GASDYNAMICS OF THE SOLAR WIND INTERACTION WITH THE INTERSTELLAR MEDIUM

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Abstract. A survey of the present-day situation in gasdynamical models of solar wind interaction with the local interstellar medium is presented. A role of these models in interpreting a number of observed physical phenomena is investigated. Experimental data and possible observations are considered from the viewpoint of their interpretation on the basis of theoretical models. Our attention is concentrated on the main limitations of the gasdynamical models, in particular, two-shocks model developed by Baranov *et aL* (1981, 1982).

I. Introduction

The solar wind as the process of a supersonic expansion of solar corona was discovered theoretically by Parker (1958). Later this phenomenon was confirmed experimentally by means of spacecraft (Gringauz *et al.,* 1960; Neugebauer and Snyder, 1962). This discovery was made possible due to the contradiction between the high pressure in infinity obtained by solving the problem of the distribution of parameters in a hydrostatic solar corona and the interstellar medium pressure estimated by observations. The latter turned out to be smaller by two or more orders of magnitude.

However, Parker's solution for a spherically-symmetric solar wind and its later developments taking into account viscosity, thermal conduction, magnetic field, plasma fluctuations, temperature anisotropy, etc., have given rise to an asymptotic solution (with $r \to \infty$, where r is the distance from the Sun) for hypersonic flow with a constant radial velocity (V_{SW} = const.), and the solar wind density being $\rho_{SW} \sim 1/r^2$. On the Earth's orbit $(r = r_E)$ the solar wind velocity is, on the average, about $V_{\text{sw}} \approx 400 \text{ km s}^{-1}$.

It follows that the solar wind's dynamic pressure ($\sim \rho_{\rm sw} V_{\rm sw}^2$) tends to zero for $r \to \infty$ (static pressure p_{SW} of solar wind also tends to zero as $p_{SW} \ll \rho_{SW} V_{SW}^2$ in hypersonic flow). Since the pressure p_{∞} of the interstellar medium, into which the solar wind flows, is finite (though very low) a new theoretical problem appeared, that is the problem of conjugation of the solutions for the solar wind and the interstellar medium. At some heliocentric distance the solar wind's pressure becomes too low to push it self further into the interstellar medium and as a result, the solar wind must be damped.

The location, shape, and size of the damped region depend on the sum of the static, dynamic, and magnetic field pressures in the interstellar medium as well as on the cosmic-ray pressure of the Galaxy. Hence, to find the basic mechanism of the damping of the solar wind at large distances from the Sun, one must know the density, temperature, the ionization degree of interstellar medium in the neighbourhood of the solar system, its bulk velocity relative to the Sun, the direction and value of the magnetic field, the characteristics of cosmic-ray fluxes and so on.

In connection with this, two terms have appeared in literature: the local interstellar medium, LISM (at distances from tens to several hundred parsecs from the Sun) and the very local interstellar medium, VLISM (the interstellar medium region contacting the solar wind boundary and directly affecting its damping). Cox and Reynolds's (1987) review and that of Bochkarev (1987) are devoted to modern concepts of the LISM and VLISM and to methods for measuring their parameters.

The problem of the interaction of the solar wind with the interstellar medium has long been a topic of interest (Davis, 1955; Parker, 1961; Axford *et al.,* 1963; Brandt, 1964; Baranov *et al.,* 1970).

Nevertheless it has become especially actual after at the beginning of the seventies, when experiments on scattered solar radiation at wave lengths λ 1216 Å and λ 584 Å proved us (Kurt, 1965; Bertaux and Blamont, 1971; Thomas and Krassa, 1971; Blum and Fahr, 1970; Fahr, 1974; Weller and Meier, 1974) that atoms H and He of LISM are moving with a supersonic velocity $V_{\infty} \sim 20$ km s⁻¹ relative to the Sun and that the direction of this motion is almost in the ecliptic plane and does not coincide with the direction of the solar motion with respect to the nearest stars (apex direction).

A fine review of these pioneer papers was made by Axford (1972). At present the scientists vanity is excited by, first, the possibility of direct investigations of the solar system perifery by the 'Voyager 1/2' and 'Pioneer 10' spacecraft and, second, the great achievements of space astronomy.

This paper is devoted to a survey of the present-day situation in gasdynamic models of the solar wind interaction with the VLISM, as well as their role in interpreting a number of observed physical phenomena. A historical review of gasdynamic models is given in Section 2. The gasdynamic model with two shocks (TSM) is described in Section 3, while in Section 4 our attention is concentrated on the main limitations of the present-day gasdynamical models. In Section 5 available experimental data and possible observations of some physical phenomena are considered from the viewpoint of their interpretation on the basic of theoretical models.

2. Historical Review

The construction of a theoretical model for the solar wind interaction with the interstellar medium which should be able to explain the physical phenomena is a significant problem for the following three reasons. First, from a purely theoretical viewpoint, the solutions for the solar wind should be consistent with those for the interstellar medium. Secondly, the correct interpretation of observable phenomena, such as the solar radiation scattering at wavelengths λ 1216 Å (for H-atoms) and λ 584 Å (for He-atoms), can only be made on the basis of an adequate theoretical model. And thirdly, an internally consistent model makes it possible to determine, through reliably measured quantities, those parameters which are measured very poorly, e.g., the degree of the VLISM ionization.

Besides, this model will also allow us to predict physical phenomena which have not

yet been observed, such as the anisotropy of radio-scintillations, the anisotropy of cosmic-rays arrival (Baranov, 1981), and so on.

Parker (1961) was the first to develop a quantitative gasdynamic model for the interaction of the interstellar medium with stellar winds, including the solar wind.

Since at that time there were no direct observations of the interstellar gas motion relative to stars, in particular the Sun, Parker investigated the following three possibilities: the outflow of a supersonic stellar wind into an interstellar gas at rest; into a subsonic translational flow of interstellar gas (the subsonic interstellar wind); and into an homogeneous magnetic field by neglecting the interstellar gasdynamical pressure. Later, a model of the solar wind interaction with a supersonic translational flow of interstellar gas (the supersonic interstellar wind) was developed by Baranov *etal.* (1970). They assumed that the interstellar gas moves relative to the Sun due to the Sun's own motion with respect to the nearest stars with a velocity of $V_{\infty} = 20$ km s⁻¹ which is supersonic for the an interstellar gas temperature of $T_{\infty} \approx 10^4$ K. In this case the vector of the relative velocity should be oriented at an angle of about $53°$ to the ecliptic plane. All these models are considered below.

2.1. SOLAR WIND OUTFLOW INTO THE HOMOGENEOUS INTERSTELLAR GAS AT REST

Consider a stationary, spherically-symmetric solar wind outflow into a space occupied by a gas at rest. The pressure of this gas is equal to $p_{\infty} \neq 0$, and its velocity is $V_{\infty} = 0$. To connect the solar wind solution with that of the interstellar medium it is necessary for solar wind pressure p and velocity V that

$$
p \to p_{\infty}, \quad V \to 0 \quad \text{with} \quad r \to \infty , \tag{2.1}
$$

where r is a distance from the Sun.

The relations (2.1) is valid if the interstellar magnetic field and galactic cosmic rays may be neglected. In this case the transition from a supersonic solar wind to the interstellar gas at rest may be realized through a spherical shock only. The Rankine-Hugoniot relations on this shock must be satisfied, which have the following form in the hypersonic limit (where $M_1 \ge 1$, $M_1 = V_{SW}/a_1$ is the Mach number and a_1 the sonic velocity in front of this shock):

$$
\frac{V_2}{V_{\text{SW}}} = \frac{\gamma - 1}{\gamma + 1}, \qquad \frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}, \qquad p_2 = \frac{2\rho_1 V_{\text{SW}}^2}{\gamma + 1} \ . \tag{2.2}
$$

Here V, ρ , p , and γ are the velocity, mass density, pressure, and specific heat ratio, respectively. The indices '1' and '2' relate to their values in front and behind the shock. The Mach number behind the shock is very small $(M_2 \ll 1)$ and, therefore, the flow may be assumed to be non-compressible (ρ_2 = const.). If relation (2.1) is satisfied, the Bernoulli integral has the following form

$$
p + \rho_2 \frac{V^2}{2} = p_2 + \rho_2 \frac{V_2^2}{2} = p_\infty \,. \tag{2.3}
$$

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Since the solar wind velocity $V_{SW} = \text{const.}$, we have, using the mass conservation equation

$$
\rho r^2 = \rho_{\rm E} r_{\rm E}^2 = \rho_1 r_1^2 \,, \tag{2.4}
$$

where the index 'E' signifies the Earth's orbit and r_1 is the heliospheric distance to the shock (below we call it the 'heliospheric shock').

