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Collimation of Atomic Beams by Resonant Laser Radiation Pressure

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Abstract. The application of the resonant light pressure created by an axially symmetrical light field for collimating atomic beams has been considered. As an example, consideration is given to the possibility of collimating an atomic beam by the light field produced by the reflection of a plane wave from the internal surface of a metal cone. It has been shown that the radiation pressure can reduce the atomic-beam transverse velocities to the value of the order of 100 cm/s which corresponds to effective temperature of about 10^{-3} K. A method for producing collimated beams of cold atoms based on simultaneous deceleration and collimation of atomic beams by resonant laser radiation pressure is proposed.

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It has recently been shown that the use of resonant light enables effective control of the velocity distribution of atomic ensembles. One important example of such a control applied to a beam of neutral atoms is deceleration and monochromatization of atoms along the beam axis [1–6]. Due to the use of the velocity monochromatization effect it is possible to produce beams of cold atoms with longitudinal velocities from 10^2 to 10^3 cm/s which correspond to effective temperatures from 10^{-1} to 10^{-3} K [2–6].

The purpose of our paper is to consider theoretically another method of control of the atomic motion in a beam, consisting of collimation of the atomic beam by the radiation pressure produced by an axially symmetrical field. The possibility of reducing the atomic-beam transverse velocities through two-dimensional radiative cooling was first pointed out in [7].

1. Basic Idea

The idea of the proposed radiation collimation of atomic beams is based on the use of resonant light-pressure forces, acted on atoms in axial-symmetric light fields, to reduce the atomic velocities across the beam axis. One of the potential schemes of a radiation collimator is shown in Fig. 1a. In this scheme the beam of two-level atoms 2 coming out of the source 1 is irradiated by the axial-symmetric light field 3, the frequency ω of which is red-shifted about the atomic transition frequency ω_0. The axial-symmetric field is created by the reflection of the plane light wave 5 from the mirror's conical surface 4 (a reflecting axicone [8, 9]).

Physically the radiation collimation consists of the following. In the axially symmetric field formed by the reflecting axicone atom is acted on by the light pressure force which, for $\omega < \omega_0$, is directed to the cone axis. Due to this force in the region of the axially symmetric field the transverse-velocity distribution of atoms is strongly narrowed. This, in its turn, reduces the angular divergence of the atomic beam drastically, i.e. beam collimation occurs.

2. Field Distribution

Consider as an example of an axially symmetric light field the field inside the reflective axicone (Fig. 1a). The external radiation coming into the conical surface $\varrho = z$ is supposed to be a plane travelling light wave

$$\mathbf{E}^i = \mathbf{e}\mathscr{E}_0 \cos(\omega t + kz), \tag{1}$$

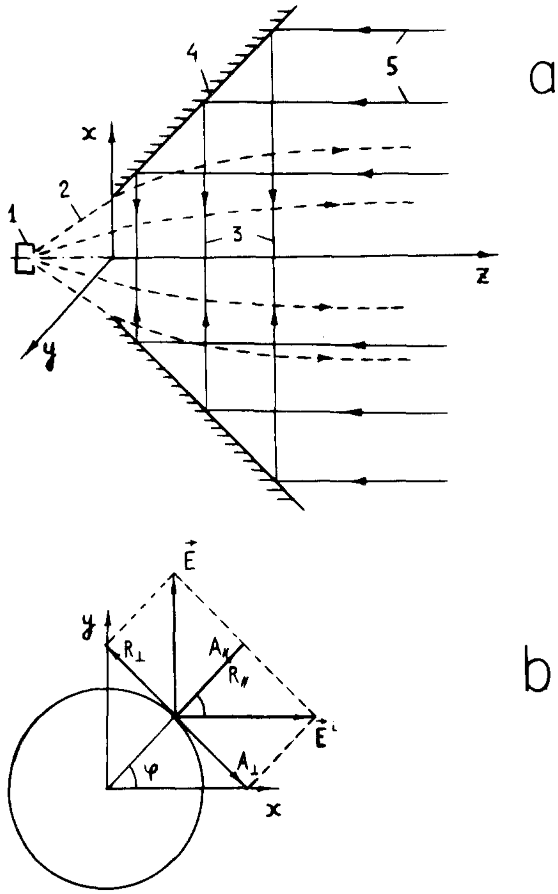

Fig. 1. (a) Scheme of a radiative collimator for an atomic beam. The angle of the vertex of the cone is 90°. (b) The reflection of the light-field vector $\mathbf{E}^i$ from a conical surface. The cone is cut by the plane xy

