Effects of Signal Detuning on Two-Photon Phase Conjugation

M. Lu and M. Sargent III
Optical Sciences Center, University of Arizona, Tucson, AZ 85721, USA

PACS: 42.55

This paper describes phase conjugation in two-photon media for signal and conjugate waves of nonsaturating intensity and pump waves of arbitrary intensity. By combining aspects of two papers by Fu and Sargent [1, 2], we study the signal and conjugate response as these waves are detuned from the pump waves. The calculation assumes that single-photon electric-dipole transitions are nonresonant, and hence can be treated using first-order perturbation theory [3]. The resulting two-level density matrix components are expanded in Fourier series as in well-known treatments of the electric-dipole two-level problem [4]. The induced signal polarization found in this way has similarities with the simpler single-photon case, but features several new contributions. Our paper consists of presenting this calculation and analyzing the polarization, reflectivity and transmission for various signal-pump detunings and pump intensities. The results are important in saturation spectroscopy, in obtaining a narrow-band retroreflector, and in studying single-mode instabilities in two-photon lasers.

We find that the signal polarization has the form

$$\mathscr{P}_1(r)=N[\tfrac{1}{2}\mathscr{E}_1(k_{aa}+k_{bb})+\tfrac{1}{2}(k_{aa}-k_{bb})(\mathscr{E}_1 d_0+\mathscr{E}_2 d_1) + 2k_{ab}^*(\mathscr{E}_3^* p_0+\mathscr{E}_2^* p_1)], \quad (1)$$

where $\mathscr{E}_1$, $\mathscr{E}_2$, and $\mathscr{E}_3$ are the signal, pump, and conjugate amplitudes, respectively; k_{aa} and k_{bb} are the electric-dipole level shifts of the upper and lower two-photon levels a and b, k_{ab}^* is the two-photon matrix-element function [3], N is the number of atoms per volume; d_0 and d_1 are Fourier components of the population-difference; p_0 and p_1 are Fourier components of the two-photon coherence. These components play critical roles in the response of the system. The p_1 term is analogous to the single-photon polarization component that yields coherent dips in forward scattering, and involves population pulsations, that is the response of the medium to the beat frequency between pump and signal waves. The p_0 term is analogous to the single-photon conjugate wave term. In addition, when the two-photon interactions Stark shift levels a and b differently, i.e., $k_{aa} \mp k_{bb}$, two new terms come into play that change the signal absorption coefficient. In particular, the p_1 and d_1 terms are both proportional to Lorentzians of the beat signal-pump frequency, with a width given by $1/T_1$, where T_1 is the two-photon population difference lifetime. Hence as the signal is detuned, these terms decay away. The first term on the RHS of Eq. (1) is linear in the wave amplitudes, and only yields a constant dispersion that has no effect on the phase conjugation. Agrawa [5] has treated signal detuning on two-photon transitions including only the p_0 term.

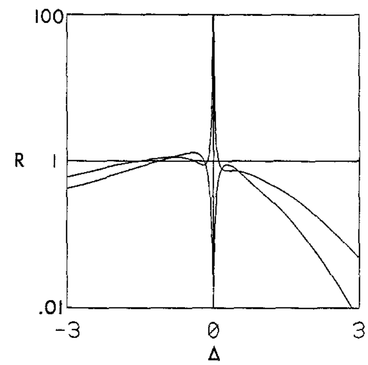

Fig. 1. Two-photon reflectivity vs signal detuning for two values of the Stark shift coefficient

The polarization [1] and its counterpart for the conjugate wave yield two-photon absorption and mode coupling coefficients for the standard coupled mode equations [1]. The corresponding reflectivities and transmissions are given by the same expressions as for the single-photon case, but with these new coefficients. Figure 1 shows an interesting set of reflectivities as functions of signal detuning. Unlike the single-photon case, a background reflectivity remains for detunings larger than $1/T_1$ due to terms independent of population pulsations. The curves reveal the onset of a Yariv-Peppe oscillation condition. They are evaluated in the limit that the Stark tuning of the resonance line is small compared to the two-photon linewidth. Our theory allows for arbitrarily large tunings (resulting in substantially more complicated expressions), and we hope to present numerical results for these cases as well.

Acknowledgement. We thank Hyatt Gibbs and Richard Shoemaker for helpful discussions.

1. T.Fu, M.Sargent III: Opt. Lett. **4**, 366 (1979)
2. T.Fu, M.Sargent III: Opt. Lett. **5**, 433 (1980)
3. See, for example, D.Grishkowsky, M.M.T.Loy, P.F.Liao: Phys. Rev. A **12**, 2514 (1975)
L.M.Narducci, W.W.Eidson, P.Furcinitti, P.C.Eteson: Phys. Rev. A **16**, 1665 (1978)
4. For a review, see M.Sargent III: Phys. Rep. **43**, 223 (1978)
5. G.P.Agrawal: Opt. Commun. **39**, 272 (1981)

Laser-Induced Continuum Structure in Multiphoton Excitation

P. E. Coleman and P. L. Knight

Optics Section, Blackett Laboratory, Imperial College, London SW7 2BZ, UK

PACS: 36.20

Multiphoton processes in which photoionization continua play a significant role are enhanced by exploiting autoionizing resonances embedded in the continua by configuration interaction [1]. *Tunable* embedding and continuum structure can be induced by radiative coupling from an unpopulated discrete state; such "pseudo-autoionizing" states [2] are shifted and broadened at will. We show that such structured continua radically alter photoexcitation lineshapes, generate Rabi oscillations via the continuum and trap population in coherent superposition states.

