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Brittle Fracture in Compression

B. COTTERELL

Department of Mechanical Engineering, The University of Sydney, Sydney, N.S.W.

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ABSTRACT

The apparent paradox of two theories of fracture depending on whether the applied load is tensile or compressive is resolved. Although in compression, fracture seems to occur when the maximum tensile stress around a hole reaches a critical value it is suggested that fracture occurs when the crack extension force at the tip of a microcrack in the neighbourhood of the crack tip reaches a critical value. An essential difference in behaviour under tension and compression is that whereas in tension the crack extension force increases with crack growth, in compression a maximum value of the crack extension force is reached with further crack growth causing a decrease in its value. If the defects, which must exist at the edge of a machined hole, are small, then the crack extension force is controlled by the maximum tensile stress at the surface of the hole. The degree of smallness is different for tension and compression. In tension, the defects must be less than one fifth of the root radius of the tip of the hole and if the hole is sharp enough, the defects will be larger than this value and the crack extension force will be given by the usual fracture mechanics expression $\mathscr{G}=\sigma^2\pi c/E$. In compression the defects must, in the limiting case, be greater than the root radius of the tip of the hole (in a more typical case, greater than twice the root radius) if the crack extension force is not to be controlled by the maximum tensile force. Such large defects are impossible since the sharpness of the root radius is limited by the defect size and thus in compression, fracture from machined notches will always be controlled by the maximum tensile stress.

1. Introduction

The original Griffith theory [1, 2] is based on the hypothesis that fracture occurs when the potential energy released by a crack exceeds the work required for the formation of the new surfaces. In the fundamental problem studied by Griffith—a plate loaded in tension that contains a crack normal to the applied load—this theory predicts that the fracture strength is inversely proportional to the square root of the crack length for an infinitely sharp crack, assuming that homogeneous linear elasticity holds even at the tip of the crack. This strength dependence has been confirmed for a wide range of materials containing sharp notches or cracks, but the physical interpretation in terms of an energy balance has never been adequately demonstrated.* As Griffith himself [1, 2] showed, the same dependence of strength on crack length can be obtained from the hypothesis that fracture occurs when the local tensile stress exceeds the cohesive strength of the material, if it is assumed that the root radius of the crack is small and independent of the crack length. There is a fundamental difficulty in applying this hypothesis in that fracture depends on the value of the stress at a point, which is influenced by the exact shape of the tip of the crack or notch and the micro-structure of the material. In many cases, fracture is initiated when the stress over a certain process zone at the tip of the crack reaches a critical level, rather than its value at one particular point. Even if, in a very brittle substance like glass, this actual process zone is very small, it is not much use in using a criterion based on stress at a point, since no two specimens are exactly alike. Thus the probability of the stress reaching a certain critical value will depend on the level of stress in the immediate neigh-

* In his first paper, Griffith [1] presented tests on glass to support his theory. He argued that it was necessary to anneal his specimens (causing the fracture strength to increase) in order to remove residual stresses and got very good agreement between the measured fracture strengths and those predicted from the surface energy measurements. Unfortunately, as he pointed out in his second paper [2], there was an error in his theory which reduced the theoretical fracture strength by 44%. He then had to argue that the annealing treatment had blunted the cracks in the glass and so had artificially increased the observed fracture strength. By decreasing the annealing time he obtained fracture strengths that agreed with the corrected theory.

bourhood. Irwin [3] has shown that the stress in the neighbourhood of the crack tip can be expressed in terms of a stress intensity factor K, which sets the boundary conditions on an interior region very near the crack tip. The stress intensity factor K can be related to the energy released when the crack grows infinitesimally (crack extension force $\mathscr{G}$). A fracture criterion that is not greatly affected by the exact shape of the crack root, is more attractive than one that is based on the maximum tensile stress at a point. However, it is on this latter approach that the theory of brittle fracture in compression rests.

While the maximum tensile stress criterion gives the same strength dependence as the maximum stress intensity factor criterion for cracks normal to a tension field, the agreement is not general. When a plate with a slit is loaded asymmetrically in tension, the fracture forms at an angle to the slit and follows a curved path which eventually becomes normal to the load (see fig. 1). The fracture path in the ideal homogeneous brittle solid is a principal stress direction [4]. Even if local inhomogeneities in a real solid do deflect the fracture it will tend to return to the ideal fracture path unless there is instability in the path direction [4–6]. However, there is an apparent paradox in the direction of the principal stress, and hence the angle at which the fracture forms, depending on whether the stress distribution around a crack, or that around an elliptical hole that degenerates into a crack, is studied [7, 8].

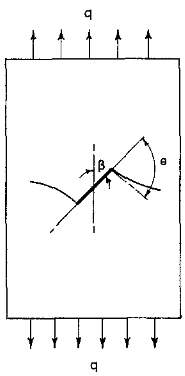

Figure 1. Crack growth from a slit under tension.