where $\mathbf{e}$ is the polarization unit vector, $\mathbf{k}=-k\mathbf{e}_z$ is the wave vector, $\mathbf{e}_z$ is the unit vector along the z axis. Let $A_{\parallel}$ and $A_{\perp}$ be the components of the vector $\mathbf{E}^i$ parallel and perpendicular to the reflecting surface, $R_{\parallel}$ and $R_{\perp}$ determine, respectively, the reflected field components. When the reflection coefficient is 1, the following relations apply [10]

$$R_{\parallel}=A_{\parallel}, R_{\perp}=-A_{\perp}. \tag{2}$$

Let, for definiteness, the incident radiation be linearly polarized along the x axis ($\mathbf{e}=\mathbf{e}_x$). Then, using the boundary conditions (2) one can find the components of reflected field $\mathbf{E}$ (Fig. 1b) near the conic surface

$$\begin{aligned} E_x&=E_x^i\cos 2\varphi\,, \\ E_y&=E_y^i\sin 2\varphi\,, \end{aligned} \tag{3}$$

where φ is the angle between the plane of incidence and the x axis.
The solution of the wave equation

$$\Delta\mathbf{E}-e^{-2}\partial^2\mathbf{E}/\partial t^2=0 \tag{4}$$

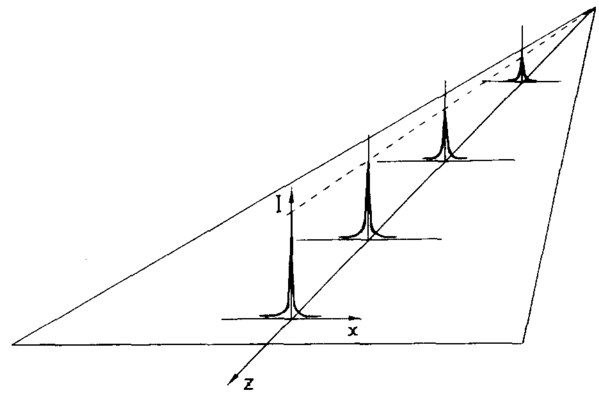

Fig. 2. The distribution of light intensity inside a reflecting axicone

satisfying (3) can be written in the form

$$\begin{aligned} E_x&=E\cos 2\varphi\cos\omega t\,, \\ E_y&=E\sin 2\varphi\cos\omega t\,, \end{aligned} \tag{5}$$

where the amplitude E is an unknown function of the cylindrical coordinates z and ϱ. When (5) is substituted into (4), we get

$$\varrho^{-1}\partial/\partial\varrho(\varrho\partial E/\partial\varrho)+(k^2-4\varrho^{-2})E+\partial^2E/\partial z^2=0\,. \tag{6}$$

The contribution of the last term to (6) is essential only for $z\sim\bar{\lambda}$. Since the coordinates $z\gg\bar{\lambda}$ are of interest, this term can be neglected. In this approximation the solution of (6) is

$$E=CJ_2(\varrho k)\,, \tag{7}$$

where $J_2(\varrho k)$ is the Bessel function, C is the constant depended on the boundary conditions (2). After having determined C, the field inside axicone can be written in a final form

$$\mathbf{E}=\mathbf{e}2(\pi kz)^{1/2}\mathscr{E}_0J_2(\varrho k)\cos\omega t\,. \tag{8}$$

Since the atomic-beam diameter is several orders larger than the wavelength ($\bar{\lambda}\sim 10\,\mathrm{cm}^{-5}$), the small vicinity in the centre of the axicone ($\varrho\sim\bar{\lambda}$) can usually be neglected in (8).
For $\varrho\gg\bar{\lambda}$ the collimating field is

$$\begin{aligned} \mathbf{E}&=\mathbf{e}2\mathscr{E}_0(2z/\varrho)^{1/2}\cos(\varrho\mathbf{e}_\varrho k-5\pi/4)\cos\omega t \\ &=\mathbf{e}\mathscr{E}_0(2z/\varrho)^{1/2}[\cos(\omega t+\varrho\mathbf{e}_\varrho k-5\pi/4) \\ &\quad+\cos(\omega t-\varrho\mathbf{e}_\varrho k+5\pi/4)]\,, \end{aligned} \tag{9}$$

where $\mathbf{e}_\varrho$ is the unit vector corresponding to the cylindrical coordinate ϱ. Thus for $\varrho\gg\bar{\lambda}$ the axially symmetric light field inside the reflecting axicone is a standing cylindrical wave or the sum of counter-propagating cylindrical waves, phase shifted by the value $\pi/2$. The intensity of the field (9) for $\varrho\gg\bar{\lambda}$ is proportional to z/ϱ

$$I \sim (z/\varrho)\mathscr{E}_0^2 .$$

The dependence of the intensity of the axially symmetric field on the transverse coordinate ϱ calculated from (8) is shown in Fig. 2.