In Fig. 1, population in j is photoionized by coupling V_j. With no dressing interaction V_k, population in j exponentially decays in time; the ion yield versus laser frequency merely maps out the continuum density of states. The interaction V_k couples discrete state k strongly to the continuum and leads to a nonlinear modification of this density of states. Photoionization of population in j to dressed continua can be analyzed nonperturbatively by (i) using the undressed states j, k, and continuum E in standard decay theory, (ii) using the structured continuum (a close-coupled superposition of k and E) and the undressed state j as in recent discussions of autoionization [3] or (iii) using fully-diagonalized dressed-states. We use (iii) and diagonalize the Hermitian Hamiltonian describing j, k, and E and their interactions, working in terms of superpositions of *stationary* states. Decay is generated by phase-mixing of the superposition. We make pole-approximations to level shifts and widths, deriving analytic expressions for the dependence of state populations on time, field strengths and detuning.

Photoionization probabilities show characteristic Fano lineshapes for small couplings V_j compared with embedding coupling V_k, reflecting multichannel interference. The Fano asymmetry parameter q characterizing lineshapes is here the ratio of the real to the imaginary part of the two-photon Rabi frequency. Recent experiments on cesium indicate [4] $q \sim 7$; our analysis indicates photoionization enhancement $\sim q^2$. Increasing V_j distorts and broadens the asymmetric lineshape. The time-evolution of the populations exhibit Rabi oscillations via the continuum with effective Rabi frequency

$$\Omega = \{[\Delta - i\pi(|\bar{V}_k|^2 - |\bar{V}_j|^2)]^2 + 4\pi^2 |\bar{V}_k|^2 |\bar{V}_j|^2 (q-i)^2\}^{1/2},$$

where $2\pi|\bar{V}_k|^2$ is the ionization width of k. For $\Delta = 0$ the bound population is *not* completely ionized as $t \to \infty$ if $V_i = V_k$: part of the population is trapped in a coherent superposition state immune to photoionization (Fig. 2). The photoelectron energy distribution similarly exhibits multichannel interference (Fano minima) and Rabi oscillations (asymmetric AC Stark doublets); this energy spectrum is also sensitive to population trapping.

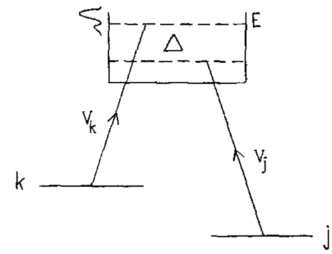

Fig. 1. Level scheme illustrating induced continuum structure

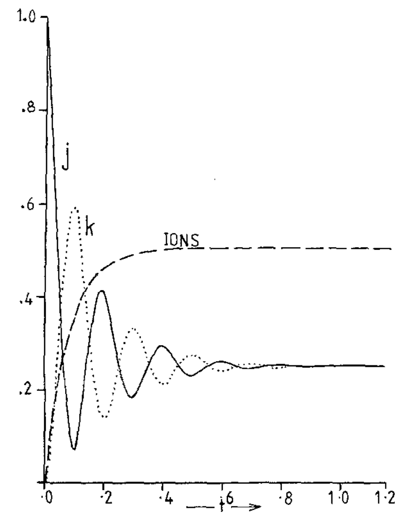

Fig. 2. Time development of population indicating coherent trapping for $q=5$, $\Delta=0$, and $\bar{V}_j = \bar{V}_k = 1$

Continua in multiphoton processes can be structured by laser excitation. Such induced structure is not merely a skewed Lorentzian: multichannel interferences are essential in the description of this system. For example, merely by turning on V_k not only can population *return* from the continuum but may never be excited at all.

1. J.A. Armstrong, J.J. Wynne: In *Nonlinear Spectroscopy*, ed. by N. Bloembergen (North-Holland, Amsterdam 1977)
2. L. Armstrong, Jr., B.L. Beers, S. Feneuille: Phys. Rev. A **12**, 1903 (1975)
3. P. Lambropoulos, P. Zoller: Phys. Rev. A **24**, 379 (1981)
K. Rzążewski, J.H. Eberly: Phys. Rev. Lett. **47**, 408 (1981)
4. Yu.I. Heller, V.F. Lukinykh, A.K. Popov, V.V. Slabko: Phys. Lett. **82** A, 4 (1981)

Enhanced Three-Wave Mixing by Autoionizing Resonances

G. Alber and P. Zoller

Theoretical Physics Institute, University of Innsbruck, A-6020 Innsbruck, Austria

