Appl. Phys. B 27, 177–181 (1982)

Applied Physics B Photophysics and Laser Chemistry

A New Method of Measuring Ultra-Short Coherent Light Pulses

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Received 22 November 1981/Accepted 8 January 1982

Abstract. A new method of pulse width measurement based on the total energy response of an etalon is presented. The theory is developed and results for several pulse shapes are given. The method is unique because no fast response detector is required and there are practically no spectral region limitations.

PACS: 42.55

The production of ultra-short light pulses in recent years has presented a challenge in measurement techniques. Generation of subpicosecond pulses from colliding pulse mode-locking of dye lasers has produced the shortest pulses in the visible region [1]. In addition, visible wavelength mode-locked laser pulses in the picosecond regime have been used with a variety of nonlinear mechanisms to generate short pulses in the infrared. Molecular gases have also been excited by mode-locked CO_2 lasers in a synchronous arrangement to generate submillimeter wavelength pulses in the 350 ps regime. Thus the availability of short pulses at a variety of carrier frequencies has been achieved. However, the techniques of measurement are fairly limited in the visible and ir regions and extremely limited in the region of wavelengths longer than 10 μm. This limitation at longer wavelengths is especially unfortunate since many interesting transient molecular responses and cooperative effects offer interesting possibilities for generating pulses shorter than 100 ps.

In the visible region measurements made with fast photodetectors and oscilloscopes have a time resolution limit of about 100 ps. Picosecond pulses have to date only been measured either in a linear fashion using streak cameras and tubes or via nonlinear effects [2–4]. Of these methods, the nonlinear optical phenomena capable of 10^{-13} s resolution has been most widely utilized. The three most extensively used concepts are those of second-harmonic generation (SHG), SHG of the second kind, and two-photon fluorescence (TPF). All of these effects can be utilized to determine the autocorrelation of a laser pulse. For TPF, it has been shown that the fluorescence along the propagation axis is proportional to

$$f(\tau)=1+2G^{(2)}(\tau),$$

where $G^2(\tau)$ is related to the pulse intensity $I(t)$ by

$$G^{(2)}(\tau)=\frac{\langle I(t)I(t+\tau)\rangle}{\langle I^2(t)\rangle}.$$

This technique is highly dependent on the experimental parameters and has been studied extensively [5, 6]. Only for well isolated coherent pulses does $G^{(2)}(0)$ equal 1 and $G^2(\tau\to\infty)$ equal 0. This results in a maximum contrast ratio of 3:1 and has been experimentally verified by Shapiro and Duguay [7]. If however the central peak is surrounded by a broad envelope such as in incomplete mode-locking, the contrast is bounded by a 2:1 ratio. Moreover, a multimode output without mode-locking can give a 1.5:1 ratio. These effects limit the use of this method to highly controlled conditions. Moreover, an estimate of the pulse shape and a degree of coherence are assumed. Pulse shapes can only be inferred from higher moment correlation functions and are virtually impossible to determine in single shot events [8–11].

The SHG measurements where the two correlated pulses have the same polarization are completely equivalent to TPF techniques. The resulting form for $G^{(2)}(\tau)$ and the various contrast ratios are all equal for

0721-7269/82/0027/0177/$01.00

both methods. A variation on SHG is second-harmonic generation of the second kind. In this measurement, the two pulses have opposite polarizations. In this arrangement it becomes possible to choose a crystal orientation in which neither pulse alone can lead to SHG. The effect is to remove the background and produce a direct measurement of $G^{(2)}(\tau)$. This technique however suffers from stringent spatial alignment constraints and birefringent group velocity effects.

In other areas of the spectrum short pulses can be measured using correlation methods in a variety of non-linear materials. These can include upconversion type systems for the infrared or direct TPF measurements. Such crystals include prousite, $AgGaS_2$, and $LiBO_3$. For short pulse measurement in the millimeter and submillimeter wavelength region, most work has relied on the nonlinear response of mixer type device such as Schottky barrier diodes and MOM junctions.