3. Atomic-Beam Collimation

The degree of atomic-beam collimation can be estimated from the equations of motion in the field, (9). A two-level atom in field is acted on by the light-pressure force which in a rate equation approximation is [11]

$$\mathbf{F}_\varrho = -\mathbf{e}_\varrho k\hbar\gamma G(\mathscr{L}_- - \mathscr{L}_+)\,[1+G(\mathscr{L}_- + \mathscr{L}_+)]^{-1}, \quad (10)$$

where $\mathscr{L}_\pm$ are the Lorentzian functions of the radial velocity

$$\mathscr{L}_\pm = \gamma^2[(\Omega \pm kv_\varrho)^2 + \gamma^2]^{-1},$$

$G = z\varrho^{-1}(d\mathscr{E}_0/\hbar\gamma)^2$ is the saturation parameter, d is the matrix element of the dipole moment, 2γ is the natural linewidth, $\Omega = \omega - \omega_0$ is the detuning of the field frequency relative the atomic transition frequency ω_0. The dependence of the light pressure force F_ϱ on v_ϱ for a small saturation parameter $G \lesssim 1$ is shown in Fig. 3a. The evolution of atomic velocity in the field (9) depends on both the radiation pressure, which narrows the transverse velocity distribution $w(v_\varrho)$ (Fig. 3b), and the atomic momentum diffusion characterized by the diffusion tensor [11, 12]. For estimation, we write down the longitudinal and the transverse diffusion coefficients as an order of magnitude

$$D_{tr}, D_{zz} \simeq 2^{-1}\hbar^2 k^2 \gamma G \cdot (\mathscr{L}_- + \mathscr{L}_+)\,[1+G(\mathscr{L}_- + \mathscr{L}_+)]^{-1} . \quad (11)$$

Assuming that the distribution $w(v_\varrho)$ is narrow, the values F_ϱ and D_{tr} can be expanded in power series in v_ϱ near the point $v_\varrho = 0$. After expansion the force and the diffusion coefficients near the point $v_\varrho = 0$ are equal to

$$F_\varrho = 4\hbar k^2(\Omega/\gamma)G(1+\Omega^2/\gamma^2)^{-1}(1+\Omega^2/\gamma^2+2G)^{-1}v_\varrho = -\beta_\varrho v_\varrho , \quad (12a)$$

$$D_{tr}, D_{zz} = \hbar^2 k^2 \gamma G(1+\Omega^2/\gamma^2+2G)^{-1} . \quad (12b)$$

The light-pressure force for small velocities, according to (12a), has the meaning of a damping force. The inverse value of the damping coefficient determines the characteristic time during which a steady-state velocity distribution $w(v_\varrho)$ is reached [13]

$$\tau_{st} = M\beta_\varrho^{-1} \sim \hbar R^{-1} , \quad (13)$$

where $R = \hbar^2 k^2/2M$ is the recoil energy.
The width of the steady-state velocity distribution $w(v_\varrho)$, for $t \gg \tau_{st}$, is determined by the effective trans-

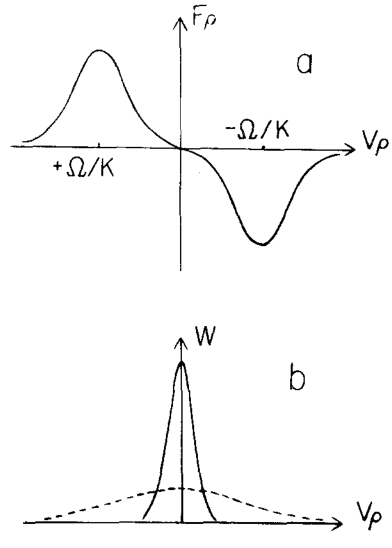

Fig. 3. (a) The light-pressure force F_ϱ as a function of the transverse velocity v_ϱ for a standing light wave. (b) The narrowing of the transverse velocity distribution $w(v_\varrho)$ as the atomic beam passes through the radiative collimator. The dashed line denotes the initial distribution. The solid curve illustrates the distribution at times $t \gg \tau_{st}$

verse temperature [13]

$$T^0_{tr} = D_{tr}/k_B\beta_\varrho = (\hbar\gamma/4k_B)\,(|\Omega|/\gamma + \gamma/|\Omega|) . \quad (14)$$