PACS: 36.10, 42.55

In this paper we present a theory of enhanced two-photon resonant three-wave mixing by resonant interaction wuth an autoionizing state. Most of the studies concerned with harmonic generation and wave-mixing have so far dealt with single electron models of atoms. The presence of electron correlation effects, manifesting themselves in autoionizing resonances, introduces qualitatively new behaviour, as it is the interplay between the configuration- and laser-atom interaction which is primarily responsible for new effects [1]. In particular, the characteristic lineshapes of the autoionizing resonances are strongly distorted by such interference effects, leading, for example, to quasi-bound dressed states when the configuration interaction becomes comparable to the coupling to the radiation field. One of the intrinsic features of this problem is, of course, multiphoton ionization via the autoionizing resonance which influences and competes with the process of wave mixing.

Adopting the model of an atomic configuration shown in Fig. 1, we derive within a semiclassical framework a set of density matrix equations coupled to the electromagnetic field, allowing both for saturation of the two-photon resonance and transition to the autoionizing state as well as propagation effects (phase matching). In the limiting case of a weak second laser these equations include as a special case previously derived results [2, 3], where our equations reduce to the ones of an effective two level system with Fano-type ionization widths and an asymmetric quadratic Stark profile. Another limiting case, which can be treated analytically, is the one of saturated transition to the autoionizing state but weak two-photon excitation, leading to the formation of dressed autoionizing states (AC Stark splitting) with pronounced asymmetric lineshapes. For both lasers having arbitrary intensities we present numerical results for the efficiency of the generated harmonic and related multiphoton ionization probabilities, as a function of the intensities and detunings of the first and second laser and typical atomic parameters. Model calculations of this type are necessary for the interpretation of experiments concerned with wave-mixing and multiphoton ionization in the alkaline earth atoms [4–6].

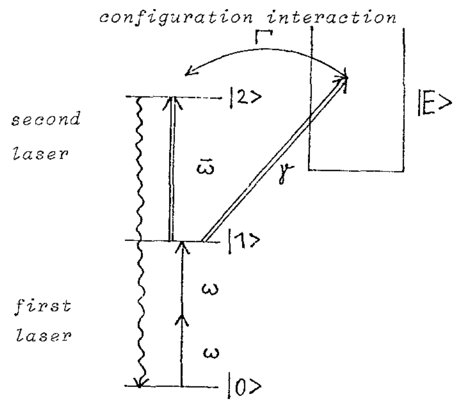

Fig. 1

1. P. Lambropoulos, P. Zoller: Phys. Rev. A **24**, 379 (1981)
2. B.L. Beers, L. Armstrong: Phys. Rev. A **12**, 2447 (1975)
3. Yu. I. Heller, A.K. Popov: Phys. Lett. **56** A, 453 (1976)
4. S.C. Wallace, G. Zdasiuk: Appl. Phys. Lett. **28**, 449 (1976)
5. S.L. Chin, D. Feldmann, J. Krautwald, K.H. Welge: J. Phys. B **14**, 2353 (1981)
6. P.P. Sorokin, J.J. Wynne, J.A. Armstrong, R.T. Hodgson: Ann. NY Acad. Sci. **267**, 30 (1976)

Nonperturbative Quantum Calculation of the Index of Refraction of Multilevel Systems

C. D. Cantrell, G. L. Peterson, and D. R. Adams

Center for Quantum Electronics & Applications, University of Texas at Dallas, TX 75080, USA

PACS: 36.20, 02

The past few years have seen a surge of interest in coherent pulse-propagation phenomena, in multilevel systems, in part as the result of the observation of four-wave mixing, phase conjugation and strong self-focusing in SF_6.

Propagation effects that involve only the dispersive properties of the medium such as the nonlinear refractive index are thus potentially important (particularly for applications such as laser isotope separation) but have received very little theoretical attention for multilevel quantum systems. In this paper we consider the propagation of a plane, nearly (but not exactly) resonant, quasi-monochromatic laser pulse through a general multilevel medium, and, after extending the two-level adiabatic-following approximation [1] to multilevel systems, give a general method for calculating the susceptibility and index of refraction of any multilevel system. Since our method includes contributions to all orders in the laser field, we are able to describe nonlinear phenomena such as self-focusing and self-phase modulation.