In this paper, we present a new method of short pulse measurement of coherent pulses using etalons. The technique is applicable in all spectral regions and does not require fast detectors. It is however sensitive to the degree of coherence and this will be treated in a later paper including experimental results.

Theory

It is a well known fact that etalons can serve as frequency filters. In particular, when there is no dispersive medium filling the etalon, the input field and output field can be linearly related in the frequency domain by

$$E_{\mathrm{OUT}}(\omega)=g(\omega)E_{\mathrm{IN}}(\omega), \tag{1}$$

where

$$g(\omega)=\frac{1}{1-re^{i\omega T_c}}. \tag{2}$$

Here r is the field amplitude reflection coefficient, ω is the frequency and T_c is the cavity roundtrip time given by $2nL/c$. The time domain output field can be found from

$$E_{\mathrm{OUT}}(t)=\int_{-\infty}^{+\infty}\frac{e^{i\omega t}E_{\mathrm{IN}}(\omega)d\omega}{1-re^{i\omega T_c}}. \tag{3}$$

When the illuminating light is a continuous coherent field, $E_{\mathrm{IN}}(\omega)=\delta(\omega_0)E_{\mathrm{IN}}^0$ and

$$E_{\mathrm{OUT}}(t)=\frac{E_{\mathrm{IN}}^0 e^{i\omega_0 t}}{1-re^{i\omega_0 T_c}}. \tag{4}$$

This gives a time-independent intensity which is a function of r, ω_0, and T_c. This result is given by

$$I(\omega, T_c, r)=I_{\mathrm{IN}}^0\frac{1}{1+r^2-2r\cos\omega_0 T_c}. \tag{5}$$

Equation (3) assumes a completely different character and produces a time-dependent intensity in the case that the illuminating field is a pulse of short duration. Of particular significance is the case where $E_{\mathrm{IN}}(\omega)$ has a Fourier transform bandwidth which is large as compared to T_c^{-1}. In this situation the integral begins to look very similar to a mode-locked laser integral and will result in intensity pulsations. From this analysis it becomes obvious that the effect that an etalon has on a coherent light field is strongly dependent on the ratio of the pulse width to the etalon round-trip time. In particular one can think of the cw case as being approached by a pulse interacting with a very short etalon.

The integral equation in (3) becomes very difficult to solve when $E_{\mathrm{IN}}(\omega)$ has any sizable structure. Therefore, we approach the desired solution by the standard method of summing transmitted waves. We begin the analysis in a general manner by assuming a single maximum input pulse shape $E_{\mathrm{IN}}(t)$. The sum of the transmission terms at the output side of the etalon is given by

$$E_{\mathrm{OUT}}(t)=(1-R^2)\sum_{N=0}^{\infty}R^{2N}e^{i\omega_0 T_c}E_{\mathrm{IN}}(1-NT_c), \tag{6}$$

where R is the electric field reflection coefficient $T_c=2nL/c$, and ω_0 is the pulse carrier frequency. This expression may be used to evaluate $E_{\mathrm{OUT}*}(t)E_{\mathrm{OUT}}(t)$ for a variety of functional forms of $E_{\mathrm{IN}}(t)$. This has been done using numerical calculations and shows that an etalon can have strong effects on the total energy

$$\varepsilon=\int_{-\infty}^{+\infty}E_{\mathrm{OUT}*}(t)E_{\mathrm{OUT}}(t)dt.$$

However, before the numerical results are presented, the case of a low reflectivity $R^2\ll 1$ etalon may be analyzed by omitting higher powers of R^2. Keeping only the linear term gives

$$E_{\mathrm{OUT}}(t)\approx(1-R^2)\left[E_{\mathrm{IN}}(t)+(R^2)e^{i\omega_0 T_c}E_{\mathrm{IN}}(t-T_c)\right]. \tag{7}$$