If the tangential stress around a crack tip is examined, it is found that there is a maximum value at an angle θ to the prolongation of the crack which is given by [9],

$$\sin\beta\,\sin\theta - \cos\beta\,(3\cos\theta - 1) = 0\,. \tag{1}$$

On this section the shear stress is zero, thus not only is this stress the maximum tangential stress at the crack tip, but it is also a principal stress—the only one that passes through the crack tip. The direction of the maximum principal stress at the edge of an elliptical hole that degenerates into a slit is given by [7].

$$\theta = \pi/4 - \beta/2\,. \tag{2}$$

The comparison of these two values, originally made by McClintock [7], is shown in fig. 2. Erdogan and Sih's experimental results [9] obtained from polymethyl methacrylate specimens agree closely to the angle predicted by the crack model. The paradox of these two conflicting results is explained when the fracture growth from an elliptical hole is examined—the fracture path changes direction in a distance of the same order as the radius of the root of the elliptical hole to propagate essentially in the same direction as a fracture propagating from a crack [8]. As the elliptical hole becomes more slender, the distinction between the fracture paths disappears.

The two hypotheses of fracture give different variation of fracture strengths with orientation, but, of course, for a given orientation the variation of strength with crack length (provided that the root radius of the slit remains constant) is the same for both theories. When Erdogan and Sih

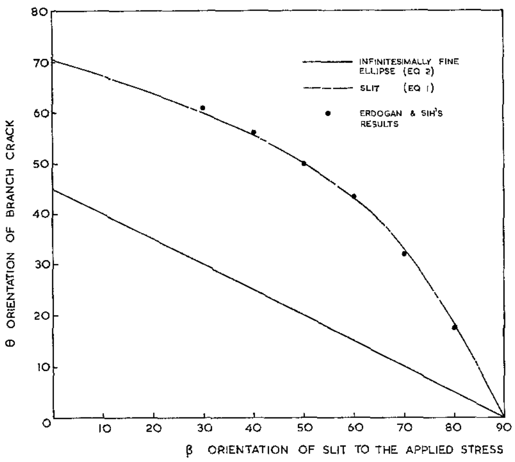

Figure 2. Direction of crack growth from a slit under tension.

performed their elegant experiments on polymethyl methacrylate [9] the stress intensity factor for a slit with an infinitesimal branch at its tip was unknown and they had to propose a modified maximum stress hypothesis, but Andersson* [10] has now solved this problem and has derived the following expression for the stress intensity factor

$$K = K_1 - iK_2 = q(\pi c)^{\frac{1}{2}} \exp\left[i\left(\beta - \frac{\pi}{2} + \theta\right)\right]\left(\frac{\pi-\theta}{\pi+\theta}\right)^{\theta/2\pi} \sin\beta \tag{3}$$

where K_1 and K_2 are the stress intensity factors for the opening and shearing modes. The result contains some surprises. When the branch opens in the direction of the maximum tangential stress at the crack tip (equation (1)) the stress intensity factor is complex and thus the prolongation of the branch is not a principal stress direction. This anomaly may be caused because the stress field can only be represented by the stress intensity factor and the inverse square root of the distance from the tip of the branch in a region around the tip of the branch that is small compared with the branch length. Thus the stress intensity factor at the end of the branch (which in general is complex) would be influenced by its curvature no matter how small the branch is, and to obtain the true stress intensity factor which would be real, indicating that the prolongation of the branch is a principal stress direction, it may be necessary to consider a curved branch. However, the shearing mode stress intensity factor is only a small fraction of

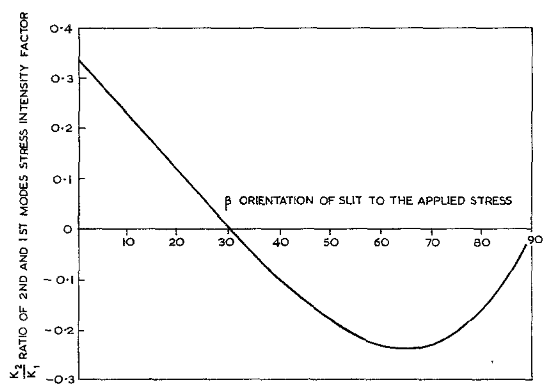

Figure 3. Ratio of shear mode to opening mode stress intensity factors for a branch crack growing in the direction θ given by eq. (1).

* Andersson's expression for the stress intensity factor for a slit with an infinitesimal branch is derived from a previous paper giving the stress intensity factor at the tips of a star shaped contour [11] which although it has been shown to contain an error that invalidates the general solution is correct for infinitesimal branches [12].

the opening mode for the straight branch model of Andersson for a branch growing in the radial direction giving the maximum tangential stress, as is shown in fig. 3, and thus it can be assumed that Andersson's result is a good approximation. Since it is the crack extension force

$$\mathscr{G} = \frac{|K|^2}{E} \tag{4}$$

that determines the fracture strength, the modulus of the stress intensity factor in equation (3) has been used to calculate the fracture strength for a slit at an angle to the applied tension given by equation (1) that is shown in fig. 4 (if θ is taken to be the value that makes K real, i.e. $\theta = \pi/2 - \beta$, the result is very little different). McClintock [7] has given the maximum tensile stress at the surface of a slender elliptical hole which in the present notation becomes

$$\sigma_\eta/q = \left(\frac{c}{\rho}\right)^{\frac{1}{2}} \sin\beta(1+\sin\beta) \tag{5}$$

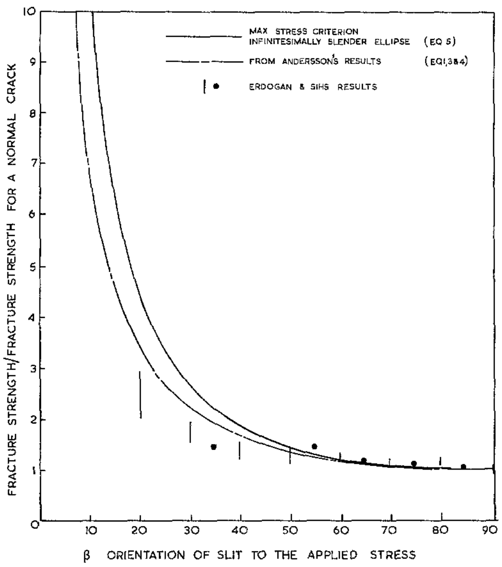

Figure 4. Fracture strength in tension.