After substituting (2.2), (2.4) into (2.3) and solving the equation obtained relative to r_1 we have finally

$$
r_1 = r_E \left[\frac{\gamma + 3}{2(\gamma + 1)} \frac{\rho_E V_{\text{SW}}^2}{p_{\infty}} \right]^{1/2} .
$$
 (2.5)

This formula determines the boundary of the hypersonic solar wind, i.e., the distance to the heliospheric shock. From the continuum equation for a noncompressible fluid (behind the heliospheric shock) we obtain

$$
Vr^2 = V_2r_1^2
$$
 or $V = \frac{\gamma - 1}{\gamma + 1}V_{sw}(\frac{r_1}{r})^2$, (2.6)

i.e., the solar wind velocity behind the heliosphere shock must decrease as the square of the distance from the Sun. In this case the conditions (2.1) are satisfied.

To estimate the order of the magnitude for r_1 let us take $V_{SW} = 4 \times 10^7$ cm s⁻¹, $p_{\infty} = 10^{-13}$ dyn cm⁻², $n_E = 5$ cm⁻³ ($\rho_E = m_p n_E$, where m_p is the proton mass), $\gamma = \frac{5}{3}$. Thus we obtain $r_1 = 350 \text{ AU}$ from (2.5), i.e., by the assumption that the solar wind outflows into the interstellar gas at rest, the distance to heliospheric shock is about some hundred of astronomical units (AU).

Here the ease considered is that of the stationary solar wind outflow into the interstellar gas at rest, which is, in general, not real because the boundary between the solar wind and interstellar gas proved to be in infinity ($V \rightarrow 0$ with $r \rightarrow \infty$). However, the solution obtained above (Parker, 1961) may be considered as a limiting case (with $t \to \infty$, where t is the time) of the well-known problem connected with interstellar bubble expansion (see, for example, Weaver *et al.,* 1977). The general picture of this flow is shown in Figure 1. Let the gas from the star located at the zero-point of the coordinate system be ejected at the initial time $t = 0$ with a radial supersonic velocity $V_s = \text{const.}$ into the interstellar gas at rest, the pressure of which is p_{∞} . The well-known solution of the arbitrary discontinuity decay problem results in separating the flow region into the following four parts: region I of the undisturbed supersonic stellar wind; region II between the contact discontinuity ($r = r_c$) and the inner shock $r = r_1$ (this region is occupied by compressed solar wind); region III occupied by the compressed (in the outer shock $r = r₂$) interstellar gas, and region IV of the interstellar gas at rest with the pressure p_{∞} .

Coupling of solutions for stellar wind (region I) and interstellar gas at rest (region IV) is effected by means of the shocks $r = r_1$ and $r = r_2$ and the contact discontinuity $r = r_c$. The whole picture shown in Figure 1 expands with time.

Fig. 1. Nonstationary picture of the interstellar bubble formation.

2.2. SOLAR WIND OUTFLOW INTO THE INTERSTELLAR GAS MOVING WITH SUBSONIC VELOCITY

The interstellar gas motion relative to the Sun gives rise to disturbing the spherical symmetry of the flow considered in Section 2.1. If we assume that there is a spherical symmetry of supersonic solar wind and that the undisturbed subsonic interstellar wind is characterized by the density ρ_{∞} , velocity V_{∞} , and pressure p_{∞} which are constants, the resulting flow will have an axial symmetry.

This case follows one of the main assumptions made by Parker (1961) for the solution of the problem, namely that the dynamic pressure of the interstellar wind is small as compared with the static value, i.e.,

$$
\rho_{\infty} V_{\infty}^2 / 2 \ll p_{\infty} \tag{2.7}
$$

or the Mach number M_{∞} of interstellar wind is much less than unit $(M_{\infty}^2 \ll 1)$.

The general view of stream lines for such a flow system is shown in Figure 2. The heliospheric shock is an almost spherical one $(r = r_1)$ because of the assumption (2.7). The characteristic length L_0 of this flow, which will be determined below, appears in this case to be much larger than r_1 ($L_0 \gg r_1$). For this reason the heliospheric shock is not shown in Figure 2. The subsonic interstellar wind interacts directly with the subsonic flow of the solar wind compressed in this shock. The outer boundary of the subsonic solar wind region is the contact discontinuity with the interstellar wind.

Fig. 2. The solar wind interaction with the subsonic interstellar wind (Parker, 1961).

Since the heliospheric shock has a spherical shape by the assumption (2.7), we can interpret the flow behind this shock as a potential and noncompressible one (the Mach number behind the shock is $M_2 \ll 1$ since $M_1 \gg 1$ in the solar wind). In the case considered we can also take the interstellar wind as noncompressible $(M_{\infty} \ll 1)$. The velocity potential introduced according to the formula

$$
\mathbf{V} = -\rho^{-1/2} \text{ grad } \varphi \tag{2.8}
$$

satisfies the Laplace equation

$$
\Delta \varphi = 0 \tag{2.9}
$$

We take $\rho = \rho_2$ for the subsonic region of the solar wind and $\rho = \rho_{\infty}$ for the interstellar wind. The boundary conditions for Equation (2.9) are

$$
-\operatorname{grad}\varphi = \rho_{\infty}^{1/2} \mathbf{V}_{\infty} \tag{2.10}
$$

with $r \rightarrow \infty$ and

$$
-\operatorname{grad}\varphi = \rho_2^{1/2} \mathbf{V}_2 \tag{2.11}
$$

for $r = r_1$ (on the shock), where ρ_2 and V_2 are the mass density and velocity of the solar wind behind the shock. The values ρ_2 and V_2 are determined by means of the Rankine-Hougoniot relations (2.2).

Since the problem formulated has axial symmetry, the unknown potential may be presented by means of the sum of potentials corresponding to a homogeneous flow and a spherically-symmetric source of a noncompressible fluid

$$
\varphi(r,\,\theta) = \rho_{\infty}^{1/2} \, V_{\infty} \, r \cos \theta + \rho_{2}^{1/2} \, V_{2} \frac{r_{1}^{2}}{r} \,, \tag{2.12}
$$

where r is the distance from the Sun (in the origin of the coordinate system), θ the angle counted from the *Oz-ards* (the interstellar wind in infinity moves at a constant velocity along the negative Oz-axis), and r_1 the location of the shock determined from (2.5). The potential (2.12) satisfies the Laplace equation (2.9) and the boundary conditions (2.10) and, approximately, (2.11). The latter is correct if we assume that $\rho_{\infty} V_{\infty}^2 \ll \rho_2 V_2^2$. As a result, the potential (2.12) is the solution of our problem.

Let us now determine the characteristic length L_0 of the flow considered. It is determined as the distance at which the solar wind will be deflected by the interstellar wind from a spherically-symmetric flow. Clearly, it is a distance, at which the kinetic energies of the solar and interstellar winds are of the same order of value, i.e., $\rho_2 V^2 \approx \rho_\infty V^2_{\infty} (\rho_2 V^2 \gg \rho_\infty V^2_{\infty})$. To estimate the value of V we shall use formula (2.6) for the spherically-symmetric case. As a result, we obtain

$$
\rho_2 V_2^2 \left(\frac{r_1}{r}\right)^4 \approx \rho_\infty V_\infty^2 \quad \text{or} \quad r \approx r_1 \left(\frac{\rho_2 V_2^2}{\rho_\infty V_\infty^2}\right)^{1/4}.
$$

It gives

$$
L_0 = r_1 \left(\frac{\rho_2 V_2^2}{\rho_\infty V_\infty^2}\right)^{1/4} \gg r_1 , \tag{2.13}
$$

i.e., the characteristic lenght of the flow considered is much larger than the distance from the Sun to the heliospheric shock. Thus we proved our statement made at the beginning of this Section.

The equations of the stream lines are

$$
-\frac{r^2\sin^2\theta}{2L_0^2}=\cos\theta+C\quad (C=\text{const.}).
$$

Evidently, on the surface dividing the solar wind and the interstellar gas (this surface is often called the heliopause), the pressure is continuous and the normal component of the velocity is equal to zero, i.e., the heliopause is a tangential discontinuity (or contact discontinuity).

Since the tangential discontinuity coincides with the stream line and the ray $\theta = 0$ intersects it, we have $C = -1$ for this stream line, i.e., the equation of the heliopause is

$$
-r^2 \sin^2 \theta = 2L_0^2 (\cos \theta - 1). \tag{2.14}
$$

Hence, we see that L_0 is the heliocentric distance to the heliopause stagnation point $(\theta = 0)$ and this distance is determined by the formula (2.14). From (2.14) we have with $r \rightarrow \infty$ and $\theta \rightarrow \pi$

$$
r\sin\theta \rightarrow 2L_0,
$$

i.e., the shape of the heliopause tends to a cylindrical surface of radius $2L₀$.