We describe the propagation of a classical, plane, quasimonochromatic electromagnetic field through a medium by using Maxwell's equations in the slowly varying amplitude and phase approximation (SVAPA) and the Schrödinger equation in the rotating-wave approximation (RWA). We work in a basis of molecular eigenstates $|mA\rangle$, in which m indicates that the zero-field energy E_{mA} is near $m\hbar\omega$ (where ω is the laser frequency), and A represents the quantum numbers that specify the state. We assume that the effects of collisions are negligible, and thus describe the multilevel systems in the propagation medium by a wave function. The effective Hamiltonian that appears in the RWA Schrödinger equation in this case has the matrix elements

$$H^{\text{eff}}_{mA;mA} = \Delta_{mA} = m\omega - E_{mA}/\hbar\,, \tag{1a}$$

$$H^{\text{eff}}_{mA;(m-1)B} = (2\hbar)^{-1}\mu_{mA;(m-1)B}\mathscr{E}\,, \tag{1b}$$

$$H^{\text{eff}}_{(m-1)B;mA} = (H^{\text{eff}}_{mA;(m-1)B})^*\,, \tag{1c}$$

where $\mathscr{E}$ is the complex amplitude of the laser field and μ is the dipole operator. The elements of the effective Hamiltonian not shown in (1) are assumed to vanish. Single- or multi-photon resonance occurs when two of the diagonal elements 1a) have the same value. When $\mathscr{E}$ is constant in time it is natural to introduce the dressed states: $H^{\text{eff}}|\lambda(\mathscr{E})\rangle = \lambda(\mathscr{E})|\lambda(\mathscr{E})\rangle$. Even when $\mathscr{E}$ varies in time, a multilevel system originally (prior to the arrival of the laser pulse) in one of the states $|m_0A_0\rangle$ will, to a good approximation, be in the dressed state $|\lambda_0(\mathscr{E})\rangle$ that is correlated with $|m_0A_0\rangle$, as

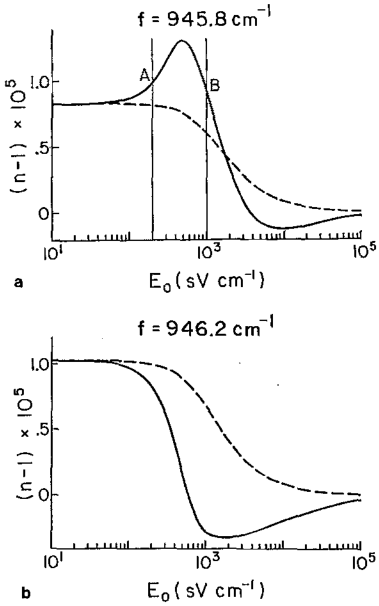

Fig. 1a and b. Dependence of $n(\mathscr{E})-1$ on E_0 for a ten-level anharmonic ladder (solid curves) compared with a two-level system (dashed curves). (For parameters, see text.) Two-photon resonance frequency $=946.0\,\mathrm{cm}^{-1}$. (a) (Left) Laser frequency $=945.8\,\mathrm{cm}^{-1}$. (b) (Right) Laser frequency $=946.2\,\mathrm{cm}^{-1}$

long as the laser frequency ω and risetime τ are such that $|\Delta\lambda_{\min}|\tau \gg 1$ where $|\Delta\lambda_{\min}| \equiv |\lambda_i(\mathscr{E}') - \lambda_j(\mathscr{E}')|_{\min}$ is the minimum (over i,j) of the difference between any two different dressed state eigenvalues, for any intermediate value of the laser field amplitude, $0 \leqq |\mathscr{E}'| \leqq |\mathscr{E}|$. We call this the multilevel adiabatic-following approximation (MLAFA).

In the MLAFA the probability amplitude for the zero-field eigenstate $|mA\rangle$ is $\tilde{c}_{mA}(t) = \langle mA|\lambda_0(\mathscr{E})\rangle \exp\text{-}i\int \lambda_0(\mathscr{E})dt$, and therefore the source term in the propagation equation for the field is

$$\mathscr{P} = 2iN \sum_{m,A,B} \langle mA|\lambda_0(\mathscr{E})\rangle \langle mB|\lambda_0(\mathscr{E})\rangle \mu_{mA;m-1,B}. \qquad (2)$$

In the MLAFA the polarization and the field have the same phase, so that the polarization adiabatically "follows" the field. The susceptibility $\chi(\mathscr{E})$ and the index of refraction are:

$$\mathscr{P}(\mathscr{E}) = i\varepsilon_0 \chi(\mathscr{E})\mathscr{E}, \qquad (3)$$