The fracture strength of a plate with a slender elliptical hole according to the cohesive strength hypothesis has been calculated from equation (5) and is shown in fig. 4. The theoretical results are not in agreement. The experimental results of Erdogen and Sih [9] (also presented in fig. 4) are in better agreement with the fracture strength predicted from Andersson's stress intensity factor [10] than that predicted from the cohesive strength hypothesis. In tension fields the original Griffith energy balance, usually expressed in terms of a critical stress intensity factor is widely accepted. It is, therefore, necessary to look at the paradoxical situation of brittle fracture being explained by two distinct theories depending on whether the load is in tension or compression.

1.1. *Fracture from a Single Open Crack under Compression*

Only Hoek and Bieniawski have tested open cracks under compression [13]. They made seven biaxial compression tests on 6 inch square by $\frac{1}{4}$ inch thick plate glass specimens with slits orientated in the least favourable direction. While these results support the cohesive strength hypothesis, they were not thought to give adequate confirmation of the hypothesis and a similar experimental programme was undertaken.

The test specimens were 6 inch square by $\frac{1}{2}$ in. thick plates of annealed glass with elliptical holes (nominally major axis 0.5 inches, minor axis 0.05 inches) ultrasonically machined at various angles. The holes were machined using 400 grade silcon carbide powder which significantly increased the root radius of the ellipse from the nominal value of 0.0025 inches to a measured average value of 0.005 inches (see fig. 5) In order to minimise bending stresses, $\frac{1}{2}$ inch thick plates were used in preference to the $\frac{1}{4}$ in. thick plates that Hoek and Bieniaswki [13] used. A uniaxial compression rig (see fig. 6) designed by Ergun [14] at Imperial College was used in the compression tests (a brief description is given in the appendix). Two tensile tests were made

Figure 5. Microphotograph of root of elliptical hole.

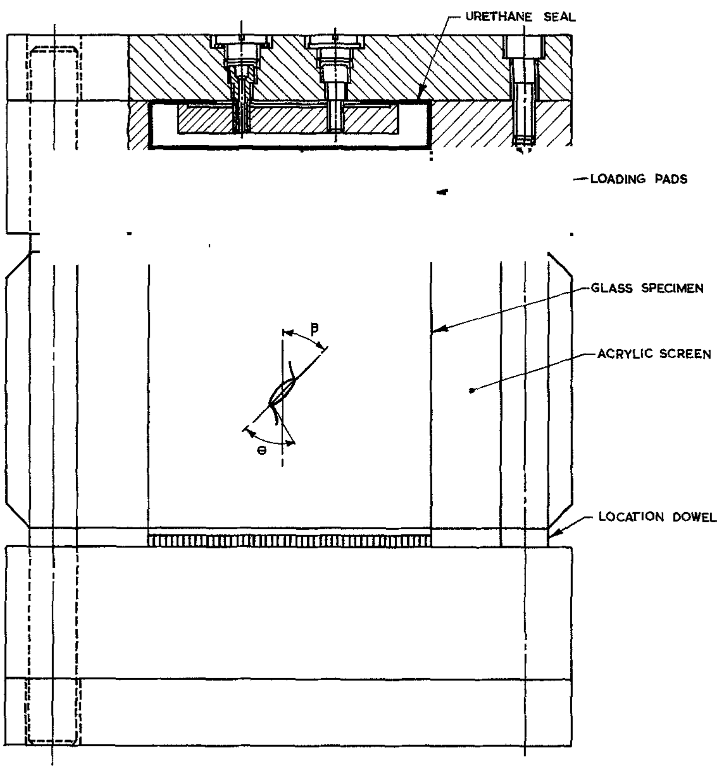

Figure 6. Compression loading apparatus and specimen.

on plates containing the same elliptical holes orientated normally to the applied load, using the "whipple tree" arrangement described by Hoek and Bieniawski [13].

The initial direction of fracture in the compression tests is shown in fig. 7. The fracture initia-

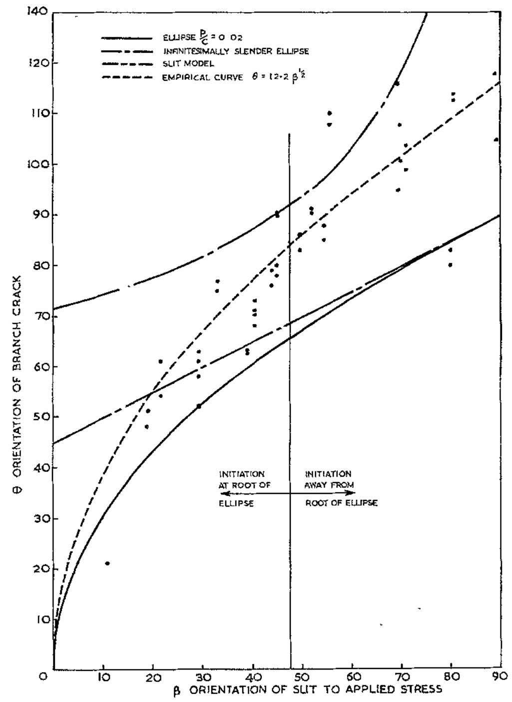

Figure 7. Direction of crack growth from a slit under compression.

