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Conditions for the Harmonic-like and Efficient Amplitude Modulation of the cw Gaussian Laser Beam by Means of a Mechanical Chopper

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Abstract. The values of the chopping disc slot and mark space widths relative to the radius of the Gaussian beam are found, which achieve optimum harmonic-like and efficient amplitude modulation of the cw laser beam. The simple approximation for the waveform of the modulated laser power valid for these optimum or near optimum values of the chopping disc parameters is presented.

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Amplitude modulation of the cw laser beam in the TEM_{00} mode by means of a mechanical chopper is frequently employed in photophysics, particularly in photoacoustic spectroscopy [1–4]. Harmonic-like modulation of the exciting laser radiation entering the photoacoustic cell is a desirable feature especially in the case of an acoustically resonant cell. It would therefore be convenient to have a criterion which, when fulfilled, ensures such a modulation. The analysis of the waveforms of the amplitude modulated normalized transmitted laser power presented in [5] shows that in certain cases the resulting waveforms are similar to harmonic function. Unfortunately, the demand for harmoniclike modulation taken alone leads to reduced laser power modulation depth. This is undesirable since the modulation efficiency is also an important experimental requirement. It is the purpose of the present paper to combine these two conflicting requirements with the model given in [5], which reproduces accurately and conveniently the waveform of the chopped cw laser light in the TEM_{00} mode. Such a treatment (Sect. 1) results in conditions of practical interest, which must be fulfilled if both imposed conditions are to be achieved in a manner satisfactory for experimental needs. Additionally, in Sect. 2 we present a simple approximation for the waveform of the normalized laser power valid when conditions for harmonic-like and efficient modulation are fulfilled or nearly fulfilled.

1. Theory

Consider a cw laser light in the TEM_{00} mode propagating along the direction normal to the plane containing moving parallel slots and mark spaces [Ref. 5, Fig. 2]. Throughout this analysis we shall use the notation introduced in [5]. It is advantageous for the present purpose to introduce following dimensionless parameters and variables

$$b' \equiv b/a, \quad \beta' \equiv \beta/a, \quad t' \equiv Vt/a. \tag{1}$$

Thus the slot and mark space widths (b and β, respectively) are measured in units of the radius a of the Gaussian laser beam while the time t is measured in units of a/V (V being the velocity of the system of parallel slots and mark spaces). In this way we reduce the number of variables present in the problem. The normalized transmitted laser power as a function of time [Ref. 5, Eqs. (3) and (5)] then becomes

$$\frac{P(t')}{P_0} = \frac{1}{2}\sum_k \left\{ \operatorname{erf} 2^{1/2}\left[k(b'+\beta') - t' + \frac{b'}{2}\right] - \operatorname{erf} 2^{1/2}\left[k(b'+\beta') - t' - \frac{b'}{2}\right]\right\}. \tag{2}$$

This function is periodic with the period

$$T' = b' + \beta', \tag{3}$$

and since it is even it can be expanded in a cosine Fourier series [6]. The expansion coefficients $A_0/2$, A_1, A_2, A_3, ... can be calculated numerically, with the aid of (2) using well-known expressions [6]. Particularly, in this case we have [5]

$$A_0/2 = \overline{P(t')}/P_0 = b'/(b' + \beta'). \tag{4}$$

Inserting $t' = 0$ and $t' = T'/2$ in (2), one obtains expressions for the largest (P_{max}/P_0) and smallest (P_{min}/P_0) normalized transmitted laser power, respectively. One should note that these two quantities depend only on two parameters: b' and β'.

After this preliminaries we can write the condition for the harmonic-like amplitude modulation of the laser light by means of a mechanical chopper in the form

$$J \equiv \frac{2}{T'} \int_0^{T'/2} \left| \frac{P(t')}{P_0} - \left[\frac{A_0}{2} + A_1 \cos\left(\frac{2\pi t'}{T'}\right) \right] \right| dt' = \min. \tag{5}$$

This quantity gives the average discrepancy between the function $P(t')/P_0$ and its harmonic approximation, which is represented with the first two terms of the Fourier expansion. It is required that it shall be a minimum. The condition (5) when applied alone leads to reduced laser power modulation depth. This is undesirable since in photoacoustics the modulation efficiency is also an important experimental requirement. Therefore we additionally demand the largest possible modulation depth:

$$(P_{max} - P_{min})/P_0 = \max. \tag{6}$$

It will be shown in this paper that the simultaneous fulfilment of the two conflicting demands (5) and (6) is possible, in a manner quite satisfactory for the experimental needs, by careful selection of the b' and β' values.