The minimum transverse temperature is reached at $\Omega = -\gamma$

$$T^{\min}_{tr} \approx \hbar\gamma/k_B . \quad (15)$$

Using (14) it is possible to find the collimation angle near the point where beam comes out of the axicone

$$\theta = (2k_B T^0_{tr}/M)^{1/2}/\bar{v}_z , \quad (16)$$

where $\bar{v}_z$ is the average atomic velocity along the z axis. The ultimate value of the collimation angle according to (15) equals

$$\theta^{\min} = (2\hbar\gamma/M)^{1/2}/\bar{v}_z . \quad (17)$$

For a beam of Na^{23} atoms irradiated by laser radiation on the transition $3S-3P$ ($\gamma = \pi \cdot 10^7\,s^{-1}$) and for $\bar{v}_z = 5 \times 10^4$ cm/s the minimum collimation angle $\theta^{\min}$ is 10^{-3}. These numerical estimations show conclusively that using an axially symmetric light field it is possible to decrease effectively the transverse velocities of the atomic beam.
Consider now the spatial atomic diffusion transverse to the z axis. The diffusion coefficient characterizing the atomic diffusion across the z axis can be estimated from (15) and (12) using the Einstein relation. Assuming the collimating intensity to be small ($G \lesssim 1$), we have

$$C_{tr} = k_B T^0_{tr}\beta_e^{-1} \approx \gamma\bar{\lambda}^2 . \quad (18)$$

The beam divergence arised due to spatial diffusion during the interaction time $t_{int}=z/\bar{v}_z$ is

$$\sigma_{tr}=(C_{tr}t_{int})^{1/2}z^{-1}\approx\bar{\lambda}(\gamma/z\bar{v}_z)^{1/2}. \quad (19)$$

For the case of sodium atoms and for $z=10$ cm, $\lambda=5890$ Å the value of σ_{tr} is equal to 7×10^{-5} and is small in comparison with θ^{min}. Thus, the numerical examples show that the irradiation of an atomic beam by an axially symmetric field (9) can be used to collimate the atomic beam and prevent the loss of atoms in the transverse direction.

4. Collimation and Cooling of an Atomic Beam

An atomic beam decelerated by the pressure of counter-propagating laser radiation always undergoes a transverse spread because of diffusion of atomic velocities in a light field [6]. As a result, the width of the atomic beam in the process of its deceleration is increased by 10 to 10^2 times [14]. Using along with the decelerating laser beam an axially symmetric light field, the wave vectors of which are directed perpendicular to the atomic beam (Fig. 1a), one can reduce considerably the atomic beam defocusing. Let the axially symmetric field be formed by a reflecting axicone and the decelerating light field is given in the form of a plane running light wave

$$\mathbf{E}_{dec}=\mathbf{e}\mathscr{E}_1\cos(\omega_1 t+kz). \quad (20)$$

The atoms in (20) are acted on by a light-pressure force directed along the z axis [15]

$$\mathbf{F}_z=-\mathbf{e}_z\hbar k\gamma G_1[1+G_1+(\Omega_1+kv_z)^2\gamma^{-2}]^{-1}, \quad (21)$$

where $\Omega_1=\omega_1-\omega_0$ is the frequency detuning about the atomic transition frequency ω_0. $G_1=(1/2)(d\mathscr{E}_1/\hbar\gamma)^2$ is the saturation parameter for the decelerating light field. The diffusion of atomic momenta (20) is described by the diffusion tensor [12]. The longitudinal and transverse diffusion coefficients are equal of the order of magnitude

$$D^1_{tr}, D^1_{zz}\simeq 2^{-1}\hbar^2k^2\gamma G_1[1+G_1 +(\Omega_1+kv_z)^2\gamma^{-2}]^{-1}. \quad (22)$$