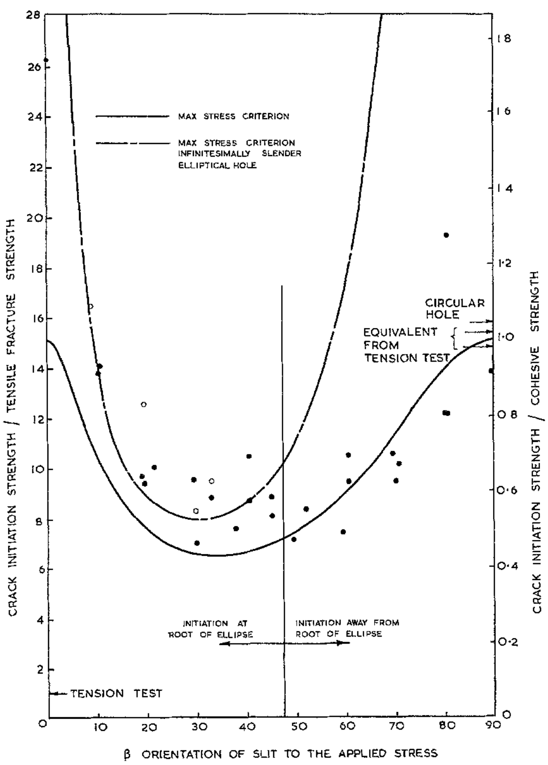

Figure 8. Crack initiation strength in compression.

tes at the mid thickness of the plate. There is some double curvature in the fracture surface—it not only curves in the plane of the plate so that it becomes parallel to the applied compression, but it is also slightly curved normal to the plate surface to form a saddle surface. Thus the stress distribution near the elliptical hole is not strictly two-dimensional. When the hole is orientated at less than 47° to the applied load, the fracture initiates at or very near to the tip of the elliptical hole. At greater orientations the fracture initiates at a distance of several tip radii from the tip of the hole. As with the uni-axial compression tests of Hoek and Bieniawski [13], fracture propagation usually commenced with the sudden appearance of a fair sized crack. Normally a crack would appear at one end of the elliptical hole first, to be followed by a mirror image crack after a delay of a few seconds; in some cases the second crack did not form until the load was increased. The crack initiation strengths of the plates are shown in fig. 8 as a fraction of the average strength in tension of a plate with an elliptical hole normal to the applied load. In those tests where the second crack did not form until the load was increased, the load at the initiation of the second crack is shown as an open point.

1.2. *The Cohesive Strength Theory of Compression Fracture.*

This theory has been presented many times [2, 13, 15], but a brief resume is convenient. The tangential stress σ_η around the boundary of an elliptical crack due to the uniaxial compressive stress q is given by Timoshenko [16]

$$\sigma_\eta/q = -\frac{\sinh 2\xi_0 + \cos 2\beta - e^{2\xi_0}\cos 2(\beta-\eta)}{\cosh 2\xi_0 - \cos 2\eta} \tag{6}$$

where ξ, η are the elliptical coordinates and ξ_0 is the value of ξ on the crack boundary. For slender elliptical holes $\xi_0 = (\rho/c)^{\frac{1}{2}}$ where ρ is the radius of curvature of the tip of the hole and c the half length of the hole. Equation (6) will give the approximate stress around a hole of approximately elliptical form if $(\rho/c)^{\frac{1}{2}}$ is substituted for the elliptical coordinate ξ_0. It is usually assumed that the maximum boundary stress occurs near the tip of the elliptical hole where η is small and if ξ_0 is also small equation (6) becomes

$$\sigma_\eta/q_c = -\frac{2(\xi_0 \sin\beta - \eta\cos\beta)\sin\beta}{\xi_0^2+\eta^2}. \tag{7}$$

By differentiation, the position of the maximum tensile stress is defined by

$$\frac{\eta}{\xi_0} = \frac{\sin\beta+1}{\cos\beta}. \tag{8}$$

The orientation of the fracture at the tip of the elliptical hole is then given by

$$\theta = \pi/4 + \beta/2. \tag{9}$$

This result is shown in fig. 7. If it is not assumed that both ξ_0 and η are small the position of the maximum tensile stress is defined by

$$e^{2\xi_0}\sin 2\beta - e^{2\xi_0}\cosh 2\xi_0 \sin 2\beta\cos 2\eta - \sinh 2\xi_0(1-e^{2\xi_0}\cos 2\beta)\sin 2\eta = 0. \tag{10}$$

This equation has been solved for the approximate elliptical holes used in the experiments by substituting $(\rho/c)^{\frac{1}{2}}$ for ξ_0 and the resulting fracture angle is presented in fig. 7.

