Appl. Phys. B 27, 99–104 (1982)

Applied Physics B Photophysics and Laser Chemistry


# Source Encoding for Partially Coherent Optical Processing

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Received 10 September 1981/Accepted 2 October 1981

**Abstract.** A relation between spatial coherence function and source encoding intensity transmittance function is presented. Since the spatial coherence is depending upon the information processing operation, a strictly broad spatial coherence function may not be required for the processing. The advantage of the source encoding is to relax the constraints of strict coherence requirement, so that the processing operation can be carried out with an extended incoherent source. Emphasis of the source encodings and experimental demonstrations are given. The constraint of temperal coherence requirement is also discussed.

**PACS:** 42.30, 42.80

The use of coherent light enables optical processing systems to carry out many sophisticated information processing operations [1, 2]. However, coherent optical processing systems are contaminated with coherent artifact noise, which frequently limits their processing capabilities. Recently, attempts of using an incoherent source to carry out complex information processing operations had been pursued by several investigators [3–6]. The basic limitations of using incoherent source for partially coherent processing is the extended source size. To achieve a broad spatial coherence function at the input plane of an optical information processor, a very small source size is required. However, such a small light source is difficult to obtain in practice. We have, nevertheless, shown in recent published papers [7–10] that there are information processing operations which can be carried out with incoherent source. In other words, a strictly broad coherence requirement may not be needed for some optical information processing operations.

In this paper, we shall describe a linear transformation relationship between spatial coherence function and source encoding intensity transmittance function. Since the spatial coherence requirement is depending upon the information processing operation, a more relaxed coherence function may be used for a specific processing operation. By Fourier transforming this coherence function, a source encoding intensity transmittance function may be found.

The purpose of source encoding is to reduce the coherent requirement, so that an extended incoherent source can be used for the processing. In other words, the source encoding technique is capable of generating an appropriate coherence function for a specific information processing operation and at the same time it utilizes the available light power more effectively. We shall illustrate examples that complex information processing operation can actually be carried out by an encoded extended incoherent source. Experimental illustrations with this source encoding technique are also included.

## Source Encoding with Spatial Coherence

We shall begin our discussion with the Young's experiment under extended incoherent source illumination, as shown in Fig. 1. First, we assume that a narrow slit is placed at plane $P_1$ behind an extended source. To maintain a high degree of spatial coherence between

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0721-7269/82/0027/0099/$01.20

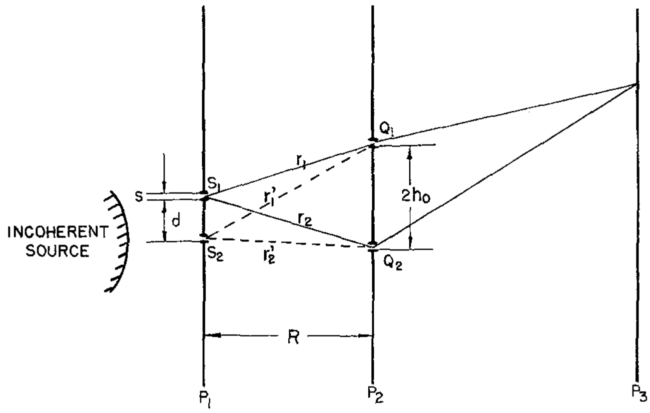


Fig. 1. Young's experiment with extended source illumination

the slits $Q_1$ and $Q_2$ at $P_2$, it is known that the source size should be very narrow. If the separation between $Q_1$ and $Q_2$ is large, then a narrower slit size $S_1$ is required. Thus, to maintain a high degree of spatial coherence between $Q_1$ and $Q_2$, the slit width should be [11]

$$w \leqq \frac{\lambda R}{2h_0}, \tag{1}$$

where $R$ is the distance between planes $P_1$ and $P_2$, and $2h_0$ is the separation between $Q_1$ and $Q_2$ (Fig. 1).