2.3. THE SOLAR WIND DECELERATION BY THE MAGNETIC FIELD OF THE INTERSTELLAR MEDIUM

The third case considered by Parker (1961) relates to the possibility of the solar wind being decelerated by a homogenous magnetic field \mathbf{B}_{∞} , with a pressure $B_{\infty}^2/8\pi$ much larger than the static and dynamic interstellar medium pressures and also larger than the cosmic-ray pressure of the Galaxy. The spherical symmetry of the solar wind's outflow into the interstellar medium is distorted by the magnetic field, because its decelerating effect is different for various directions. The magnetic Reynolds number of the solar wind is large $(R_m \gg 1)$. Therefore, the magnetic field of the interstellar medium is drived out by the solar wind plasma due to the 'infreezing' effect. Thus a cavity is formed which is elongated along the magnetic field \mathbf{B}_{∞} and is filled by the solar wind plasma (see Figure 3).

However, for solving this problem the following assumption was made by Parker (1961). He assumed that the solar wind's deceleration from a supersonic to a subsonic flow takes place through a spherical symmetric shock, in spite of the fact that the magnetic field distorts, in the general case, the spherical symmetry of this flow. Under this assumption, a solution of the problem exists, if the solar wind's stagnation pressure p_0 behind the shock is changed in a very small interval

Fig. 3. The outflow of the solar wind into a homogeneous magnetic field of the interstellar medium (Parker, 1961). HP is the heliopause.

It seems to us that this solution is not realized in nature and it will not be considered below.

We should like only to note that in the case considered the heliocentric distance to the heliospheric shock in a direction perpendicular to \mathbf{B}_{∞} may be estimated by means of formula (2.5). For this purpose we must introduce in that formula the magnetic pressure $B_{\infty}^2/8\pi$ instead of the static gas pressure p_{∞} . In this case the assumption about spherical symmetry of the shock is not important.

2.4. THE SOLAR WIND INTERACTION WITH THE HYPERSONIC INTERSTELLAR WIND

As was noted in the beginning of Section 2, Baranov *et al.* (1970) developed a gasdynamic model of the solar wind's interaction with the supersonic interstellar medium. A qualitative picture of the flow considered is shown in Figure 4. Two shocks are formed: the bow shock (BS in Figure 4), which is the shock through which the inter-

Fig. 4. A qualitative picture of the solar wind's interaction with a supersonic interstellar wind (Baranov *et al.,* 1970). BS is the bow shock, CD is the contact discontinuity, HS is the heliospheric shock.

stellar wind passes and is decelerated by solar wind, and the heliospheric shock (HS), which is the shock through which the solar wind passes and is decelerated by the interstellar wind. The dotted line CD shows the contact discontinuity separating the compressed interstellar wind from the compressed solar wind.

The heliospheric shock must approach the axis of symmetry with $\theta \rightarrow \pi$ (in the wake of this flow), and its heliocentric distance in this direction may be estimated on the basis of formula (2.5). This qualitative conclusion (which was later confirmed by numerical results) is the consequence of the fact that the solar wind velocity (\sim 400 km s⁻¹) is much larger than the interstellar wind velocity (\sim 20 km s⁻¹) and, therefore, the solar wind's outflow into the anti-apex direction must be just the same as the solar wind's outflow into the interstellar medium at rest.

Below, this model is called the two-shocks model (TSM). It was first proposed and calculated by Baranov *et aL* (1970) under the assumption of cylindrical symmetry and a negligible value of the distance between the shocks BS and HS when compared to their distance from the Sun (we shall use a spherical coordinate system in which the Sun is in its origin and the ray $\theta = 0$, where θ is the polar angle, coincides with the direction to the apex).

The latter assumption may be corrected if, first, the interstellar wind flow is hypersonic, i.e., $M_{\infty} \gg 1$ (the undisturbed solar wind is always hypersonic behind the Earth's orbit) and, second, if the region of the flow considered is not far from the direction to the apex (the thin layer approximation is not correct in the anti-apex direction).

Let us assume that the compressed layer between the shocks BS and HS can be considered as a discontinuity surface across which the average velocity of the gas does not change. In this case we consider the momentum conservation law in the gas layer in the directions normal and tangential to the layer (below in this section we shall follow the papers by Baranov and Krasnobaev (1971, 1977) in which the TSM by Baranov *et al.* (1970) is generalized to take into account the interstellar magnetic field \mathbf{B}_{∞} parallel to the interstellar wind velocity V_{∞}). This leads to

$$
\rho_{\infty} V_{\infty n}^2 + \frac{B_{\infty \tau}^2}{8\pi} - \frac{B_{\infty n}^2}{8\pi} = \rho_1 V_{\text{SWn}}^2 + \frac{mV_l}{2\pi r R_{\kappa} \sin \theta},
$$
\n
$$
\frac{d}{dl}(mV_l) = 2\pi r \sin \theta \left(\rho_{\infty} V_{\infty n} V_{\infty \tau} + \rho_1 V_{\text{SWn}} V_{\text{SWr}} - \frac{B_{\infty n} B_{\infty \tau}}{4\pi}\right).
$$
\n(2.15)

Here, m is the mass of the gas from the solar wind and the interstellar medium flowing into the layer per unit time, R_{κ} is the radius of curvature of the unknown surface replacing the compressed layer of gas between the shocks BS and HS (the last term of the first equation of (2.15) represents a centrifugal force affecting the gas layer), V_I is the average (over the area of the cross-section of the layer) velocity along the layer, r and θ are polar coordinates, the indexes 'n' and ' τ ' are related to projections on directions normal and tangential to the discontinuity surface. The values of m and R_{κ} are determined by the formulae:

$$
m = \pi r^2 \rho_{\infty} V_{\infty} \sin^2 \theta + 2\pi r^2 \rho_1 V_{\text{SW}} (1 - \cos \theta),
$$

\n
$$
R_{\kappa} = \frac{(r^2 + r^2)^{3/2}}{r^2 + 2r^2 - rr^{\prime \prime}}, \quad r = r(\theta),
$$
\n(2.16)

where a prime denotes the polar angle θ derivative.

It should be remarked that Equations (2.15) do not include the pressure p. This is because $\rho V^2 \gg \pi$, which is satisfied for hypersonic flow.

Equations (2.15) and (2.16) represent the generalization of a Newtonian approximation for a thin layer, which is often used in hydro-aeromechanics for calculations of the hypersonic flow around blunt bodies (Cherny, 1959). More strictly these equations were obtained by Giuliani (1982), who generalized them for the nonstationary case and for the case when an arbitrarily directed magnetic field is present.

Equations (2.15) and (2.16) were also used by Dyson (1975) for investigating the stellar wind interaction with globulas in H_{II} regions.

Let us take into account the geometric relations between θ and the angle between the direction of the interstellar wind velocity V_{∞} and the tangent to the unknown discontinuity surface and also with the angle between a radius-vector and the direction, normal to the discontinuity surface. After excluding V_i , m, and R_r from (2.15), (2.16) we obtain a nonlinear differential equation of the third-order for determining the form $r = r(\theta)$ of the discontinuity surface

$$
rr''' = \frac{1}{F_2} \left(F_1 - \frac{F_2'}{F_3} \right) F_3^2 + 2rr' + 3r'r'' \,. \tag{2.17}
$$

Here

$$
F_1 = 2\pi r \sin \theta (r^2 + r'^2)^{1/2} \left[\frac{(\rho_{\infty} V_{\infty}^2 - B_{\infty/8\pi}^2) (r \cot \theta + r') (r - r' \cot \theta)}{(r \cot \theta + r')^2 + (r - r' \cot \theta)^2} + \rho_1 V_{\text{SW}}^2 r r' (r^2 + r'^2)^{-1} \right],
$$
\n
$$
F_2 = 2\pi r \sin \theta (r^2 + r'^2)^{3/2} \left[\frac{(\rho_{\infty} V_{\infty}^2 - B_{\infty/8\pi}^2) (r \cot \theta + r')^2}{(r \cot \theta + r')^2 + (r - r' \cot \theta)^2} - \rho_1 V_{\text{SW}}^2 r^2 (r^2 + r'^2)^{-1} + \frac{B_{\infty}^2}{8\pi} \right],
$$
\n(2.18)

$$
F_3 = r^2 + r'^2 - rr''.
$$

There are two boundary conditions for Equation (2.17)

$$
r = r_* , \qquad r' = 0 \quad \text{with} \quad \theta = 0 , \tag{2.19}
$$

where r_* is the heliocentric distance of the unknown discontinuity surface along the axis of symmetry. The value r_* is determined by the equations

$$
\rho_1 V_{\text{SW}}^2 = \rho_{\infty} V_{\infty}^2 - \frac{B_{\infty}^2}{8\pi}, \qquad \rho_1 V_{\text{SW}} r_{*}^2 = \rho_{\text{E}} V_{\text{SW}} r_{\text{E}}^2 = \text{const.}
$$