The maximum tensile stress can now be obtained from knowledge of the position at which it occurs. If the elliptical hole is very slender, equation (7) gives the maximum tensile stress as

$$\sigma_\eta/q_c = \left(\frac{c}{\rho}\right)^{\frac{1}{2}}\sin\beta(1-\sin\beta). \tag{11}$$

The maximum tensile stress for a slender elliptical hole normal to an applied tensile stress q_t is

$$\sigma_\eta/q_t = 2\left(\frac{c}{\rho}\right)^{\frac{1}{2}}. \tag{12}$$

Thus if fracture occurs when the maximum tensile stress reaches the cohesive strength of the material, the ratio of fracture strengths is

$$(q_c/q_t) = 2/\sin\beta(1-\sin\beta)\,. \tag{13}$$

This well known curve is presented in fig. 8. If the elliptical hole is not slender, equation (6) must be used to obtain the maximum tensile stress with the value of η obtained from equation (10). The maximum tensile stress for an elliptical hole normal to an applied tensile stress q_t is

$$\sigma_\eta/q_t = \left[1+2\left(\frac{c}{\rho}\right)^{\frac{1}{2}}\right]. \tag{14}$$

The ratio of the fracture strengths is presented in fig. 8 for the approximately elliptical holes used in the experiments. The maximum tensile stress at the edge of the elliptical hole can be calculated from equation (14). This maximum stress is sometimes rather loosely called the "cohesive strength" although this value is naturally orders of magnitude less than the true cohesive strength of a defect free solid. The right-hand scale of fig. 8 has been constructed using this definition of the "cohensive strength" of glass.

1.3. *The Griffith Energy Theory of Compression Fracture*

Before developing this theory it is necessary to recognise that fracture depends on the variation of crack extension force with crack growth as well as on its value for the original notch or the existing crack. The main features of the variation of crack extension force can be seen from the study of crack growth from a circular hole under tension and compression. Bowie [17] has given the crack extension force for a circular hole with radial cracks. In fig. 8. the crack extension force is presented as a function of the crack length for a circular hole under uniaxial tension. When the crack is small, the crack extension force can be given approximately by the expression for a crack at the edge of a semi-infinite plate [18] under the stress $3q$

$$\mathscr{G} = 1.25(3q)^2\,\pi L/E\,. \tag{15}$$

On the other hand, if the crack is large then the crack extension force can be given approximately by a crack of length $2(L+a)$ under a uniform stress q

$$\mathscr{G} = q^2\pi(L+a)/E\,. \tag{16}$$

Both of these approximate relationships are shown in fig. 9. One can see that together these two straight line approximations can be used to give a quite accurate description of the crack extension force for any crack length. Now if a test is made on a brittle material with an inherent defect at the edge of the hole less than approximately $a/10$, then fracture will occur when the crack ex-

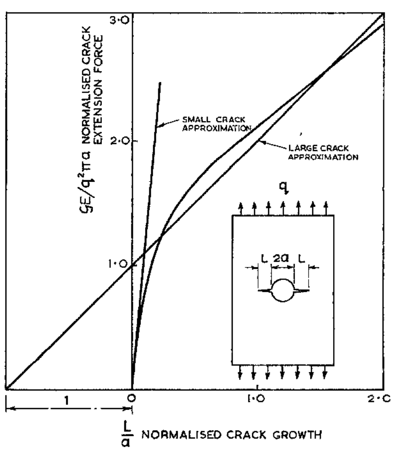

Figure 9. Crack extension force for crack growth from a circular hole under tension.

tension force given by equation (15) reaches a critical value and, providing that the defect size is the same, the fracture strength will be independent of the size of the hole. Pragmatically one could say that fracture occurs when the maximum tensile stress reaches the cohesive strength of the material. Only if the defect size is greater than $a/10$ will be the strength be size-dependent.

The variation of crack extension force with crack growth from a circular hole under uniaxial compression is different (see fig. 10). When the crack is small the crack extension force can again be given approximately by the expression for a crack at the edge of a semi-infinite plate this time under a stress q.

$$\mathscr{G} = 1.25 q^2 \pi L/E\,. \tag{17}$$

When the crack is large the crack extension force can be obtained from the expression for a crack of length $2(L+a)$ opened by the normal stress σ that acts across the line of the crack before it forms. This stress is [16]

$$\sigma = \frac{q}{2}\left[5\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right] \tag{18}$$

where r is the distance from the centre of the hole. Paris and Sih [19] give the expression for the stress intensity factor

$$K = \frac{1}{\pi(L+a)^{\frac{1}{2}}} \int_{-(L+a)}^{+(L+a)} \sigma \left[\frac{L+a+x}{L+a-x}\right]^{\frac{1}{2}} \mathrm{d}x\,. \tag{19}$$

This integral expression has been evaluated numerically and the crack extension force $\mathscr{G} = K^2/E$ is shown in fig. 10. In this case only the approximation for a small crack is at all realistic. If the initial defect-size is less than approximately $a/10$ the fracture will initiate when the crack extension force given by equation (17) reaches its critical value, the fracture will then propagate suddenly until it reaches the stable portion of the crack extension force diagram. For large defects the fracture is always stable. Thus if the defect size is less than $a/10$, the fracture strength is independent of the size of the hole whether the load is tensile or compressive. However, for the elliptical hole the critical defect size is not the same for tension and compression.