Let us now consider two narrow slits of $S_1$ and $S_2$ located in source plane $P_1$. We assume that the separation between $S_1$ and $S_2$ satisfied the following path length relation:

$$r_1' - r_2' = (r_1 - r_2) + m\lambda, \tag{2}$$

where the $r$'s are the respective distances from $S_1$ and $S_2$ to $Q_1$ and $Q_2$, as shown in the figure. $m$ is an arbitrary integer, and $\lambda$ is the wavelength of the extended source. Then the interference fringes due to each of the two source slits $S_1$ and $S_2$ would be in phase. A brighter fringe pattern can be seen at plane $P_3$. To further increase the intensity of the fringe pattern, one would simply increase the number of source slits in appropriate locations in the source plane $P_1$ such that every separation between slits satisfied the coherence or fringe condition of (2). If separation $R$ is large, i.e., $R \gg d$ and $R \gg 2h_0$, then the spacing $d$ between the source slits becomes,

$$d = m\frac{\lambda R}{2h_0}. \tag{3}$$

From the above illustration, we see that by properly encoding an extended source, it is possible to maintain the spatial coherence between $Q_1$ and $Q_2$, and at the same time it increases the intensity of illumination. Thus, with a specific source encoding technique for a given information processing operation may result a better utilization of an extended source.

To encode an extended source, we would first search for a spatial coherence function for an information processing operation. With reference to an extended source optical processor of Fig. 2, the spatial coherence function at input plane $P_2$ can be written [11]

$$\Gamma(\mathbf{x}_2, \mathbf{x}_2') = \iint S(\mathbf{x}_1) K_1(\mathbf{x}_1, \mathbf{x}_2) K_1(\mathbf{x}_1, \mathbf{x}_2')\, d\mathbf{x}_1, \tag{4}$$

where the integration is over the source plane $P_1$, $S(\mathbf{x}_1)$ is the intensity transmittance function of a source encoding mask, and $K_1(\mathbf{x}_1, \mathbf{x}_2)$ is the transmittance function between source Plane $P_1$ the input plane $P_2$, which can be written

$$K_1(\mathbf{x}_1, \mathbf{x}_2) \simeq \exp\left[i\left(2\pi\frac{\mathbf{x}_1\mathbf{x}_2}{\lambda f}\right)\right]. \tag{5}$$

By substituting $K_1(\mathbf{x}_1, \mathbf{x}_2)$ into (4), we have

$$\Gamma(\mathbf{x}_2 - \mathbf{x}_2') = \iint s(\mathbf{x}_1) \exp\left[i2\pi\frac{\mathbf{x}_1}{\lambda f}(\mathbf{x}_2 - \mathbf{x}_2')\right] d\mathbf{x}_1. \tag{6}$$

From the above equation, we see that the spatial coherence function and source encoding intensity transmittance function forms a Fourier transform pair

$$s(\mathbf{x}_1) = \mathscr{F}[\Gamma(\mathbf{x}_2 - \mathbf{x}_2')], \tag{7}$$

and

$$\Gamma(\mathbf{x}_2 - \mathbf{x}_2') = \mathscr{F}^{-1}[s(\mathbf{x}_1)], \tag{8}$$

where $\mathscr{F}$ denotes the Fourier transformation operation. If a spatial coherence function for an information processing operation is provided, then the source encoding intensity transmittance function can

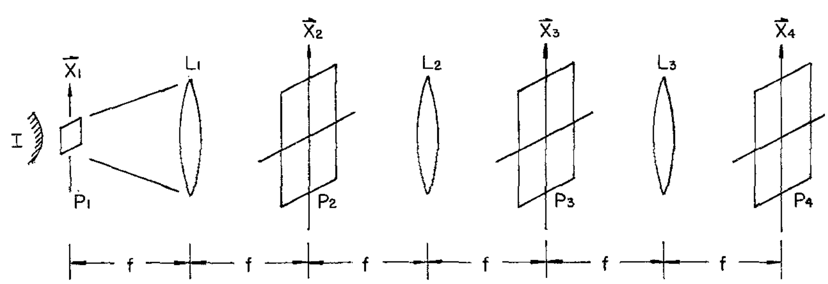


Fig. 2. Partially coherent optical processing with encoder extended incoherent source. (I: extended incoherent source, $L_1$: collimation lens, $L_2$ and $L_3$: transform lenses)

be determined through Fourier transformation of (7). We note that the source encoding function $S(\mathbf{x}_1)$ can consist of apertures or slits of any shape. We further note that in practice $S(\mathbf{x}_1)$ should be a positive real function which satisfies the following physical realizable condition:

$$0 \leqq S(\mathbf{x}_1) \leqq 1 . \tag{9}$$

For example, if a spatial coherence function for an information processing operation is

$$\Gamma(x_2 - x_2') = \text{rect}\left\{\frac{|x_2 - x_2'|}{A}\right\}, \tag{10}$$

where $A$ is an arbitrary positive constant, and

$$\text{rect}\left\{\frac{x}{A}\right\} = \begin{cases} 1, & |x| \leqq A, \\ 0, & \text{otherwise}, \end{cases}$$

then the source encoding intensity transmittance would be

$$S(x_1) = \text{sinc}\left(\frac{\pi A x_1}{\lambda f}\right). \tag{11}$$

Since $S(x_1)$ is a bipolar function, therefore it is not physically realizable.