$$n(\mathscr{E}) = [1 + \chi(\mathscr{E})]^{1/2}. \qquad (4)$$

We have calculated the nonlinear refractive index $n(\mathscr{E})$ for a ten-level nondegenerate anharmonic ladder, and find striking departures from two-level behavior. Anharmonic energy levels $\omega_n = n\omega_0 + n(n-1)X$ and harmonic transition moments $\mu_{n,n+1} = \sqrt{n+1}\,\mu_0$, with $\omega_0 = 948\,\mathrm{cm}^{-1}$, $X = -2\,\mathrm{cm}^{-1}$, and $\mu_0 = 4 \times 10^{-19}$ e.s.u., were used in our numerical calculations. The dependence of $n(\mathscr{E})$ on $\mathscr{E}$ for a laser frequency near the two-photon resonance at $946\,\mathrm{cm}^{-1}$ is shown in Fig. 1 for the ten-level ladder and for a system consisting of the two lowest levels of the anharmonic ladder. The decrease of $n(\mathscr{E})-1$ to zero for large fields in both cases is the result of saturation of the transition, which results in a polarization $|\mathscr{P}|$ that asymptotically approaches a certain constant value, giving a susceptibility χ that decreases in $\mathscr{E}$. At intermediate field strengths the dependence of $n(\mathscr{E})$ on $\mathscr{E}$ is completely different in the two-level and multilevel cases. In particular, the increase of $|n(\mathscr{E})-1|$ over its zero-field value, which is strikingly evident for the anharmonic ladder, is impossible for a two-level system under the adiabatic-following approximation [1]. The widely used expansion $n(\mathscr{E}) \cong n_0 + \frac{1}{2}n_2|\mathscr{E}|^2$ may be much less useful for a multilevel system than for a two-level system, since (as shown in Fig. 1) the derivative ($\propto n_2$) of the curve of n versus $|\mathscr{E}|^2$ is not only non-constant, but changes sign for the multilevel system at field strengths for which $n(\mathscr{E})$ for a two-level system can still be described well by a single n_2. Thus for a multilevel system, a pulse with the maximum amplitude A (Fig. 1a) will show self-focusing or defocusing behavior opposite to that of a pulse with amplitude B. The fact that initially $|n(\mathscr{E})| > |n(0)|$ in some cases for a multilevel system (near a multi-photon resonance), but $|n(\mathscr{E})| < |n(0)|$ for a two-level system, has been verified by analytical calculations for a three-level system.

The range of frequencies where the phenomenon can be observed depends on the magnitudes and signs of the transition moments and detunings. As the laser frequency sweeps through a two-photon resonance Δ_2 changes sign, causing $|n(\mathscr{E})-1|$ to increase initially with increasing $|\mathscr{E}|$ for ω on one side of the resonance and to decrease for ω on the other side, as seen from Fig. 1. Thus a marked change in the dependence of $n_2 \equiv 2dn/d(E_0^2)$ on E_0^2, as measured (for example) by the method of Phipps et al. [2], should be evident as the laser frequency is swept through a two-photon resonance. The latter behavior is not expected in a classical calculation [3]. The prediction that $n(\mathscr{E})-1$ shows a minimum as a function of $\int|\mathscr{E}|^2dt$ at the $P(20)$ CO_2 laser line is based on a completely different physical effect (the anharmonic shift of the dominant resonant absorption frequency with increasing fluence) than the multi-photon resonance that gives rise to the minimum in $n(\mathscr{E})-1$ shown in Fig. 1. Of the two points in Fig. 1 where $n_2(E_0^2)=0$, one is stable (meaning that small radial intensity variations are damped out) while the other (at higher intensity) is unstable. An anomalously strong dependence of transmitted beam diameter on intensity may be observed near the unstable point.

The dependence of $|n(\mathscr{E})-1|$ on $|\mathscr{E}|$ and ω predicted here may be observable near the strong, narrow, isolated two-photon resonance at $944.44\,\mathrm{cm}^{-1}$ from the ground state to the A_{1g} sublevel of the $v_3 = 2$ vibrational state in SF_6 that was recently observed in low-temperature spectra [4]. Should a minimum in $n(\mathscr{E})-1$ as a function of E_0 be found experimentally for CO_2 $P(20)$ irradiation, it may be the result of the proximity of this strong two-photon resonance under conditions like those of Fig. 2a. Simpler molecules such as OCS that have absorption bands near CO_2 laser lines, and atomic systems (in the visible spectrum), may also be convenient for observation of the effects predicted here.

1. D. Grischkowsky: Phys. Rev. A **7**, 2096 (1973)
2. C.R. Phipps, Jr., S.J. Thomas, D.E. Watkins: In *Proceedings of the International Conference on Lasers '79*, ed. by V.J. Corcoran (STS Press, 1980)
3. J.R. Ackerhalt, H.W. Galbraith, J.C. Goldstein: Opt. Lett. **6**, 377 (1981)
4. S.S. Alimpiev et al.: CLEO Paper FN-6 (1981)

Phase Conjugation and Probe Gain in Bichromatically Pumped Two-Photon Resonant Systems