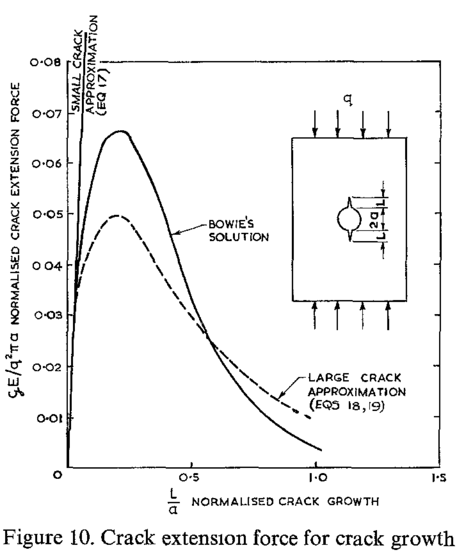

Figure 10. Crack extension force for crack growth from a circular hole under compression.

Approximate crack extension curves can be constructed for the elliptical hole using similar techniques. For a slender elliptical hole normal to an applied tension the maximum tensile stress at the boundary of the hole is given by equation (12) and the crack extension force for a small crack is

$$\mathscr{G} = 5q^2 \left(\frac{c}{\rho}\right) \pi L/E \,. \tag{20}$$

When the crack is large, the crack extension force is

$$\mathscr{G} = q^2 \pi (L+c)/E \,. \tag{21}$$

Assuming that the crack extension force diagram is given by the same portions of the straight line approximations as in the case of the circular hole, the crack extension force will be as shown in fig. 11. Since it has been assumed that $c/\rho \gg 1$, equation (20) will apply if $L/\rho < \frac{1}{5}$. Even if $L/\rho > \frac{1}{5}$, L/c can still be small and equation (21) will be approximately

$$\mathscr{G} = q^2 \pi c/E$$

which is, of course, the normal expression for a thin slit. The inequality $L/\rho > \frac{1}{5}$ must be satisfied in practice by making the root of the notch very sharp, and is, in part, the reason why limits on notch sharpness are specified in fracture toughness tests.

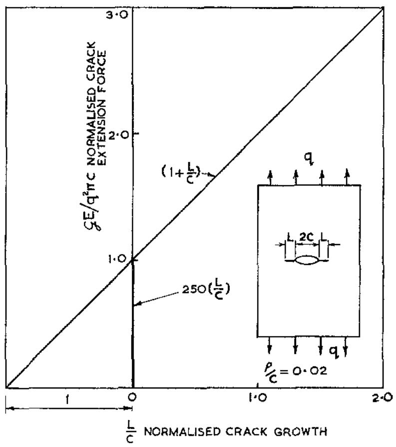

Figure 11. Crack extension force for crack growth from an elliptical hole under tension.

In compression, a small crack will grow under the maximum tensile stress σ_η at the boundary of the elliptical hole, and the crack extension force is

$$\mathscr{G} = 1.25\sigma_\eta^2 \pi L/E \,. \tag{23}$$

The straight line portions of the crack extension force diagram for compression (see fig. 12) have been calculated from equation (23) for the particular elliptical holes used in the experiments. The fracture path is almost straight, but the direction is not accurately predicted by the analysis of the stress distribution around either an elliptical hole or an infinitesimally fine slit [9]. An empirical relationship between the orientation of the hole and the direction of fracture is (see fig. 7)

$$\theta = 12.2\, \beta^{\frac{1}{2}} \,. \tag{24}$$

Andersson's calculation of the stress intensity factor for a crack with a branch at an angle to it [10, 11] is only valid for an infinitesimal branch, but provided that the branch is small it will give an approximate representation of the true stress intensity factor. The crack extension force obtained from Andersson's stress intensity factor is presented in fig. 12 for a crack growing in the direction given by equation (24). The approximate crack extension force diagram for the growth of a crack from an elliptical hole loaded in compression is given by the straight line relationship of equation (23) for small cracks and by the curves for large cracks. Taking a hole orientated at 30° to the applied load, as a typical case, it is instructive to calculate the maximum crack length for which crack growth is controlled by the maximum stress at the edge of the hole. When the elliptical hole is slender, the maximum stress is given by equation (6) and for

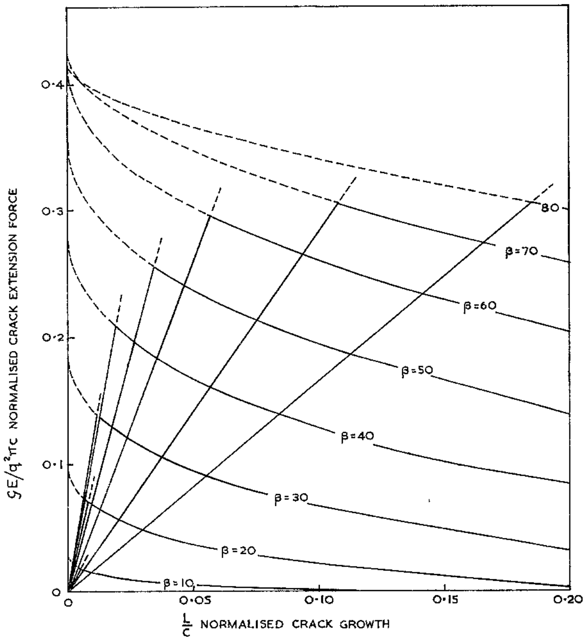

Figure 12. Crack extension force for crack growth from an elliptical hole under compression.

