated by ORE and POWELL (1949) to be of the form

$$F(\eta) = \frac{2}{\pi^2 - 9}\left[\frac{\eta(1-\eta)}{(2-\eta)^2} - \frac{2(1-\eta)^2}{(2-\eta)^3}\ln(1-\eta) + \frac{2-\eta}{\eta} + \frac{2(1-\eta)}{\eta^2}\ln(1-\eta)\right]. \tag{52}$$

The function $F(\eta)$ is shown in Figure 7; $F(\eta)$ is normalized so that

$$\int_0^1 F(\eta)\, d\eta = 1. \tag{53}$$

The average background γ-ray spectra from two- and three-photon annihilations near rest are shown in Figure 8 for $X = 5\ \mathrm{g/cm^3}$ and $nL_{\mathrm{eff}} = 3 \times 10^{21}\,\mathrm{cm}^{-2}$, along with the spectra discussed previously.

8. Discussion

Non-directional observations of the cosmic X-ray spectrum by METZGER *et al.* (1964), have indicated a continuum spectrum well above the theoretical predictions calculated here for both the continuum and line spectra from the galactic halo.

The theoretical analysis presented here would indicate that it would be more profitable to concentrate on the possible detection of the 0.51 MeV line spectrum in the direction of the galactic disk. The results of Metzger *et al.* indicate that the background continuum X-rays have a flux of about 6×10^{-5} cm^{-2} sec^{-1} sr^{-1} keV^{-1} at 0.51 MeV.

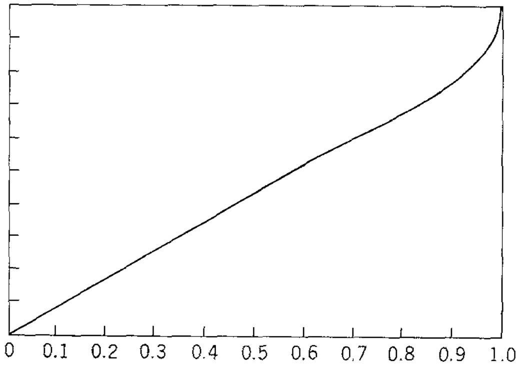

Fig. 7. Energy spectrum of γ-rays resulting from the three-photon annihilation of an electron and a positron (from ORE and POWELL (1949)).

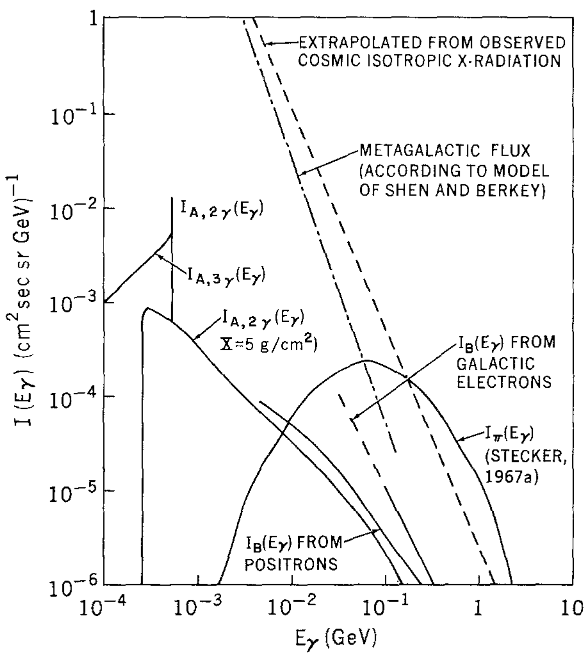

Fig. 8. Background γ-ray spectra from various astrophysical processes (see discussion in text). Subscripts A, B, and π indicate annihilation, bremsstrahlung and neutral-pion decay γ-ray spectra respectively. Intensities are for an average value over the galactic halo of $\langle nL_{\text{eff}}\rangle = 3\times10^{21}$ cm^{-2} and a mean positron travel path of 5 g/cm^2. The line spectrum at 5.1×10^{-4} GeV is due to two-photon annihilations at rest; the continuous spectrum below 5.1×10^{-4} GeV is due to three-photon positronium annihilations at rest. The continuous spectrum starting at 2.5×10^{-4} GeV is due to the free two-photon annihilations of cosmic-ray positrons having energies greater than 5 keV.

Thus, an instrument with a resolution of 5 keV (the predicted width of the 0.51-MeV line) would be able to measure a line imposed on that continuum with a flux of about 3×10^{-4} cm^{-2} sec^{-1} sr^{-1}. Such an intensity is at least an order of magnitude higher than the highest fluxes predicted in Table IV for the halo, but may not be unreasonable for the disk. The AGS from η^+ positrons in the disk is proportional to $\langle nL_{\text{eff}} \rangle$ under the assumption that these positrons fill the halo and the disk with an equal intensity. From Table I we would therefore expect a disk flux one or two orders of magnitude above the halo flux. When we consider the positrons arising from C, N, O-β-decay chances for observing the AGS-line may be even more favorable. Since most of these positrons annihilate at rest after traversing 3 g/cm^3 (see footnote on p. 593), it is possible that they may be trapped in the disk. In this case, the line flux in any direction would be simply proportional to the positron production rate along the line-of-sight. The result would be a disk intensity at least two orders of magnitude above the value given in Table IV, i.e., 4×10^{-4} cm^{-2} sec^{-1} sr^{-1}, which is a large enough value to be detectable. Taking into account the possibility that the true galactic CNO cosmic-ray spectrum may be significantly higher between 20 and 1000 MeV/nucleon, the AGS line spectrum in the disk may be possibly above 10^{-3} cm^{-2} sec^{-1} sr^{-1}. (Metzger *et al.* give an upper limit of about 10^{-3} cm^{-2} sec^{-1} sr^{-1} for the 0.51-MeV line. However, this is an average over the celestial sphere since their detector had no directionality.) It may therefore be concluded that annihilation-γ-rays from the galactic halo may remain forever masked by a metagalactic continuum. However, an 0.51-MeV line from the disk may well be detectable. It is most reasonable to assume that this line is formed predominantly by the annihilation of positrons arising from cosmic-ray C, N, O-β-decay. Under this assumption, the intensity of this line becomes a sensitive measure of the galactic cosmic-ray flux below 1000 MeV/nucleon.

Acknowledgments

The author would like to acknowledge and thank Dr. Reuven Ramaty, Dr. Frank C. Jones and Dr. Richard Drachman for many fruitful discussions, Dr. Ramaty for supplying the author with much helpful information on positron production in the galaxy and Mr. Joseph Bredekamp for programming the numerical calculations essential to this paper.

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