## Temporal Coherence Requirement

There is, however, a temporal coherence requirement for incoherent source. In optical information processing operation, the scale of the Fourier spectrum varies with wavelength of the light source. Therefore, a temporal coherence requirement should be imposed on every processing operation. If we restrict the Fourier spectra, due to wavelength spread, within a small fraction of the fringe spacing $d$ of a complex spatial filter (e.g., deblurring filter), then we have,

$$\frac{P_m f \Delta\lambda}{2\pi} \ll d, \tag{12}$$

where $1/d$ is the highest spatial frequency of the filter, $P_m$ is the angular spatial frequency limit of the input object transparency, $f$ is the focal length of the transform lens, and $\Delta\lambda$ is the spectral bandwidth of the light source. The spectral width or the temporal coherence requirement of the light source is, therefore,

$$\frac{\Delta\lambda}{\lambda} \ll \frac{\pi}{h_0 P_m}, \tag{13}$$

where $\lambda$ is the center wavelength of the light source, $2h_0$ is the size of the input object transparency, and $2h_0 = (\lambda f)/d$.

In order to gain some feeling of magnitude, we provide a numerical example. Let us assume that the size of the object is $2h_0 = 5$ mm, the wavelength of the light source is $\lambda = 5461$ Å, and we take a factor 10 for (13) for consideration, that is

$$\Delta\lambda = \frac{10\pi\lambda}{h_0 P_m}. \tag{14}$$

Several values of spectral width requirement $\Delta\lambda$ for various spatial frequency $P_m$ are tabulated in Table 1.

Table 1. Source spectral requirement

| $\frac{P_m}{2\pi}$ [lines/mm] | 0.5 | 1 | 5 | 20 | 100 |
|---|---|---|---|---|---|
| $\Delta\lambda$ [Å] | 218.4 | 109.2 | 21.8 | 5.46 | 1.09 |

From Table 1, we see that, if the spatial frequency of the input object transparency is low, a broader spectral width of light source can be used. In other words, if higher spatial frequency is required for an information processing operation, then a narrower spectral width of light source is needed.

## Examples of Source Encoding

We shall now illustrate examples of source encoding for partially coherent processing operations. We would first consider the correlation detection operation [12].

In correlation detection, the spatial coherence requirement is determined by the size of the detecting object (i.e., signal). To insure a physically realizable encoded source transmittance function, we assume a spatial coherence function over the input plane $P_2$ is

$$\Gamma(|\mathbf{x}_2 - \mathbf{x}_1|) = \frac{J_1\left(\frac{\pi}{h_0}|\mathbf{x}_2 - \mathbf{x}_2'|\right)}{\frac{\pi}{h_0}|\mathbf{x}_2 - \mathbf{x}_2'|}, \tag{15}$$

where $J_1$ is a first-order Bessell function of first kind, and $h_0$ is the size of the detecting signal. A sketch of the spatial coherence as a function of $|x_2 - x_2'|$ is shown in Fig. 3a. By taking the Fourier transform of (15), we obtain the following source encoding intensity transmittance function,

$$S(|\mathbf{x}_1|) = \text{cir}\left\{\frac{|\mathbf{x}_1|}{w}\right\}, \tag{16}$$

where $w = (f\lambda)/h_0$ is the diameter of a circular aperture as shown in Fig. 3a,

$$\text{cir}\left\{\frac{|\mathbf{x}_1|}{w}\right\} \triangleq \begin{cases} 1, & 0 \leqq |\mathbf{x}_1| \leqq w \\ 0, & \text{otherwise}, \end{cases}$$