$\beta = 30°$ is $\frac{1}{4}\,(c/\rho)^{\frac{1}{2}}$. The maximum crack extension force can be obtained from Andersson's expression for the stress intensity factor for a slit with an inifinitesimal branch and for $\beta = 30°$ is $0.1872\, q^2 \pi c/E$. Thus the limit on the crack length will be given approximately by the expression

$$0.0781\, q^2 \pi \left(\frac{c}{\rho}\right) L = 0.1872\, q^2 \pi c \qquad (25)$$

or $L/\rho = 2.4$. In fig. 13 the maximum crack growth that will be controlled by the maximum stress at the edge of the hole is shown as a function of the orientation of the hole.

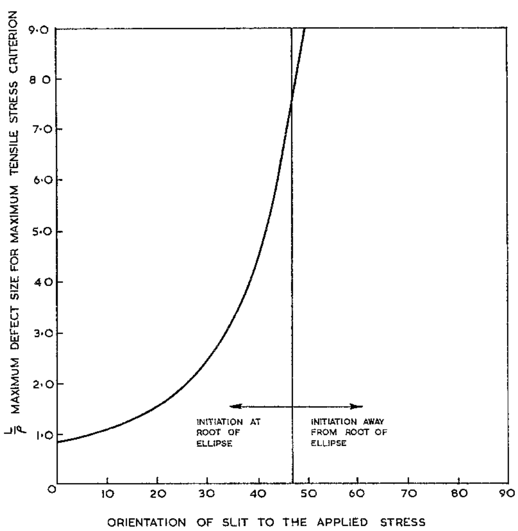

Figure 13. Limiting size of crack growth under the maximum tensile stress at the edge of an elliptical hole.

Thus if the initial defects at the root of the elliptical hole are greater than $\rho/5$, fracture in tension will occur when Griffith's crack extension force (equation 22) reaches a critical value. However, in compression, the defects would need to be greater than the root radius of the elliptical hole (see fig. 13) for fracture to be independent of the radius of the tip of the hole. Since the root of the hole will be limited by the inherent defects to a sharpness no greater than about the magnitude of the defects, it is most likely that in all cases of compression tests on elliptical holes that the crack extension force will be given by equation (23). Thus assuming that the inherent defect-size is constant, the fracture strength in compression will be dependent on the maximum tensile stress at the edge of the hole. Fracture will take place apparently when the maximum stress reaches the "cohesive strength" of the material, but actually the crack extension force, as expressed by equation (23), will have reached its critical value. In tension provided that the root of the slit is sharp, it is most likely that the crack extension force will be given by the normal expression (equation (22)) and that fracture will not depend on the maximum stress.

2. Discussion

Examination of the root of the elliptical hole machined in the glass specimens (fig. 6) shows that the maximum defect size is less than $\rho/5$. Thus fracture should, from the above argument, occur when the nominal maximum stress at the edge of the elliptical hole reaches a critical value whether the applied load is tensile or compressive. In fig. 10, although there is considerable scatter, the results do conform essentially to the maximum stress theory. The results on the circular hole under compression and the elliptical hole under tension agree well with the compression tests. Some evidence that fracture is governed by the maximum stress also comes from the observation that cracks normally appear suddenly, that is they are unstable when they initiate and suddenly propagate to the stable portion of the crack extension force diagram.

Thus the conclusion is that crack initiation in a plate containing a slit loaded by a uniaxial compression stress q will occur when the crack extension force

$$\mathscr{G} = 1.25\, q^2 \sin^2\beta (1 - \sin\beta)^2 \frac{c \cdot L}{\rho E} \tag{26}$$

obtains a critical value (where c is the half crack length, ρ the tip root radius, β the orientation of the crack to the applied stress, and L the length of microcracks at the edge of the hole). If the defect size remains constant this condition may be empirically interpreted as a critical stress criterion, but the maximum tensile stress is in no way a measure of the true cohesive strength of the material. Since defects in any machining process limit the sharpness of any slit so that it is inconceivable to have the root radius sharper than the size of the defects of the machining process, it is impossible for the initiation of fracture from a machined slit in compression to occur when the crack extension force is controlled by the length of the slit and its orientation alone. Only for the sharpness of a natural crack can a theory of an infinitely sharp slit be applied and then, because in compression the crack would close, the problem is no longer one of an open slit. Thus Andersson's expression [10] for the stress intensity factor for a slit at an angle to the applied stress (equation (3)) has no relevance to the initiation of fracture in compression.

In tension the crack extension force for slit with natural defects at its end becomes independent of the root radius at the tip of the slit, if the radius is less than five times the defect size. Such a sharpness can be achieved for a machined slit, thus for a sharp slit normal to the applied stress the relevant crack extension force is

$$\mathscr{G} = q^2 \pi c / E \,. \tag{27}$$

If the slit is at an angle to the applied stress, Andersson's expression [10] for the stress intensity factor (equation (3)) can be used as an approximation with the direction of the branch crack taken to be the direction of maximum tangential stress (equation (1)).