We take (5) and (6) into account by requiring that the combined quantity

$$K = K(b', \beta') \equiv (P_{max} - P_{min})/P_0 - J < 1, \tag{7}$$

shall be a maximum. In this way the two conditions (for the harmonic-like and efficient amplitude modulation) are given the same weight. For any given values of b' and β' one can find the corresponding value of K numerically using (2, 4, 5, and 7). The quantity K is function of two variables and we found its maximum using a Monte Carlo sampling procedure. As a result we obtained that the values

$$b' = \beta' = 2.770, \tag{8}$$

give the maximum K value of $K_{max} = 0.9570$. The values of other relevant parameters for this important particular case are

$$\begin{aligned} A_0/2 &= 0.5, \\ A_1 &= 0.5421, \\ A_2 &= 0, \\ A_3 &= -0.0499, \\ J &= 0.0318, \\ P_{max}/P_0 &= 0.9944, \\ P_{min}/P_0 &= 0.0056. \end{aligned}$$

Thus, in the case when (8) is fulfilled, the average discrepancy between the function $P(t')/P_0$ and its harmonic approximation is $< 3.2\%$ and furthermore the Fourier coefficient A_1 is more than an order of magnitude larger than $|A_3|$. This shows that, on inserting the values $b' = \beta' = 2.770$ into (2), one obtains a $P(t')/P_0$ curve which is, for experimental applications, sufficiently close to the harmonic function.

Additionally, the numerical calculation of the integral appearing in the Parseval's equality gives

$$\frac{A_0^2}{2} + \sum_{n=1}^{\infty} A_n^2 = \frac{4}{T'} \int_0^{T'/2} \left(\frac{P(t')}{P_0}\right)^2 dt' = 0.79633 \tag{9}$$

while, on the other hand, $A_0^2/2 + A_1^2 + A_3^2 = 0.79632$. We see that the first three nonvanishing terms practically exhaust the sum appearing in the Parseval's equality (9), showing that the corresponding terms in the Fourier expansion give, in this case, a satisfactory approximation for the function $P(t')/P_0$. We shall make use of this fact in Sect. 2 in order to obtain a simple approximation for the $P(t')/P_0$ curves.

The physical meaning of the result (8) is clear; in order to get harmonic-like and at the same time efficient amplitude modulation of the cw laser light in the TEM_{00} mode by means of a mechanical chopper one must ensure that the slot and mark space widths are equal and moreover that they are 2.770 times larger than the radius of the Gaussian beam. The fulfilment of these conditions gives an optimum between the two opposite demands (we can improve one only at the expense of the other; this can be achieved, if needed, by giving different weights to the two conditions appearing in (7) and then finding the maximum of the modified K function). The obtained equality between the b' and β' values, (8), is logical enough. Only then the widths of the maximum and the minimum parts of the $P(t')/P_0$ curve are equal and since this is also the case with its harmonic approximation, the discrepancy between the two is then minimum. Figure 1 shows the behaviour of the relevant quantities P_{max}/P_0, P_{min}/P_0, J and K which enter into (7) in the case $b' = \beta'$. The quantity K which is maximized has a broad maximum

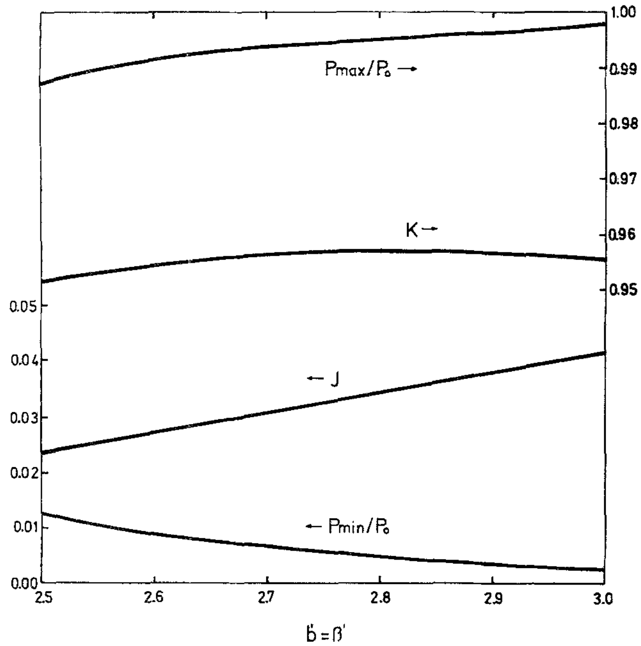

Fig. 1. Functional dependence of P_{max}/P_0, P_{min}/P_0, J and K on $b'=\beta'$ values

centered at $b'=\beta'=2.770$ and therefore the condition (8) is not a stringent one. For experimental applications the use of any $b'=\beta'$ values in the vicinity of (8) will do almost as good as the optimum value. In practice, one can realize the conditions (8) either by adjusting the slot and mark space widths of the chopping disc for the given value of the radius of the Gaussian beam, or by modifying the radius of the laser beam with suitable optics.