Govind P. Agrawal
City College of New York, USA

PACS: 36, 42.55

Resonant phase conjugation through four-wave mixing has been considered using one-photon as well as two-photon processes [1–3]. In most of the previous work, however, the counter-propagating pump beams are assumed to have identical frequency and polarization. The resulting standing-wave nature of the pump gives rise to spatial holes burnt in atomic populations which reduce considerably the phase-conjugation efficiency [1].
In this paper we consider the case when the counter-propagating pump beams are distinct. More specifically, we assume that they have nondegenerate frequencies ω_1 and ω_2 such that the sum $\omega_1+\omega_2$ is in two-photon resonance with the nonlinear medium. For simplicity the individual frequencies ω_1 and ω_2 are taken to be far from one-photon resonances so that the nonlinear medium can be modeled as an effective two-level system. A weak probe wave at one of the frequencies, say at ω_1, generates the conjugate wave at the other frequency ω_2.
The description of the four-wave-mixing process requires the use of the nonlinear susceptibility derived for the two-frequency case [4]. The phase-conjugate reflectivity R is then obtained following a generalization of the procedure adopted by Abrams and Lund [1]. Pump depletion is ignored and the plane-wave approximation is made to simplify the analysis. The resulting expression for R, defined in terms of the absorption and the coupling coefficients, α_n and κ_n $(n=1,2)$, incorporates the absorptive, the dispersive, and the Stark-shift effects. It should be noted that because of bichromatic pumping the four-wave-mixing process is not exactly phase-matched. However, the mismatch is negligible if the frequency difference $\omega_1-\omega_2$ is small ($\sim$100 MHz) and the angle between the pump and probe beams is small.
The effects of bichromatic pumping are manifested through the dependence of α_n and κ_n on the individual pump intensities I_1 and I_2. Several new features arise when the intensities I_1 and I_2 are varied independently. The most notable among them is that in a certain range of I_1 and I_2 the absorption coefficient α_1 becomes negative, thereby indicating the possibility of probe gain. An exploration of the parameter space shows that it is possible to achieve $R>1$. The intensity-dependent Stark-shift is found to affect the phase-conjugation process in a significant way. Our results are obtained for a homogeneously broadened system. The Doppler broadening is, however, expected to play a minor role because a two-photon resonance is being probed by counter-propagating pump beams.
The proposed method of using distinct pump beams has the advantage of probing a two-photon resonance with relatively high efficiency. Furthermore, in view of the different frequencies of the probe and the conjugate waves, heterodyne techniques can be readily employed. The case wherein each pump wave is in one-photon resonance with the nonlinear medium requires the use of a three level system [3]. Qualitatively similar features are found to hold for this case as well.

1. R.L. Abrams, R.C. Lind: Opt. Lett. **2**, 94 (1978); **3**, 205(E) (1978)
2. T. Fu, M. Sargent III: Opt. Lett. **5**, 433 (1980)
3. G.P. Agrawal: IEEE J. QE-**17**, 2335 (1981)
4. G.P. Agrawal, C. Flytzanis: Phys. Rev. Lett. **44**, 1058 (1980); Phys. Rev. **24**, 3173 (1981)

Dispersive Effects in Magnetically Induced Mode Locking

M. Lindberg and B. Ståhlberg
Research Institute for Theoretical Physics, University of Helsinki, SF-00170 Helsinki 17, Finland

PACS: 42.80

Magnetic mode locking on every mode, every second one and every third one has been reported in previous experimental investigations [1, 2]. Using the semiclassical laser theory by Lamb [3] and the Zeeman laser theory by Sargent et al. [4], we study the locking phenomena.
Our model consists of a two-level amplifier and a Zeeman splitted three-level absorber system. The laser oscillates on three to five longitudinal modes. Following the general approach as presented in [4] we calculate the intensities and the relative phase angles of the modes. Performing the numerical integration of the coupled

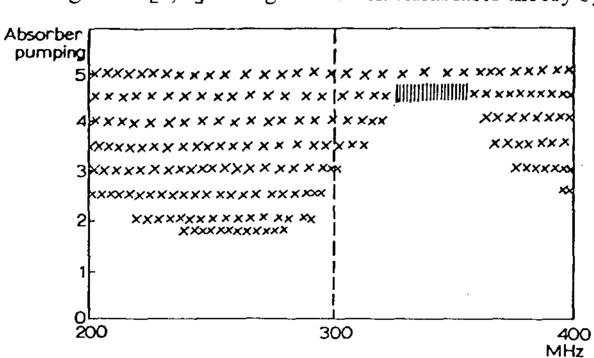

Fig. 1. Result obtained by semiclassical theory. The crosses indicate locked operation in the plane of the absorber pumping parameter and magnetic splitting Δ. The vertically shaded region shows unstable behaviour

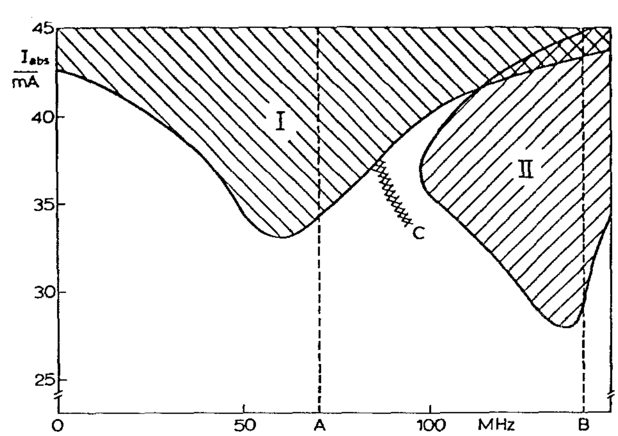

Fig. 2. Experimentally obtained locking regions in the plane of the absorber pumping current and the magnetic detuning. The regions are explained in the text. The laser output power is 20 mW and absorber Ne pressure is 300 Pa

In Fig. 2 we show our experimental results obtained with the He-Ne equipment described in [1, 2]. Region I represents locking on every mode and region II locking on every second mode. The pumping current I_{abs} is analogous with the absorber pumping parameter before. When the corresponding regions in Figs. 1 and 2 are identified we expect from our theoretical treatment locking on every mode to reappear around the Curve C on the right side of the first mode-crossing resonance A. Because of the near vicinity of region II our experimental system displays no locking around C. Still closer to the second mode-crossing B we obtain locking on every second mode.
In their general behaviour the theoretical and experimental results agree. Both clearly show that the phenomenon is caused by dispersive effects of the three-level resonances.