Thus, although apparently fractures in compression and tension from open slits are explained by two separate theories, it is seen that the initiation of fracture, in both cases, occurs when the crack extension force reaches a critical value, but that in compression this crack extension force is always dependent on the root radius of the slit.

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Appendix

The uniaxial compression loading rig.

The main difficulty in compression tests is to obtain a uniform stress distribution. Ergun [14] has solved the problem by loading the specimen with 48 steel loading pads $\frac{1}{8}$ in. × 1 in. × 2 in. one side of each pad has 0.004 in. thick P.T.F.E. tape glued to its surface to reduce the friction. The load is transmitted to the loading pads by a urethane box seal which is filled with hydraulic oil. The seal and the loading pads are housed in steel shoes which are located with one another by 1 in. diameter dowels. The plate is lightly clamped between two pieces of acrylic sheet which both locate the plate in a central position and also protect the operator from the danger of any flying glass. The whole rig (see fig. 6) loaded in a hydraulic compression machine.

With this system of loading the mean stress through the plate thickness is uniform (as observed by photo elasticity) within a distance equal to the plate thickness. Bending stresses are not eliminated entirely. Strain gauge tests on an aluminium specimen (which has approximately the same Young's Modulus as glass) showed that along the specimen the maximum bending stress was 11% of the mean stress. Since it was observed that the fracture initiated from the mid thickness of the plate, it is assumed that these bending stresses did not affect the fracture strength.

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RÉSUMÉ

On a solutionné le paradoxe apparent de deux théories de la rupture correspondant respectivement à des sollicitations de traction et de compression.

En compression, la rupture semble survenir lorsque la tension maximum au voisinage d'une cavité atteint une valeur critique. On suggère néanmoins que la rupture se produit au moment où la force d'extension de la fissure à l'extrémité d'une microfissure atteint, au voisinage de celle-ci, une certaine valeur critique. A ce stade, la différence essentielle de comportement sous traction et sous compression réside dans le fait que, sous traction, la force d'extension de la fissure augmente avec la propagation de la fissure, tandis que, sous compression, on atteint une valeur maximum de la force d'extension qui tend ensuite à décroître lorsque se propage la fissure.

Si les défauts, toujours présents en bordure d'un trou foré, sont de petite dimension, la force d'extension de la

fissure est limitée par la tension maximum à la surface du trou. La dimension critique de ces défauts est différente, selon qu'on est en traction ou en compression. En traction, les défauts doivent être inférieurs au cinquième du rayon de courbure du trou. Si ce rayon est assez petit, la dimension des défauts excèdera cette valeur et la force d'extension de la fissure sera déduite de l'expression classique de mécanique de rupture $G=\sigma^2 \Pi C/E$.

En compression, les défauts doivent être, aux limites, plus grands que le rayon de courbure du trou (voir le double de ce rayon) pour autant que la force d'extension de la fissure ne soit pas contrôlée par la tension maximum. De telles dimensions de défauts ne sont pas possible à envisager, car l'acuité du rayon de courbure est de toute manière limitée à la dimension des défauts usuels. De la sorte, une rupture fragile par compression au départ d'entailles usinées sera toujours dépendante de la tension maximum.

ZUSAMMENFASSUNG

Das scheinbare Paradox der beiden Theorien über das Bruchverhalten bei Zug- oder bei Druckbeanspruchung wurde gelöst. Trotzdem bei einer Druckbelastung der Bruch scheinbar dann eintritt wenn die maximale Zugspannung in der Umgebung des Loches einen kritischen Wert erreicht, wird unterstellt, daß der Bruch dann eintritt, wenn die Bruchausweitungskraft an der Spitze eines Haarrisses in seiner Umgebung einen kritischen Wert erreicht. Der grundlegende Unterschied des Bruchverhaltens unter Druck- und Zugbelastung besteht darin, daß bei Zugbelastung die Rißausweitungskraft mit der Rißausbreitung ansteigt, während bei einer Druckbelastung ein Maximum für die Rißausbreitungskraft erreicht wird, welche dann bei weiterem Fortschreiten des Risses wieder zu geringeren Werten abfällt. Wenn die Fehlstellen, welche unweigerlich am Rande eines Bohrloches auftreten klein sind, wird die Rißausweitungskraft durch die maximale Zugspannung an der Lochoberfläche kontrolliert. Die kritische Größe der Fehlstellen ist unterschiedlich für Druck- und Zugbelastung. Bei Zugbelastung muß die Fehlstelle kleiner sein als ein Fünftel des Lochdurchmessers; falls die Krümmung klein genug ist, so sind die Abmessungen der Fehlstellen größer als dieser Wert und die Rißausweitungskraft wird durch den in der Bruchmechanik üblichen Ausdruck $G=\sigma^2 \Pi C/E$ gegeben. Bei Druckbelastung müssen die Fehlstellen im Grenzfall größer als die Krümmung der Lochspitze sein (in einem typischeren Fall mehr als zweimal so groß) wenn die Rißausweitungskraft nicht durch die maximale Zugspannung bestimmt sein soll. Solche groben Fehlstellen sind unmöglich da die Schärfe der Wurzelkrümmung durch die Abmessungen des Defektes begrenzt ist. Folglich wird im Fall von Druckbelastung der Bruch an bearbeiteten Kerben immer durch die maximale Zugspannung bestimmt.