The diffusion coefficient D^1_{tr} is responsible for the transverse spread of atomic beam. The coefficient D^1_{zz} characterizes the atomic momentum diffusion along the z axis. As a result of joint action of radiation pressure and longitudinal momentum diffusion a steady-state narrow velocity distribution around the average velocity $\langle v_z\rangle$ is formed for times $t\gg\tau_{st}$. The value $\langle v_z\rangle$ satisfies the condition [16]

$$|k\langle v_z\rangle+\Omega_1|\gamma^{-1}\gg(1+G_1)^{1/2}, \quad (23)$$

which means that the change of atomic velocity in the process of velocity monochromatization exceeds the characteristic velocity interval due to the light-pressure force. The force F_z and the diffusion coefficients D^1_{zz} and D^1_{tr} for a velocity-monochromatized atomic beam can be expanded in the powers of the local velocity $u=v_z-\langle v_z\rangle$

$$\begin{aligned}F_z&=-\hbar k\gamma L_0+2\hbar k^2\gamma L_0(k\langle v_z\rangle+\Omega_1)^{-1}u\\&=-\hbar k\gamma L_0-\beta_z u,\\D^1_{zz}, D^1_{tr}&=2^{-1}\hbar^2k^2\gamma L_0,\end{aligned} \quad (24)$$

where

$$L_0=G_1\gamma^2(k\langle v_z\rangle+\Omega_1)^{-2}.$$

In (24) the first term $-\hbar k\gamma L_0$ is the average force which decelerates the atomic ensemble and the second term is the damping force which narrows the longitudinal velocity distribution. If a collimating light field, (9), is added to the decelerating laser radiation (20), at times $t\gg\tau_{st}$ a narrow velocity distribution will be formed in all directions. Assuming that the joint action of (9) and (20) is additive, we estimate the width of the monochromatized velocity distribution from a formula analogous to (14)

$$T_{tr}=(D_{tr}+D^1_{tr})/k_B\beta_\varrho,\ T_z=(D_{zz}+D^1_{zz})/k_B\beta_z. \quad (25)$$

Taking into account (12) and (24), we get finally

$$T_{tr}=T^0_{tr}(1+b/2), \quad (26a)$$

$$T_z=T^0_z(1+2/b). \quad (26b)$$

Here T^0_z is the longitudinal temperature of atoms that, without an axial-symmetric field (9) is equal to [16]

$$T^0_z\simeq(\hbar/4k_B)|k\langle v_z\rangle+\Omega_1|. \quad (27)$$

The parameter b is

$$\begin{aligned}b&=G_1[\Omega^2+(1+2G)\gamma^2]/G(k\langle v_z\rangle+\Omega_1)^2\\&\approx(G_1/G)(\Omega/\Omega_1)^2.\end{aligned} \quad (28)$$

Since the transverse temperature is linearly dependent on b, and T_z is inversely proportional to b, the optimal value of b is about unity.

The efficiency of radiative collimation can be illustrated by comparing the expression for the transverse temperature of atoms, (26a), with that due to the transverse momentum diffusion in the absence an axially symmetric field [16]

$$T^d_{tr}\simeq(\hbar/k_B)|k\langle v_z\rangle+\Omega_1|. \quad (29)$$

As it seen from Fig. 4a, showing the dependence of T_{tr} and T^d_{tr} on the average velocity $\langle v_z\rangle$, the transverse temperature T^d_{tr} increases beyond all bounds and T_{tr} tends to the constant $\sim T^0_{tr}$.

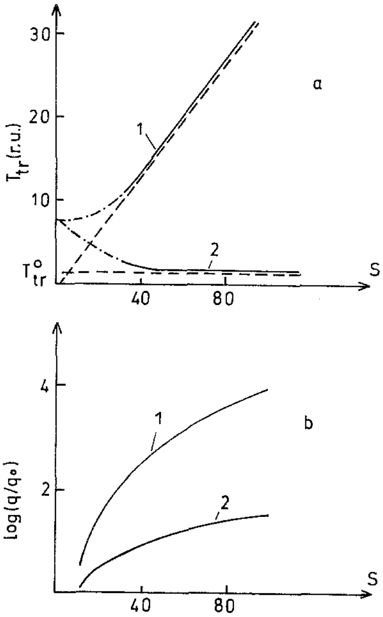

Fig. 4. (a) The transverce temperature of atomic beam as a function of the average atomic velocity $\langle v_z\rangle$ ($S=|k\langle v_z\rangle+\Omega_1|/\gamma$) for detuning $\Omega_1=-100\gamma$: 1-collimating laser field is switched off, 2-collimating field is switched on. The dashed lines show the asymptotical behaviour of the temperature. The point-dashed parts of the curves are not described by macroscopic theory. (b) The spatial width of atomic beam decelerated by laser radiation as a function of the average velocity for parameters $\Omega_1=-100\gamma$, $G_1=10^3$, $\hbar\gamma/R=10^3$, $\lambda=5890$ Å. 1-collimating field is switched off, 2-collimating field is switched on