Here the first equation follows from Equations (2.15), (2.16) with $\theta \rightarrow 0$ and the second is the continuity equation for a spherically-symmetric solar wind. The second condition of (2.19) is a symmetry condition of the problem. To obtain the third boundary condition we must note that Equation (2.17) has a singular point ($\theta = 0$, $r = r_*$), which is a 'saddle'. Therefore, there is a unique integral curve emerging from the singularity and determining our solution. Let us make an expansion of the function $r = r(\theta)$ in a Taylor series in the vicinity of this point

$$
r(\theta)=r(0)+\frac{r''(0)}{2!}\,\theta^2+\cdots.
$$

Then from (2.18) with $\theta \rightarrow 0$ we have $F_1 \approx \theta^2$, $F_2 \approx \theta^3$, $F_3 \approx 1$. In order that r''' be limited it is necessary to require the vanishing terms of the order θ^2 between the parentheses at the right-hand side of Equation (2.17). Setting the coefficient of θ^2 in the expansion of this parentheses term in θ equal to zero have

$$
[r''(0)]^2 + (5M_A^2 - \frac{9}{2})r_*r''(0) + 2r_*^2(1 - M_A^2) = 0,
$$

where $M_A = V_{\infty}/V_A$ is the Alfvén Mach number $(V_A = B_{\infty}/\sqrt{4\pi\omega_{\infty}}$ is the Alfvén velocity of the interstellar wind). From the last equation we obtain the third boundary condition for Equation (2.17)

$$
\frac{r''(0)}{r_*} = -\frac{1}{2}(5M_A^2 - \frac{9}{2}) + \sqrt{\frac{1}{4}(5M_A^2 - \frac{9}{2})^2 - 2(1 - M_A^2)}.
$$
 (2.20)

The numerical solution of Equation (2.17) with the boundary conditions (2.19), (2.20) is shown in Figure 5 for $M_A > 1$. The values of $r = r(\theta)$ in Figure 5 are given in astronomical units (AU) for $\rho_{\infty} = 10^{-24}$ g cm⁻³, $V_{\infty} = 20$ km s⁻¹, $\rho_{\rm E} = 3 \times 10^{-24}$ g cm⁻³, $V_{\rm sw} = 400$ km s⁻¹ (Baranov and Krasnobaev, 1971).

Fig. 5. The shape and position of the heliopause in a Newtonian thin layer-approximation (Baranov *et al.,* 1970).

From Figure 5 we see that the heliocentric distance to the discontinuity surface, separating the undisturbed solar and interstellar winds, increases with increasing magnetic field (with decreasing M_A). Obviously, this effect results from the tension of the magnetic field lines because the vectors B_{∞} and V_{∞} are parallel in the case considered (if \mathbf{B}_{∞} is normal to \mathbf{V}_{∞} the boundary surface must approach the Sun due to the magnetic pressure). The dotted line in Figure 5 was calculated for $\mathbf{B}_{\infty} = 0$.

To justify the use of the hydrodynamic approximation for the solution of the problem Baranov *etal.* (1970) assumed that the interstellar wind consists of fully ionized hydrogen. However, after the discovering of neutral particles penetrating into the solar system from the very local interstellar medium it was necessary to improve the model considered above, in particular, to investigate the flow structure in the region between the shock waves BS and HS (the thin layer approximation simplifies the problem but makes it impossible to calculate this structure).

Wallis (1975) qualitatively showed, that the charge exchange effect of H-atoms with protons in the region between BS and CD (protons of the VLISM origin) can greatly influence the structure and the location of the interface between the solar and interstellar winds.

The following numerical results (Baranov *et al.,* 1979; Baranov and Ruderman, 1979) gave rise to the conclusion that the region between the shocks BS and HS (Figure 4) is not thin, i.e., it is not described on the basis of the Newtonian approximation, and, besides, it is a good 'filter' for the penetration of H-atoms from the very local interstellar medium into the solar wind.

A self-consistent problem of the solar wind's interaction with the supersonic interstellar wind taking into account the effect of resonance charge exchange was considered by Baranov *etal.* (1981, 1982). These investigations showed that it is necessary to correct the interpretation of the solar L_{α} -scattered radiation, which did not take into account the flow structure in the region between BS and HS.

3. Two-Shocks Gasdynamic Model (TSM)

The interaction model of the solar wind with the very local interstellar medium considered in Section 2.4 was based on the assumption that the hypersonic solar wind is damped mainly in the charged component (called below the plasma component) of the interstellar medium (electrons and protons). It is this assumption that makes it possible to describe the model using Euler's equations, firstly, due to the large Coulomb crosssections of charged particles; secondly, due to the possibility of their scattering by 'collective' processes in the plasma and, thirdly, because of the possible 'hydrodynamization' of the plasma by means of Larmor rotation of charged particles in a magnetic field.

However, solar $L\alpha$ -scattering experiments showed that there is a penetration of hydrogen atoms into the solar wind from the interstellar medium. The resonance charge exchange and photoionization processes give rise to a situation in which some of these atoms turn into protons. In this case the 'new' protons change the total plasma **compo-** nent's momentum and energy by means of their 'pick-up' process. In such a way these processes can act on the solar wind's deceleration, distorting its spherical symmetry (Grzedzielski and Ratkiewicz, 1975) as well as on the structure of the interface separating the solar wind and the plasma of the very local interstellar medium. Conversely, the solar wind's interaction with the plasma component of that medium can act on the H-atoms penetration from it into the solar system.

Wallis (1975), Baranov *et aL* (1979), Baranov and Ruderman (1979), Ripken and Fahr (1983), Fahr and Ripken (1984), Fahr *et al.* (1986a) took into account the last effect without considering the effects of neutral particles on the interaction of the plasma components, i.e., the theoretical models were not self-consistent.

For developing a self-consistent model of the flow, represented qualitatively in Figure 4 (the possible trajectory of the H-atoms penetration into the solar wind from the very local interstellar medium is illustrated with a dotted line), Baranov *et al.* (1981, 1982) assumed that this flow is stationary, has axial symmetry and is described by the gasdynamic equations without viscosity and heat conduction.

If we introduce a spherical coordinate system with its center at the Sun and the symmetry axis Oz in the direction opposite to that of the velocity vector $V_∞$ of the interstellar medium, the problem will be two-dimensional, all parameters depending on r and θ only (r is the distance from the Sun and θ is the polar angle). For the plasma component there are the momentum equations

$$
v_r \frac{\partial v_r}{\partial r} + v_\theta \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = v_c U_r ,
$$

\n
$$
v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + \frac{1}{\rho r} \frac{\partial p}{\partial \theta} = v_c U_\theta ,
$$
\n(3.1)

the continuity equation

$$
\frac{\partial \rho v_r r^2}{\partial r} + \frac{r}{\sin \theta} \frac{\partial \rho v_\theta \sin \theta}{\partial \theta} = 0 \tag{3.2}
$$

the energy equation

$$
\frac{1}{r^2} \frac{\partial}{\partial r} \left[\left(\frac{5kT}{m_p} + \frac{v_r^2 + v_\theta^2}{2} \right) \rho v_r r^2 \right] + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left[\left(\frac{5kT}{m_p} + \frac{v_r^2 + v_\theta^2}{2} \right) \rho v_\theta \sin \theta \right] = \rho v_c (U_r v_r + U_\theta v_\theta) + \rho v_c \left(\frac{v_r^2 + v_\theta^2}{2} + \frac{3kT_H}{m_H} - \frac{3kT}{m_p} \right)
$$
\n(3.3)

and the equation of state

$$
p = \frac{2\rho k}{m_p},\tag{3.4}
$$

where v_r , v_θ are the plasma velocity components along the radius-vector and polar angle respectively, T is the temperature, p the pressure, ρ the mass density, k is Boltzmann's constant.