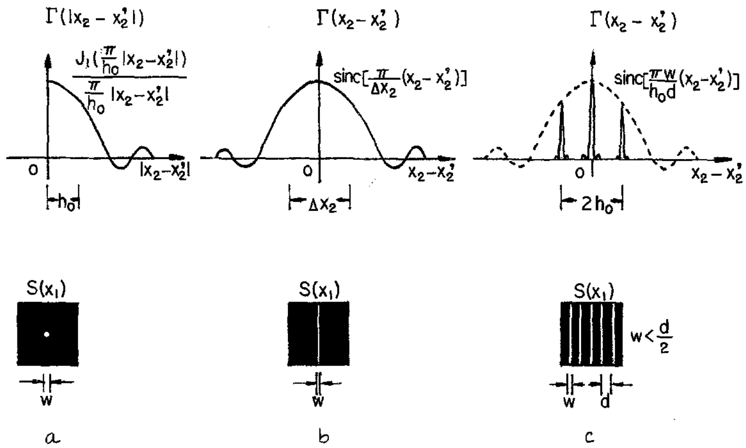


Fig. 3a–c. Examples of spatial coherence requirements and source encodings. [$\Gamma(x_2-x_2')$: spatial coherence function, $S(x_1)$: source encoding transmittance]. (a) For correlation detection, (b) for smeared image deblurring, and (c) for image subtraction

$f$ is the focal length of the collimating lens and $\lambda$ is the wavelength of the extended source. As a numerical example, we assume that the signal size is $h_0=5$ mm, the wavelength is $\lambda=5461$ Å, focal length is $f=300$ mm, then the diameter $w$ of the source encoding aperture should be about 32.8 μm or smaller.

We now consider smeared image deblurring [13] operation as our second example. We note that the smeared image deblurring is a 1−D processing operation and the inverse filtering is a point-by-point processing concept such that the operation is taking place on the smearing length of the blurred object. Thus, the spatial coherence requirement is depending upon the smearing length of the blurred object. To obtain a physically realizable source encoding function, we let the spatial coherence function at the input plane $P_2$ be

$$\Gamma(|x_2-x_2'|)=\operatorname{sinc}\left(\frac{\pi}{\Delta x_2}|x_2-x_2'|\right), \tag{17}$$

where $\Delta x_2$ is the smearing length. A sketch of (17) is shown in Fig. 3b. By taking the Fourier Transform of (17), we obtain

$$S(x_1)=\operatorname{rect}\left\{\frac{|x_1|}{w}\right\}, \tag{18}$$

where $w=(f\lambda)/(\Delta x_2)$ is the slit width of the source encoding aperture, as shown in Fig. 3b, and

$$\operatorname{rect}\left\{\frac{|x_1|}{w}\right\}=\begin{cases}1, & 0\leqq|x_1|\leqq w,\\ 0, & \text{otherwise}.\end{cases}$$

For a numerical illustration if the smearing length is $\Delta x_2=1$ mm, the wavelength is $\lambda=5461$ Å, and the focal length is $f=300$ mm, then the slit width $w$ should be about 163.8 μm or smaller.

We would now consider image subtraction [14] for our third illustration. Since the image subtraction is a 1−D processing operation and the spatial coherence requirement is depending upon the corresponding point-pair of the images, thus a strictly broad spatial coherence function is not required. In other words, if one can maintain the spatial coherence between the corresponding image points to be subtracted, then the subtraction operation can take place at the output image plane. Therefore, instead of utilizing a strictly broad coherence function over the input plane $P_2$, we would use a point-pair spatial coherence function. Again, to insure a physically realizable source-encoding transmittance, we would let the point-pair spatial coherence function be [10]

$$\Gamma(|x_2-x_2'|)=\frac{\sin\left(\frac{N\pi}{h_0}|x_2-x_2'|\right)}{N\sin\left(\frac{\pi}{h_0}|x_2-x_2'|\right)}\operatorname{sinc}\left(\frac{\pi w}{h_0 d}|x_2-x_2'|\right), \tag{19}$$

where $2h_0$ is the main separation of the two input object transparencies at plane $P_2$, $N\gg1$ a positive integer, and we note that $w\ll d$. Equation (19) represents a sequence of narrow pulses which occur at $|x_2-x_2'|=nh_0$, where $n$ is a positive integer, and their peak values are weighted by a broader sinc factor, as shown in Fig. 3c. Thus, we see that a high degree of spatial coherence is maintained at every point-pair between the two input object transparencies. By taking the Fourier transformation of (19), we obtain the following source encoding intensity transmittance

$$S(|x_1|)=\sum_{n=1}^{N}\operatorname{rect}\left\{\frac{|x_1-nd|}{w}\right\}, \tag{20}$$

where $w$ is the slit width, and $d=(\lambda f)/h_0$ is the separation between the slits. It is clear that (20) represents $N$ number of narrow slits with equal spacing $d$, as shown in Fig. 3c. As a numerical example, we let the separation of the input objects $h_0=10$ mm, the wavelength $\lambda=5461$ Å, the focal length of the collimator $f = 300$ mm, then the spacing $d$ between the slits is 16.4 μm. The slit width $w$ should be smaller than $d/2$, or about 1.5 μm. If the size of the encoding mask is 2 mm square, then the number of slits $N$ is about 122. Thus we see that with the source encoding it is possible to increase the intensity of the illumination $N$ fold, and at the same time it maintains the point-pair spatial coherence requirement for image subtraction operation.