1. B.Ståhlberg, K.Weckström, S.Stenholm: Opt. Commun. **30**, 209 (1979)
2. B.Ståhlberg, K.Weckström: Phys. Scripta **22**, 483 (1980)
3. W.E.Lamb, Jr.: Phys. Rev. A **134**, 1429 (1964)
4. M.Sargent III, W.E.Lamb, Jr., R.L.Fork: Phys. Rev. **164**, 436 (1967)

differential equations we obtain their time dependence. When the relative phase angles become constant we interpret mode locking to occur.
The theoretical calculations predict that the locking phenomenon is sensitive to the dispersive part of the three-level resonances. In Fig. 1 we show results obtained by three modes and the following parameters: amplifier line widths = 50 MHz, absorber line widths = 20 MHz, Doppler width = 500 MHz, detuning = 20 MHz, mode spacing = 300 MHz and amplifier pumping = 60 (arbitrary dimensionless units). The magnetic Zeeman splitting is

$$\Delta = 2gB\mu_B/h,$$

where linear polarization imposes $\Delta m = \pm 1$. The crosses indicate locked operation. When the magnetic splitting Δ is less than the mode-spacing (300 MHz) locking occurs with lower absorber pumping (arb. units) than when Δ is somewhat larger than 300 MHz.

Advances in Modelling of Near-Resonant Light Propagation

P. D. Drummond
Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627
F. P. Mattar
Aerodynamics Laboratory, Polytechnic Institute of NY, Brooklyn, NY 11201, USA

PACS: 42.80

There has been recent progress in modelling the propagation of light in media of various kinds. The advances that have been made permit accurate description of propagating optical pulses under conditions that may require attention to any or all of the following:
1) nonlinear and time-varying interaction with the medium via rate equations or Maxwell-Bloch equations,
2) stochastic initial conditions, and
3) non-plane-wave diffractive effects requiring all four dimensions (X-Y-Z-T) to be treated independently. These theoretical techniques are applicable to a large number of problems of interest in Quantum Electronics including optical bistability, self-focusing, superfluorescence, amplified spontaneous emission, and self-induced transparency. The technique and some examples of applications will be described.

Resonance Fluorescence in Mixed Coherent/Chaotic Fields

G. P. Hildred and R. K. Bullough
Department of Mathematics, UMIST, Manchester M60 1QD, England
S. S. Hassan
Ain Shams University, Faculty of Science, Applied Mathematics, Cairo, Egypt

PACS: 42.80

As part of a general programme investigating the quantum properties of N 2-level atoms of transition frequency ω_0 driven by an incident field we have adapted Agarwal's model master equation for the reduced density matrix [1] to the case when the driving field is a mixed coherent/chaotic field. For $N = 1$ results will be compared against a more fundamental analysis [2] which avoids the decorrelation of matter and field operators implicit in the model master equation.
For $N = 1$ we have solved the master equation *exactly* for a single mode coherent state/broad band thermal field within Markoff, Born, and rotating wave approximations. The single coherent mode has frequency ω_L. Expressions are obtained for the transient and steady state atomic inversion and dipole moment; the total scattered radiation intensity and the elastically scattered radiation

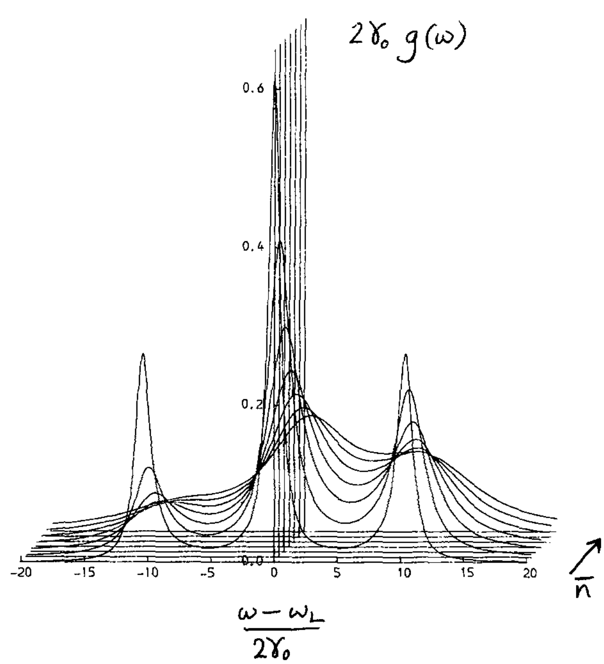

Fig. 1. Fluorescent spectrum for $\Omega/\gamma_0 = 5$, $\Delta/\gamma_0 = 3$, $\bar{n} = 0, 0.5, 1, 1.5, 2, 2.5, 3$