The fight-hand sides in Equations (3.1) and (3.3) describe the exchange of momentum and energy between protons and H-atoms due to collisions accompanied by charge exchange. For their mathematical describing (Holzer, 1972, Grzedzielski and Ratkiewicz, 1975) we use the following formulas:

$$
v_c = n_{\rm H} \sigma U_{*}, \qquad U_{*} = \left[U_r^2 + U_\theta^2 + \frac{128k(T + T_{\rm H})}{9\pi m_{\rm H}} \right]^{1/2}, \tag{3.5}
$$

$$
U_r = -(v_r - v_{rH}), \qquad U_\theta = -(v_\theta - v_{\theta H}),
$$

where σ is the effective charge exchange cross-section (\approx 2 to 6 \times 10⁻¹⁵ cm²) is, in general, a function of the relative velocity, the index 'H' relates to the H-atoms, n_H is the number density, T_H the temperature, v_c the frequency of collisions for the resonance charge exchange processes.

Let us assume now that the H-atoms do not change their velocity (V_{∞}) and temperature (T_{∞}) , which are equal to those of the very local interstellar medium plasma component. In addition, we ignore the secondary H-atoms produced by charge exchange (the process of photoionization is not taken into account).

Then the motion of the hydrogen atoms in the region between BS and the Earth's orbit is described by the equations

$$
v_{rH} = -V_{\infty} \cos \theta, \qquad v_{\theta H} = V_{\infty} \sin \theta, \qquad T_H = T_{\infty},
$$

$$
V_{\infty} = \text{const.}, \qquad T_{\infty} = \text{const.},
$$

(3.6)

$$
\frac{1}{r^2}\frac{\partial}{\partial r}(m_{\rm H}n_{\rm H}v_{r{\rm H}}r^2)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(m_{\rm H}n_{\rm H}v_{\theta{\rm H}}\sin\theta)=-\rho v_c.
$$

The boundary conditions of the problem are the Rankine-Hugoniot relations on the shock waves BS and HS, the condition of equality of the pressures and the no-flow condition for the plasma component (a vanishing normal component of the plasma velocity) on the contact discontinuity CD. Besides, it is necessary to apply the symmetry conditions on the stagnation line $\theta = 0$ and to assume the spherically-symmetric values of the solar wind parameters at $r = r_{\rm E}$. These assumptions are convenient since wellknown experimental data on the parameters of the solar wind are available in the region of the Earth's orbit. It should be noted, that the solar wind has axial but no spherical symmetry at $r > r_E$ due to resonance charge exchange processes. We must also apply the condition $n_{\text{H}} = n_{\text{H}\infty}$ on the bow shock BS.

Five dimensionless parameters determine the problem formulated above (the specific heat ratio is $\gamma = \frac{5}{3}$ for the plasma component)

$$
M_{\infty} = \frac{V_{\infty}}{a_{\infty}}, \qquad K = \frac{\rho_{\rm E} V_{\rm SW}^2}{\rho_{\infty} V_{\infty}^2} = \frac{n_{\rm E} V_{\rm SW}^2}{n_{e\infty} V_{\infty}^2} \quad (n_{e\infty} \approx n_{p\infty}),
$$

$$
q = \frac{\rho_{\rm H\infty}}{a} = \frac{n_{\rm H\infty}}{n}, \qquad \varphi = \sigma r_{\rm E} n_{e\infty} \sqrt{K}, \qquad \chi = \frac{T_0}{T},
$$
 (3.7)

where φ characterizes the resonance charge exchange process, χ is the ratio of the stagnation temperatures for the solar wind and the VLISM, M_{∞} is the interstellar wind Mach number, K is the ratio of the solar wind kinetic energy at $r = r_E$ and the kinetic energy of the VLISM. The parameter q characterizes the degree of ionization of the VLISM.

This self-consistent problem was solved by Baranov *et al.* (1981, 1982). Calculations showed a very weak dependence on the parameter χ , hence, we can neglect it. In addition, all characteristic lenghts, for example, the heliocentric distances to BS, CD, and HS turn out to be similar in terms of \sqrt{K} . This result significantly extends the region of application of the problem. Figures 6 and 7 demonstrate the shapes and the positions of the bow shock BS, the contact discontinuity CD and the heliospheric shock HS as a function of the Mach number M_{\odot} and the parameter q for $\varphi = 0.8$ (the linear dimensions in these figures are divided by $r_{\rm E} \sqrt{K}$).

It can be seen from the data in Figure 6 that a change in the Mach number of the interstellar wind affects only the size of the region between the bow shock BS and the contact discontinuity CD and has hardly any influence on the position and size of the region between CD and the heliospheric shock HS in the solar wind. As can be seen from Figure 7, the region between BS and HS moves strongly toward the Sun with increasing parameter q. For $q \ge 10$ the heliospheric shock HS can approach the orbits of the planets. For $q = 0$ these results coincide with those of a paper by Baranov *et al.* (1979).

Calculations also showed that the charge exchange in the region between BS and CD contributes basically to the obtained results. That effect was qualitatively predicted by Wallis (1975).

In Figure 8 we have plotted the degree of 'survival' (or 'extinction') of the H-atoms as a function of the dimensionless parameter $\varphi(M_\infty = 2, q = 1; 10)$ on the axis of symmetry ($\theta = 0$). This value is determined by the formula

$$
N = \frac{n_{\text{H}\infty} - n_{\text{HHS}}}{n_{\text{H}\infty}} \tag{3.8}
$$

where n_{HHS} is the number density of H-atoms at the heliospheric shock (HS). Calculations showed the approximate equality $n_{\text{HHS}} \approx n_{\text{HCD}}$, where n_{HCD} is the number density of H-atoms at the contact discontinuity CD, i.e., the region between BS and CD

Fig. 6. Effect of the Mach number M_{∞} on the position and shape of the discontinuity surfaces.

is the main 'filter' for the penetration of H-atoms into the solar wind from the VLISM. The lines n_H/n_{exc} = const. in the solar wind and the intensity of the scattered solar La-radiation along the axis of symmetry ($\theta = 0$) are given in Figure 9 with the following values of the parameters: $n_{\text{e}\infty} = 0.04 \text{ cm}^{-3}$, $M_{\infty} = 2$, $V_{\infty} = 20 \text{ km s}^{-1}$, $n_{\text{E}} = 5 \text{ cm}^{-3}$, V_{sw} = 400 km s⁻¹, σ = 6 \times 10⁻¹⁵ cm². In Figure 9(b) (Ermakov, 1983) the solid and dotted lines give the intensity of the L α -scattered radiation as a function of $n_{H_{\infty}}$ obtained on the basis of the results of Figure 9(a), and by using the exponential formula (Fahr, 1974)

$$
n_{\rm H} = n_{\rm H\infty} \exp\left(-\frac{r_{\rm E}^2 \sigma n_{\rm E} V_{\rm SW}}{rV_{\infty}}\right).
$$

Fig. 7. Effect of the degree of ionization of the VLISM on the position and shape of the discontinuity surfaces.

It is seen that the interpretation of the H-atoms number density in the very local interstellar medium on the basis of $L\alpha$ -scattering experiments depends significantly on the theoretical model (see also, Fahr and Ripken, 1984).

In Figures 10 and 11 we have plotted the distributions of the velocity (absolute value) and pressure along the shock waves (BS and HS) and the contact discontinuity CD (as functions of the polar angle θ) for $M_{\infty} = 2$ and $\varphi = 0.8$. In Figure 10 the velocity is divided by V_{∞} in the region between BS and CD and by V_{SW} in the region between CD and HS; the symbols CD_1 and CD_{11} refer to the outer and inner sides of the contact discontinuity CD. The pressure in Figure 11 is divided by $\rho_{\infty} V_{\infty}^2$.

As can be seen from Figure 11, the pressure at the bow shock depends weakly on the number density $n_{\text{H}\infty}$ of the H-atoms in the very local interstellar medium (the pressure behind the bow shock at its tip depends only on the Mach number M_{∞} , and the

Fig. 8. The rate of H-atoms 'survival' (or 'extinction') as a function of the parameter φ .

dependence of the pressure at the bow shock for $\theta > 0$ on the parameter q is due to changes in the shape of this shock due to changes of q). At the same time the pressure changes appreciably in the region between BS and CD and, in particular, on the contact discontinuity, as a result of changes in q . Because of this, appreciable pressure gradients arise in the direction across the layer of the compressed interstellar wind in the case of a gas flow with a high number density of neutral particles.

In the region between CD and HS, which contains gas from the solar wind, a change in q leads to a uniform change of the pressure in the complete shock layer. The calculations show that the temperature distributions in the shock layers between BS and HS behave similarly. The calculations also show that the velocity of the supersonic solar wind is decreasing and the temperature is increasing before the heliospheric shock (HS) due to the resonance charge exchange processes. However, the deflection of the solar wind flow from spherical symmetry is not significant.