## Experimental Results

In this section, we would illustrate two examples as obtained from the source encoding technique. The first experimental illustration is the result obtained for smeared photographic image deblurring with encoded incoherent source as shown in Fig. 4. In this experiment a Xenon arc lamp with a green interference filter was used as extended incoherent source. A single slit mask of about 100 μm was used as a source encoding mask. The smeared length of the burred image was about 1 mm.

Figure 5 shows an experimental result obtained from image subtraction operation with encoded incoherent source. In this experiment, a mercury arc lamp with a green filter was used as an extended incoherent source. A multislit mask was used to encode the light source. The slit width $w$ is 2.5 μm and the spacing between slits was 25 μm. The overall size of the source encoding mask was about $2.5 \times 2.5$ mm$^2$. The mask contains about 100 slits.

From these experimental results, we see that the constraint of strictly broad spatial coherence requirement may be alleviated with source encoding techniques so that it allows the optical information processing operation can be carried out with extended incoherent source.

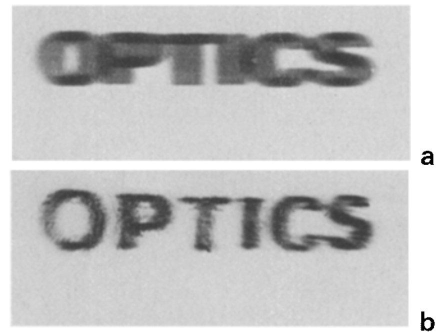


Fig. 4a and b. Photographic image deblurring with encoded extended incoherent source. (a) Input blurred object and (b) deblurred image

## Conclusion

We have derived a Fourier transform relationship between the spatial coherence function and the source encoding intensity transmittance function. Since the coherence requirement is depending upon the nature of a specific information processing operation, a strictly broad coherence requirement may not be needed in practice. The basic advantage of the source encoding technique is to alleviate the constraints of the strict coherence requirement imposed upon the optical information processing system, so that the information processing can be carried out with encoded extended incoherent source. The use of incoherent source to carry out the optical processing operation has the advantage of suppressing the coherent artifact noise. In addition, the incoherent processing system is usually simple and economical to operate. Finally, we would stress that the source encoding technique may be extended to white-light optical processing operation, a program is currently under investigation.

*Acknowledgement.* We wish to acknowledge the support of the U.S. Air Force Office of Scientific Research Grant AFOSR-81-0148.

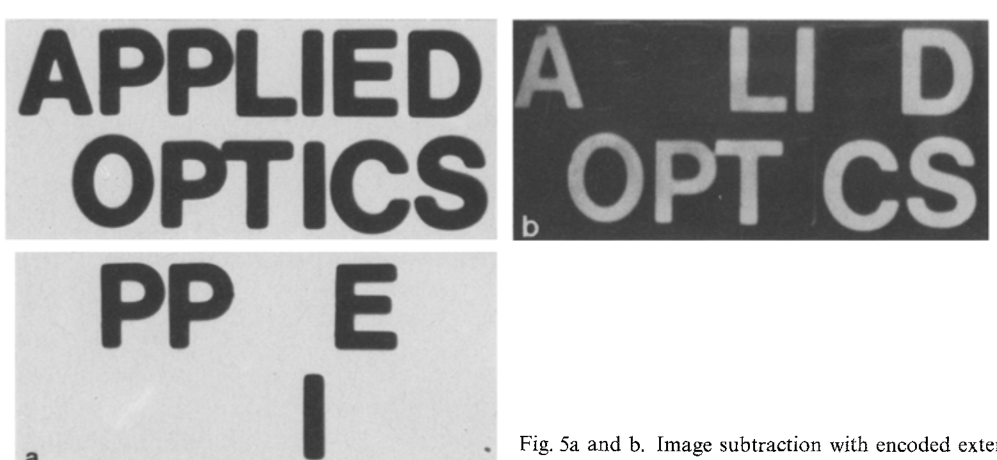


Fig. 5a and b. Image subtraction with encoded extended incoherent source. (a) Input object transparencies and (b) subtracted image

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