2. Approximation for the $P(t)/P_0$

The results obtained in the previous section, namely the close fulfilment of the Parseval's equality (9) and the broad maximum of the quantity K which is maximized (Fig. 1) show that it is possible to give a simple and yet very accurate approximation for the often needed normalized transimitted laser power as a function of time. This approximation is given by the first three nonvanishing terms of the Fourier series and it is valid only when the conditions (8) for the harmonic-like and efficient amplitude modulation of the cw laser light are fulfilled or nearly fulfilled. One obtains simply

$$\frac{P(t)}{P_0} \simeq \frac{1}{2}(1+m\cos\omega t+n\cos 3\omega t) \tag{10}$$

with

$$m \equiv 2A_1\,, \quad n \equiv 2A_3\,, \quad \omega \equiv 2\pi/T = \pi V/b\,. \tag{11}$$

Table 1. Expansion coefficients m and n appearing in (10) together with the quantity L given by (12) as functions of $b=\beta$. Linear interpolation gives non tabulated values with sufficient accuracy

$\frac{b}{a}=\frac{\beta}{a}$	m	$-n$	$L\times 10^3$
2.50	1.0452	0.0718	0.58
2.52	1.0484	0.0739	0.63
2.54	1.0516	0.0759	0.68
2.56	1.0548	0.0780	0.74
2.58	1.0578	0.0800	0.79
2.60	1.0608	0.0821	0.85
2.62	1.0638	0.0842	0.91
2.64	1.0667	0.0863	0.97
2.66	1.0695	0.0884	1.04
2.68	1.0723	0.0905	1.11
2.70	1.0750	0.0925	1.18
2.72	1.0777	0.0946	1.26
2.74	1.0803	0.0967	1.34
2.76	1.0829	0.0988	1.42
2.78	1.0854	0.1009	1.50
2.80	1.0879	0.1030	1.59
2.82	1.0903	0.1051	1.68
2.84	1.0927	0.1071	1.78
2.86	1.0950	0.1092	1.87
2.88	1.0973	0.1113	1.98
2.90	1.0995	0.1133	2.08
2.92	1.1017	0.1154	2.19
2.94	1.1039	0.1175	2.30
2.96	1.1060	0.1195	2.41
2.98	1.1081	0.1216	2.52
3.00	1.1101	0.1236	2.64

The expansion coefficients m and n, together with the quantity

$$L \equiv \frac{2}{T}\int_0^{T/2}\left|\frac{P(t)}{P_0}-\frac{1}{2}(1+m\cos\omega t+n\cos 3\omega t)\right|dt\,, \tag{12}$$

which gives the average discrepancy between the function $P(t)/P_0$ and its approximation (10), are tabulated in the interval from $b=\beta=2.5a$ to $b=\beta=3a$ which is likely to be of practical interest in photoacoustic applications (Table 1).

The approximation (10) is to be compared with [Ref. 5, Eq. (1)] when the harmonic modulation function

$$f(t)=\cos\omega t\,, \tag{13}$$

is inserted, as is often done in theoretical treatments. The present analysis reveals that [Ref. 5, Eq. (1)] is unable to reproduce the $P(t)/P_0$ curves with sufficient accuracy. The reason for this is simple; the $P(t)/P_0$ curves are not harmonic, they are only (more or less) harmonic-like. One must introduce, the additional correcting term $c\cos 3\omega t$ to obtain (10) and even then the approximation is valid only under conditions stated above.

3. Conclusion

In order to achieve harmonic-like and efficient amplitude modulation of the cw laser radiation in the TEM_{00} mode by means of a mechanical chopper it is necessary that the slot and mark space widths of the chopping disc are equal and moreover that they are 2.77 times larger than the radius of the Gaussian laser beam. Furthermore, when these two conditions are satisfied or nearly satisfied the simple approximation (10) for the often needed normalized transmitted laser power as a function of time is valid.

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