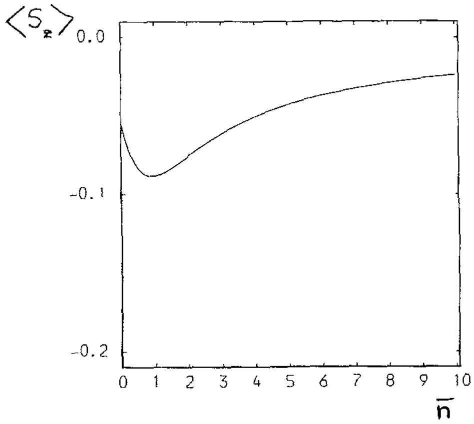

Fig. 2. Steady state $\langle S_z \rangle$ for $\Omega/\gamma_0=1$, $\Delta/\gamma_0=0$ versus $\bar{n}$

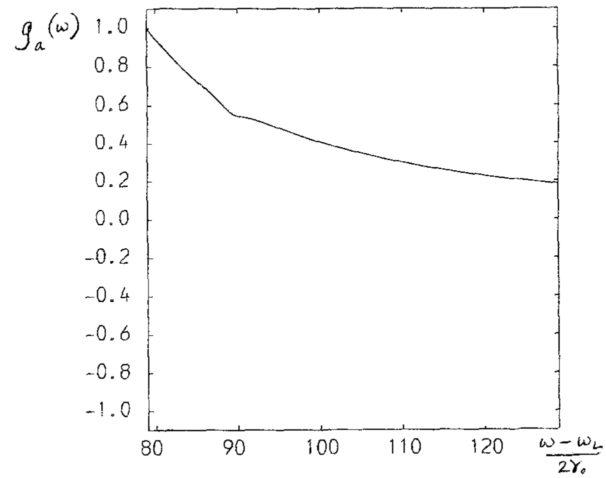

Fig. 3. Absorption spectrum of a weak probe field for $\Omega/\gamma_0=20$, $\Delta/\gamma_0=20$, $\bar{n}=0$, $N=2$

intensity; the intensity autocorrelation function and the power spectrum of the inelastically scattered radiation. The cubic equation, whose roots determine the separation and linewidths of the Stark splitting, is found to be identical to the case of a coherent driving field [3] except for the replacement of the Einstein A-coefficient, $2\gamma_0$, by an effective decay coefficient $2\gamma_0(1+2\bar{n})$ where $\bar{n}$ is the mean photon number of the broad band thermal field at the atomic transition frequency. The Stark spectrum for strong coherent fields is asymmetric (Fig. 1) when the laser-atom detuning $\Delta=(\omega_L-\omega_0)$ and $\bar{n}$ are nonzero; the asymmetries are compared with those in the phase diffusion model of Kimble and Mandel [4] and the previous results [2]. The ratio of the peak heights of the Stark spectrum is found to be 1:3:1 when $\Omega^2>[\gamma_0(1+2\bar{n})]^2$ and $\Delta=0$; Ω is the laser Rabi frequency. In the steady state we find that for certain values of Ω, Δ, and γ_0 the atomic inversion and the total and inelastically scattered radiation intensities initially decrease with increasing $\bar{n}$ and pass through a minimum before increasing to their saturation values as $\bar{n}\to\infty$ (Fig. 2). In contrast the elastically scattered radiation intensity decreases monotonically with increasing $\bar{n}$ – a feature explained by the two different interactions between the atoms and the incident field. The corresponding master equation for $N \geqq 2$ identical two-level atoms driven by both a single-mode coherent field and a broad band thermal field has been solved numerically under the Markoff, Born, and rotating wave approximations and neglecting dipole-dipole interactions (Dicke model [5]). Graphical comparisons are made between the numerical results for N atoms and the analytical results for a single atom in order to reveal co-operative atomic effects in the N atome case.

The effect of $\bar{n}$ and Δ on the first pair of additional side bands in the Stark spectrum and on the dips (Fig. 3) in the absorption spectrum of a weak probe field which arise at frequencies $\omega=\omega_0 \pm \sqrt{(4\Omega)^2+(2\Delta)^2}$ in both cases is examined. Present results for $N=2$ (Fig. 3) are expected to show enhanced dips and amplification of the weak probe for large enough N and weak enough chaotic field.

1. G.S. Agarwal: Springer Tracts in Modern Physics, Vol. 70 (Springer, Berlin, Heidelberg, New York 1974) pp. 105–107
2. See discussion in Phil. Trans. Roy. Soc. A **293**, 232–236 (1979)
3. B.R. Mollow: Phys. Rev. **188**, 1969 (1969)
4. H.J. Kimble, L. Mandel: Phys. Rev. **13**, 2123 (1976); **15**, 689 (1979)
5. See Hassan et al.: Appl. Phys. B **28**, (in press, 1982)