4. The Limitations of the Gasdynamicai Models

The most important limitations of the two-shocks model (TSM), considered in Section 3, are connected with the assumptions introduced by Equations (3.5) and (3.6) for hydrogen atoms. In particular, it is assumed that H-atoms, moving from the VLISM with velocity V_{∞} , disapper through their charge exchange with protons. However, a hydrogen atom, 'born' in this process, moves with the velocity of the 'killed' proton. These 'fresh' H-atoms are not taken into account in the TSM. The 'born' H-atom can again be effected by charge exchange and so on (see, for example, Burgin, 1983). The effects of multiple resonance charge exchange can give rise to violation of the assump-

Fig. 10. **The absolute value of the velocity along the shocks BS** and HS and the contact discontinuity CD.

Fig. 11. The distribution of pressure along the shocks BS and HS and the contact discontinuity CD.

tions introduced in deriving the expressions (3.5) , for example, the assumption on the Maxwell distribution function of the hydrogen atoms (Holzer and Banks, 1969; Holzer, 1972).

In Figure 12 we have plotted the right-hand side of the continuity equation (3.6) calculated by Monte-Carlo method without assuming a Maxwellian distribution function for the H-atoms (Malama, 1987) as a function of the distance from the Sun.

Fig. 12. The right-hand side of Equation (3.6), calculated on the basis of formulas (3.5) (solid curve) and on the basis of the Monte-Carlo method for the case of the spherically-symmetric solar wind model (Malama, 1987).

In do doing it was supposed that the solar wind parameters are known from the hypersonic one-dimensional solution $(v_r = V_{sw} = \text{const.}, v_\theta = 0, \rho = \rho_E r_E^2/r^2)$. For comparison we show solid lines calculated by Equations (3.5). We see from Figure 12 that the processes of multiple charge exchange is especially significant for large polar angles (the results of the two shocks model, considered in Section 3, are given only for $\theta \le 90^{\circ}$) and at heliocentric distances $r \le 10$ AU, i.e., in the region where the solar $L\alpha$ -scattered radiation is formed.

Another effect which can change our ideas about the structure of the interface separating the solar wind and the interstellar medium, is that connected with the instability of the contact discontinuity CD.

The number density of the compressed solar wind (in the region between HS and CD) is much less than the number density of the compressed interstellar wind (in the region between BS and CD) if the heliocentric distance of the contact discontinuity is $r \gtrsim 50$ AU. Therefore, at low polar angles θ (in the vicinity of the stagnation point, where the velocities tangential to CD are small) a Rayleigh-Taylor instability of the contact discontinuity may develop. For large polar angles θ the contact discontinuity is transformed into a classical tangential discontinuity. As we see from Figure 10, the difference of the velocities at the contact discontinuity for $V_{\infty} = 20$ km s⁻¹, $V_{\text{SW}} = 400$ km s⁻¹ and $\theta = 30^{\circ}$ is equal to \sim 70 km s⁻¹. In this case a Kelvin-Helmholtz instability may develop. A qualitative analysis of these effects was made in the review paper of Fahr *et al.* (1986).

However, it is to be noted that the contact discontinuity has in practice a finite thickness which may be due to viscosity, thermal conductivity, current layers which are forming in the presence of magnetic fields, and so on. Besides the form of the flow around the contact surface CD does not satisfy the classical conditions, which are required for the formation of Rayleigh-Taylor and Kelvin-Helmholtz instabilities.

Hence, it appears that the problems of the stability and structure of the contact discontinuity are not solved.

The two-shocks model considered in Section 3, does not take into account the effects of the interstellar magnetic field and the galactic cosmic rays in spite of the fact, according to modern estimates, their pressures may be comparable to the static and dynamic pressures of the interstellar wind (Afford and Ip, 1986). In particular, the interstellar magnetic field can give rise to a three-dimensional picture of the interstellar gas flow around the solar wind. Until now there are no adequate gasdynamical models in which all these effects are taken into account.

Fahr *et al.* (1986b) started an attempt to estimate the magnetic field effect on the form of the heliosphere (the region bounded by the heliospheric shock HS). But their thin-layer approximation, as is seen from the results of Section 3, is not realistic for the problem considered, although it can give us a qualitative picture of the possible heliospheric deformations due to the effects of the magnetic fields.

It is necessary to draw attention to the limitations connected with the description of the considered physical phenomenon on the basis of the hydrodynamical equations. A rigorous substantion of these equations is based on the assumption that the mean free path of the particles *l* is small as compared with the characteristic length $L(l \ll L)$ of the physical phenomenon. The coulomb free path of the charged particles is about $l \sim 1$ AU if (for example) $T_{\infty} \sim 10^4$ K and $n_{\text{e}\infty} \sim 0.04$ cm⁻³. This value is much less than the characteristic length $L \sim 100 \text{ AU}$ of a contact discontinuity which is an 'obstacle' for the interstellar wind. But the solar wind flow around the same 'obstacle' with $T_{SW} \sim 10^5$ K and $n_{SW} \sim 10^{-3}$ cm⁻³ (for $r \sim 50$ AU) gives rise to the reverse inequality $l \ge L$, i.e., the continuity condition, rigorously speaking, is violated.

Nevertheless, it should be noted that when two plasma flows penetrate into each other a beam instability can arise. The scattering of charged particles on generated fluctuations represente an effective mechanism of collisions excluding the existence of multiple-speed streams of the ionized gas and the interpenetration of single-speed streams into each other. In this case the scattering by such 'collective' processes in a plasma is much more effective than Coulomb scattering.

The magnetic field can also convert the plasma component into a hydrodynamical system, if the Larmor radius is small as compared with L.

Thus we can hope that the use of the hydrodynamic equations for describing the phenomena considered is correct. In this case the bow shock BS can be considered as a classical (collisional) shock while the heliospheric shock HS is a collisionless shock.

Here it is be noted that, as we seen from the results of Section 3, the solar wind's deflection from spherical symmetry due to the effect of charge exchange of H-atoms does not change the axial symmetry of the problem. However, the latitudinal dependence of the outflow of the solar wind from the solar corona makes necessary the construction of a three-dimensional model. Such a model has not yet been developed. Qualitatively, the effect of the latitude dependence of the solar wind on the shape of the heliospheric shock was investigated by Suess *et aL* (1987).

We should like to draw attention to another more theoretical problem connected with the formation of the heliospheric shock. Wallis (1971) investigated the deceleration of the solar wind due to resonance charge exchange of the hydrogen atoms from the very local interstellar medium using a spherically-symmetric model. It was shown that it is theoretically possible for a transition of the solar wind from supersonic to subsonic flow without a heliospheric shock originating. The same problem was also investigated by Holzer (1972), who took into account the processes of photoionization.

One of the main assumptions made in the quoted papers is that about the instantaneous pick-up of protons, produced by resonance charge exchange and photoionization processes by the solar wind. Later, these papers were generalized by Isenberg (1986), who took into account the possibility of multiple-speed streams in the solar wind.

From a mathematical point of view the problem of a smooth deceleration of the solar wind to subsonic velocities is reduced, for the one-dimensional model, to two problems. It is necessary, first, to investigate the kind of singular points, in which the Mach number of the solar wind equals unity, and, secondly, the gas flow stability in the vicinity of these singularities (Kulikovsky and Slobodkina, 1967, 1982).

These problems were investigated by Baranov and Ivlev (1989) on the basis of equations suggested by Holzer (1972) and Isenberg (1986). It was shown that there is a theoretical (but unlikely) possibility of the solar wind being decelerated without the formation of a heliospheric shock.

Scattering experiments of the solar radiation showed us that the very local interstellar medium (VLISM) is moving relative to the Sun with a velocity $V_{\infty} \approx 20$ to 30 km s⁻¹. Therefore the model of the solar wind's outflow into the interstellar gas at rest (Section 2.1) is not real. We think that Parker's model of the solar wind's interaction with the subsonic interstellar wind (Section 2.2) is also not real, because its basic assumption is the condition (2.7). For a VLISM temperature $T_{\infty} \approx 10^4$ K, determined also by the solar radiation scattering experiments, and $V_{\infty} \approx 20$ to 30 km s⁻¹ a reverse of the inequality to (2.7) takes place, i.e., $\rho_{\infty} V_{\infty}^2 > 2p_{\infty}$. The application of Parker's

model (Section 2.2) for the interpretation of the penetration of H-atoms into the solar wind (Ripken and Fahr, 1983) is often founded on the assumption that the velocity of the interstellar wind is less than the interstellar magnetosound or Alfvén velocity (for example, $V_{\infty} \le a_{A\infty} = B_{\infty}/\sqrt{4\pi n_{e\infty}m_p}$. But in this case $\rho_{\infty}V_{\infty}^2 \lesssim B_{\infty}^2/4\pi$ and it is necessary to use a magnetohydrodynamic model rather than a gasdynamic one. The magnetohydrodynamic model of the solar wind's interaction with the interstellar medium has not yet been developed.