In conclusion, from the Einstein relation we can estimate the radius of the atomic beam decelerated and collimated by the pressure due to the resonant laser radiation during the time t_{int}

$$q=(t_{\text{int}}k_B T_{tr}/\beta_\varrho)^{1/2}. \tag{30}$$

Assuming that the time t_{int} is equal to the deceleration time of the atomic beam, t_{dec}, [16]

$$t_{\text{dec}}=2\hbar R^{-1}|\Omega_1+k\langle v_z\rangle|^3/3\gamma^3G_1, \tag{31}$$

and the transverse temperature T_{tr} is equal to $T_{tr}^{\min}$ we find the minimum radius of the decelerated atomic beam

$$q^{\min}=\lambda\!\!\!^{-}\gamma^{-1}|\Omega_1+k\langle v_z\rangle|\,[\hbar|\Omega_1+k\langle v_z\rangle|/6RG_1]^{1/2}. \tag{32}$$

For comparison, one can estimate the cross dimension of an atomic beam decelerated without collimation. The beam width determined by the atomic momentum diffusion in (20) can be calculated as the distance covered by a particle in the time t_{dec} with the velocity $(2k_BT_{tr}^d/M)^{1/2}$

$$q^d\simeq\lambda\!\!\!^{-}\gamma^{-3}|\Omega_1+k\langle v_z\rangle|^3[\hbar|\Omega_1+k\langle v_z\rangle|/R]^{1/2}/3G_1. \tag{33}$$

Figure 4b shows the dependence of $q^{\min}$ and q^d on the average velocity $\langle v_z\rangle$. As it can be seen, the axial-symmetric field decreases considerably the spatial diffusive spread of the decelerated atomic beam.

Thus, the above consideration shows that the use of the radiation pressure on atomic beams enables two problems to be solved. First, to reduce the transverse temperature of an atomic beam up to the ultimate value (15). Second, to attain a minimum divergence of an atomic beam.

References

1. V.I. Balykin, V.S. Letokhov, V.I. Mishin: Pis'ma Zh. Eksp. Teor. Fiz. (in Russian) **29**, 561 (1979); Zh. Eksp. Teor. Fiz. (in Russian) **51**, 692 (1980)
2. V.I. Balykin, V.S. Letokhov, V.G. Minogin: Zh. Eksp. Teor. Fiz. (in Russian) **80**, 1779 (1981)
3. S.V. Andreev, V.I. Balykin, V.S. Letokhov, V.G. Minogin: Pis'ma Zh. Eksp. Teor. Fiz. (in Russian) **34**, 463 (1981); Zh. Eksp. Teor. Fiz. (in Russian) **82**, 1429 (1982)
4. W.D. Phillips, H. Metcalf: Phys. Rev. Lett. **48**, 596 (1982)
5. J.V. Prodan, W.D. Phillips, H. Metcalf: Phys. Rev. Lett. **49**, 1149 (1982)
6. V.I. Balykin, V.S. Letokhov, V.I. Sidorov: Zh. Eksp. Teor. Fiz. (in Russian) **86**, 2019 (1984)
7. T.W. Hänsch, A.L. Schawlow: Opt. Commun. **13**, 68 (1975)
8. S. Fujiwara: J. Opt. Soc. Am. **52**, 287 (1962)
9. M. Riox, R. Tremblay, P.A. Be'langer: Appl. Opt. **17**, 1532 (1978)
10. M. Born, E. Wolf: *Principles of Optics* (Pergamon Press, Oxford 1964)
11. V.G. Minogin: Zh. Eksp. Teor. Fiz. (in Russian) **80**, 2231 (1981)
12. V.G. Minogin: Zh. Eksp. Teor. Fiz. (in Russian) **79**, 2044 (1980)
13. S. Chandrasekhar: Rev. Mod. Phys. **15**, 1 (1943)
14. T.V. Zueva: Kvantovaya Electron (in Russian) (in press, 1984)
15. A. Ashkin: Phys. Rev. Lett. **25**, 1321 (1970)
16. V.S. Letokhov, V.G. Minogin, T.V. Zueva: Opt. Commun. **38**, 225 (1981); Zh. Eksp. Teor. Fiz. (in Russian) **81**, 84 (1981)