At present, as it seems to us, the two-shocks model (TSM) considered in Section 3, describes most adequately the real physical phenomenon. In particular, only the TSM by Baranov *et al.* (1981, 1982) takes into account the effect of the hydrogen atoms on the motion of the plasma component. Certainly, this model must be generalized to take into account the interstellar magnetic field, the cosmic rays of the Galaxy, multiple charge exchange processes, the deflection of the solar wind from one-dimensional flow, and so on.

Everywhere below we shall use the results of the TSM for the interpretation of experimental data.

5. Theory and Observations

As was mentioned above, scattering experiments of the solar radiation show (see, for instance, the reviews of Kurt, 1981; Burgin, 1981) that atoms of helium and hydrogen are moving with a supersonic velocity $V_{\infty} \ge 20$ km s⁻¹ relative to the Sun. The vector of this velocity, in this case, is almost in the ecliptic plane and does not coincide with the direction of the solar motion with respect to the nearest stars, as it was assumed in the first paper of Baranov *et al.* (1970). It is clear that this fact is explained by the own motion of the very local interstellar medium (VLISM).

Table I, adopted from a paper of Ripken and Fahr (1983), presents the range within which the parameters of the atoms of H and He may vary in the VLISM. This range was determined from experimental data on scattering of solar radiation (average values are given in brackets) and adopted at a Workshop on the problem of the interstellar gas in interplanetary space (Lindau, F.R.G., June 18-20, 1980).

Here it is to be noted that, as was shown in Section 3 (see also Figure 8), the region between BS and CD is the specific 'filter' via which part of the H-atoms, moving from

the VLISM with velocity V_{∞} , is absorbed through resonance charge exchange on the shocked interstellar wind protons. Therefore, it is necessary to relate the values of the parameters of the H-atoms, showed in the table, to heliopause (contact discontinuity) rather than to the VLISM (the absorption of these atoms in the region between CD and HS, i.e., the absorption due to resonance charge exchange on the shocked solar wind protons is not significant). It is clear that the number density of the hydrogen atoms in the VLISM may be higher than the number density given in the Table. The values $n_{\text{H}\,\infty} = 0.2$ to 0.6 cm⁻³ are not in contradiction with the data on absorption of L α -emission from the nearest stars (see the review of Blum and Fahr, 1976).

The effect of a plasma 'filter' is not often taken into account in the interpretation of the scattering experiments of solar $L\alpha$ (see, for instance, the review of Bertaux, 1984).

At present we have no reliable data for the parameters of the VLISM's plasma component (e.g., protons and electrons). The mean number density of electrons in the interstellar medium $(n_{e\infty} = 0.04 \text{ cm}^{-3})$ was determined using measurements of dispersion of pulsar signals. However, this value is the result of averaging over large distances (of the order of hundred parsecs and more) and over a great number of measurements (see Manchester and Taylor, 1977). The analysis of these data gives rise to the conclusion that the number density of electron in the VLISM could be either much higher or much less than the mean value given above. There is a large spread of the data for different pulsars. For example, for the pulsar PSR 1642 - 03 we have $n_{em} = 0.21$ to 0.25 cm^{-3} (Manchester and Taylor, 1977).

The values $n_{e\infty} = 0.1$ to 0.3 cm⁻³ are obtained on the basis of analysis of experimental data from the Copernicus satellite (Grewing, 1975). Reynolds (1984) and Cox and Reynolds (1987) gave the same value of the electron number density in the LISM when they analyzed optical emission lines. However, the investigations of emission and absorption in the UV spectral region (Paresce, 1984) gave rise to values of the electron number density that are less by two and more orders of magnitude. Thus the electron number density in the interstellar medium (especially in the LISM and VLISM) is a parameter, which is measured very poorly (with an accuracy of one or more orders of magnitude). We show below that the TSM may be used as an indirect method for determining the electron number density in the VLISM (for example, through the position of the heliospheric shock). In this case a theoretical model of the solar wind's interaction with the VLISM must not contradict, first, the solar radiation scattering data (see Table I) and, secondly, the results of the electron number density measurements within the limits mentioned above.

We also use the following fact. Recently data about high-frequency signals (\sim 3 kHz), detected on board the Voyager 1 and 2 have been described. The authors of these publications (Kurth *et al.,* 1984; Suess and Dessler, 1985) suggested that the heliospheric shock HS is the source of these signals (due to the collisionless character of this shock) at the double plasma frequency $2\omega_{pe}$. They estimated the heliocentric distance of the shock as being equal to 30 to 50 AU, because of the relations

$$
\omega_{pe} \sim \sqrt{n_e}
$$
 and $n_e \sim 1/r^2$.

To explain such a comparatively small size of the heliosphere (the supersonic solar wind) Suess and Dessler (1985) used a semi-empirical (rather than classical) solar wind theory and, as a mechanism for solar wind deceleration, either the interstellar magnetic field or galactic cosmic rays. Their estimated value of the interstellar magnetic field for this purpose is twice the value commonly used for the LISM. Obviously, in this case, the magnetic field's value in the LISM turns out to be underestimated since its direction was taken perpendicular to the radial direction (in reality the LISM's magnetic field direction is unknown). Also the mechanism of the deceleration of the solar wind due to cosmic rays is still unclear. Axford and Ip (1986), using empirical relations, also concluded that it is impossible to explain the small radius of the heliosphere ($\approx 50 \text{ AU}$) either by the interstellar magnetic field or by cosmic rays only. Neither Suess and Dessler (1985) nor Axford and Ip (1986) did take into account the effect of the plasma 'filter' considered in Section 3.

An attempt to correlate the heliospheric shock position (\approx 50 AU) obtained by the Voyager 1 and 2 experiments with the data on scattering solar $L\alpha$ -radiation and the results of electron number density measurements in the LISM was made by Baranov (1986a, b) on the basis of the TSM considered in Section 3. In this case, the interstellar magnetic field and the galactic cosmic rays are not taken into account. Figure 13 shows the calculated positions of the heliospheric shock HS on the axis of symmetry as a function of the parameter $q = n_{H\infty}/n_{e\infty}$ characterizing the degree of ionization of the VLISM. These results are almost independent of the parameters φ and M_{∞} (see

Fig. 13. The position of the heliospheric shock (HS) along the axis of symmetry as a function of the degree of ionozation of the VLISM.

Equations (3.7)) in the real range of their variation (see, also Baranov *et aL,* 1982)

$$
0.4 \le \varphi \le 2.7; \qquad 1.1 \le M_{\infty} \le 4. \tag{5.1}
$$

However, as it is seen from Figure 8, the parameter φ acts on the 'survival' (or 'extinction') of hydrogen atoms (3.8) penetrating into the solar wind with the velocity V_{∞} . But it is to be noted here that there is not strong dependence of this 'survival' number N on the polar angle θ . For example, $N = 0.8$ on the axis of symmetry with $\varphi = 0.8$ and $q = 1$ (see Figure 8) and $N = 0.9$ with $\theta = 90^\circ$. For $q = 10$ we have $N = 0.45$ and $N = 0.6$ with $\theta = 0$ and $\theta = 90^{\circ}$, respectively.

Thus the heliospheric shock position r_{HS} along the axis of symmetry only depends on two parameters, K and q if the inequalities (5.1) are satisfied. In this case the 'survival' number N of hydrogen atoms depends strongly on the parameters φ and q , and the dependence of N on the polar angle θ is not strong.

A significant result, that will be used below is the similarity with respect to the parameter \sqrt{K} in the position and shape of the discontinuity surfaces BS, CD, and HS (see Section 3 and Figure 13). For given values of the dimension parameters $n_{\rm E}$, $V_{\rm sw}$, V_{∞} , and σ the dependence $r_{\rm HS}$ and N on the dimensionless parameters q, φ , and K is reduced to that on the dimension parameters $n_{e\infty}$ and $n_{H\infty}$. If we take for the solar wind at the Earth's orbit $n_E = 5 \text{ cm}^{-3}$, $V_{SW} = 400 \text{ km s}^{-1}$ and for the VLISM $V_{\infty} = 25$ km s⁻¹ we get (with $\sigma = 6 \times 10^{-15}$ cm⁻²)

$$
\sqrt{K} = \frac{35.2}{\sqrt{n_{e\infty}}}, \qquad \varphi = 3\sqrt{n_{e\infty}} \tag{5.2}
$$

Now it is easy matter to obtain the heliospheric shock position r_{H_S} and the number density of the H-atoms n_{HHS} , that penetrated through the 'filter'. We give them along the axis of symmetry ($\theta = 0$) as functions of the electron number density n_{em} and the number density $n_{H_{\infty}}$ of the VLISM hydrogen. To this end the results shown in Figures 8 and 13 must be used together with the relations (5.2). Altematively, we can determine the parameters $n_{e\infty}$ and $n_{H\infty}$ using the results of the heliospheric shock position measurements and the value of n_{HHS} , which is obtained on the basis of the L α -scattering experiments of solar radiation (see Table I).

Figure 14 presents the results of these calculations for determining r_{HS} . In particular, it is evident that for $n_{H\infty} = 0.3$ cm⁻³ we have $r_{HS} = 45$ AU if $n_{e\infty} = 0.3$ cm⁻³ (q = 1). Such a value of r_{HS} does not contradict the Voyager 1 and 2 data (Kurth *et al.*, 1984). In this case the value $N(\varphi = 1.62$ for $n_{e\infty} = 0.3$ cm⁻³, as it is seen from (5.2)), determined from Figure 8, leads to $n_{\text{HHS}} = 0.018 \text{ cm}^{-3}$. The number density of the H-atoms penetrating into the solar system turns out to be approximately one half of that obtained from data about the scattered solar $L\alpha$ -radiation (see Table I). However, this fact may be explained by the limitations of our theoretical model which does not take into account the production of H-atoms due to secondary and subsequent charge exchanges (see Section 4). This effect, as Fahr and Ripken (1984) have shown, can considerably increase the transparency of the 'filter' for H-atoms which is created by the gasdynamic interface between the solar wind and the VLISM. Concerning the rather large values

Fig. 14. The position of the heliospheric shock on the axis of symmetry $(r_{\rm HS})$ as a function of the electron number density in the VLISM.

of $n_{e\infty}$ as compared with the mean value ($n_{e\infty} = 0.04$ cm⁻³), obtained by correlating the Voyager 1 and 2 data with those of the theoretical model, they appear not to be in contradiction with the data of measurements as indicated at the beginning of this Section.

Using the weak dependence of $r_{\rm HS}$ on $n_{\rm ex}$ for $n_{\rm H\infty} \ge 0.3$ cm⁻³ (Figure 14) let us determine from Figure 13 the value $n_{\text{H}\infty}$ which is required to have $r_{\text{HS}} = 50 \text{ AU}$ with the mean value of electron number density $n_{e\infty} = 0.04$ cm⁻³. In this case we have from (5.2) $K = 176$, $\varphi = 0.6$. Then Figure 13 gives us $r_{\text{HS}} = 50 \text{ AU}$ for $q = 10$, i.e., $n_{H\infty} = 0.4$ cm⁻³, if $n_{e\infty} = 0.04$ cm⁻³. But from Figure 8 we see that the number density of the H-atoms penetrated into the solar system is too high ($n_{\text{HHS}} = 0.24 \text{ cm}^{-3}$) in this case (see Table I). This contradiction with the data of the solar $L\alpha$ -scattering experiments can not be explained neither by the limitations of the TSM considered in Section 3, nor by any another reasons.

Here it is to be noted that, as is seen from Table I, the ratio of mean number densities of helium and hydrogen is $n_{He}/n_{H} \approx 0.25$. This value, obtained in experiments on the scattering of the solar radiation at wavelengths λ 584 Å and λ 1216 Å, contradicts the helium cosmic abundance $(n_{\text{He}\infty}/n_{\text{H}\infty} \approx 0.1)$. The contradiction may easily be explained on the basis of the TSM.

Indeed (see Grzedzielski, 1983), the charge exchange cross-section of He-atoms on

protons is small as compared with the resonance charge exchange cross-section of H-atoms. Therefore the interface between the solar wind and the VLISM is a bad 'filter' for helium atoms penetrating into the solar wind, while the hydrogen atoms are absorbed in the region between BS and CD.

As was noted in Section 2.4 the heliospheric shock in the wake of heliosphere must be closed in an oval. The heliospheric shock distance to the Sun in the wake (downstream direction) is much more than that at small angles θ (upstream direction). To estimate the first distance we can use the relation (2.5), because $V_{SW} \geq V_{\infty}$. For $V_{\text{sw}} = 400 \text{ km s}^{-1}$, $n_E = 5 \text{ cm}^{-3}$, $\gamma = \frac{5}{3}$ and $p_{\infty} = 10^{-12} \text{ dyn cm}^{-2}$ we have $r_{\text{HS}} = 120 \text{ AU}$ in the downstream direction. This result may explain why the Pioneer-10 spacecraft did not observe the heliospheric shock.

If the TSM is real for the description of the solar wind interaction with the VLISM we can predict physical phenomena, which can be observed on the basis of directed relevant experiments (Baranov, 1981). One possible test for observing the relative motion of the VLISM plasma component is based on observations of the anisotropic scintillation of radiosources. This anisotropy is associated with the interstellar wind inhomogeneities flowing into the region between BS and CD, and should be observed relative to the direction of motion of the interstellar medium.

One other test for determining the character of the interaction between the solar wind and the interstellar medium is associated with the possible anisotropy of cosmic rays due to asymmetry of the heliosphere. In particular, investigations of the modulation of galactic cosmic rays in the heliospheric shock (see, for example, Chalov, 1987a, b) showed that this modulation is different in the upstream and downstream region of the heliosphere. For example, the cosmic-rays spectra obtained on the Voyager 1 and 2 and Pioneer 10 spacecraft must be different. At present one can only estimate the size of the region of modulation of the cosmic rays. This estimate, obtained on the basis of the Voyager 1 and 2 and Pioneer 10 experiments, gives rise to values from 50 to 100 AU (Webber and Lockwood, 1987; Mckibben, 1987). The size of the modulation region depends on the level of solar activity and is also badly influenced by our poor knowledge of the cosmic-rays spectrum in interstellar medium. The dimensions of the modulation region are obtained by extrapolation of the radial gradient of the cosmic-rays intensity and they do not contradict the results of the TSM considered in Section 3. We hope to investigate the galactic cosmic-rays modulation on the basis of the TSM.

6. Conclusions

(1) An analysis of the data on solar scattered radiation at wavelenghts λ 584 Å (for He-atoms) and λ 1216 Å (for H-atoms) shows that the very local interstellar medium is moving relative to the solar system with a supersonic velocity. Therefore, it is necessary to use for the interpretation of experimental data a model for the interaction of a supersonic solar wind with a supersonic interstellar wind. The only model of that kind existing at present is the two-shocks model (TSM) considered in Section 3. But this model does not consider magnetic field. The value and the direction of the magnetic fields in the very local interstellar medium (VLISM) are not well known. It is necessary to use a magnetohydrodynamic model if the inequality $\rho_{\infty} V_{\infty}^2 \leq B_{\infty}^2/4\pi$ is satisfied for the VLISM. At present such a model has not yet been developed.

(2) For a correct interpretation of the data on scattered solar $L\alpha$ -radiation it is necessary to take into account the interaction of H-atoms with the plasma component due to the process of resonance charge exchange. However, the discussion of this interaction on the basis of the TSM is not get complete. Indeed, the TSM, considered in Section 3, does not take into account the effects of multiple resonance charge exchange. This effect can give rise to a violation of the assumptions used in the derivation of the expressions (3.5) (for example, the assumption of a Maxwellian distribution function of the hydrogen atoms).

(3) If the T SM is correct for the description of the interaction of the solar wind with the VLISM, there is a possibility to predict certain physical phenomena. In particular, special experiments or an accurate treatment of available data may enable one, to discover an anisotropy of radio-scintillations or of galactic cosmic-rays spectra. This anisotropy is connected with the asymmetry of the interface separating the solar wind and the VLISM.

(4) The estimate of the electron number density $n_{e\infty} = 0.3$ cm⁻³ in the VLISM obtained from the TSM on the basis of the position of heliospheric shock derived from recent Voyager 1 and 2 data (Kurth *et al.,* 1984; Suess and Dessler, 1985) does not contradict existing data. The value of the number density of the hydrogen atoms penetrating the solar wind from the VLISM $n_{\text{HHS}} = 0.018 \text{ cm}^{-3}$ is lower than that estimated from the scattered solar L_x-radiation. This difference results from the limitations inherent to the TSM, which does not take into account multiple charge exchange.

 (5) In order to describe the interaction between the VLISM and the solar wind in a more complete and adequate way, further development of the TSM is needed, it should include the effects of the interstellar magnetic field, cosmic rays, etc.

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