The intensity is then given by

$$\frac{E_{\mathrm{OUT}}^*(t)E_{\mathrm{OUT}}(t)c\varepsilon_0}{2}$$

and results in

$$I(t)\approx\tfrac{1}{2}c\varepsilon_0(1-R^2)^2\left[E_{\mathrm{IN}}(t)+2R^2E_{\mathrm{IN}}(t)E_{\mathrm{IN}}(t-T_c)\cos\omega_0 T_c+R^4E_{\mathrm{IN}}(t-T_c)\right]. \tag{8}$$

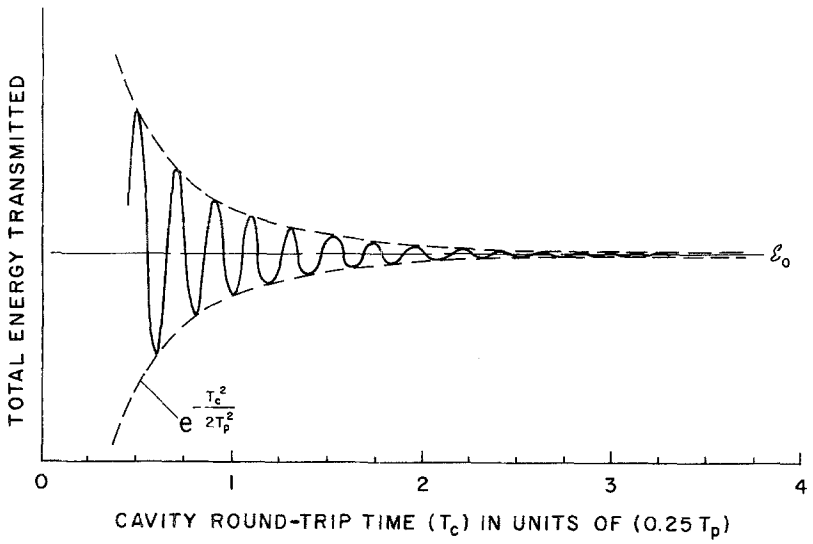

Fig. 1. Numerical results for total energy transmission of an etalon with $R=0.9$

Equation (8) can be integrated in order to determine the total energy ε which is transmitted by the etalon. The linear approximation of (7) results in a transmitted energy

$$\varepsilon=\tfrac{1}{2}c\varepsilon_0(1-R)^2\,[E^{IN}(t)dt+2R^2\cos\omega_0 T_c \cdot\int E_{IN}(t)E_{IN}(t-T_c)dt + R^4\int E_{IN}^2(t-T_c)dt]. \quad (9)$$

For $R^2\ll 1$, the R^4 term may be droped and we are left with the electric field autocorrelation term as the expression containing the cohrent effect of the etalon. Thus we have a constant term plus a term which has a resonance variation and is proportional to the electric field $G^{(2)}(T_c)$. In this sense this is not a linear process in total energy.

The expression in (9) can be evaluated in closed form for a Gaussian pulse shape and a decaying exponential for $E_{IN}(t)$. If $E_{IN}(t)$ is taken to be

$$E_{IN}(t)=E_0 e^{-t^2/T_p^2}$$

the total energy which is transmitted by the etalon is given by

$$\xi=\xi_0[1+\sqrt{2}R^2\exp(-T_c/2T_p^2)\cos(\omega_0 T_c)], \quad (10)$$

where $\xi_0=\dfrac{\sqrt{\pi c\varepsilon_0}}{4}E_0^2T_p(1-R^2)^2$.

For the pulse shape

$$E_{IN}(t)=\begin{cases}0 & t<0\\ E_0\exp(-t/T_p) & 0<t<\infty\end{cases}$$

the energy transmitted is given by:

$$\xi=\tfrac{1}{4}c\varepsilon_0E_0^2T_p[1+\sqrt{2}R^2\cos(\omega_0T_c)\exp(-T_c/T_p)]. \quad (11)$$

These expressions for the total energy ξ, reflect how the etalon can alter the total transmitted energy as a function of T_c/T_p. When the cavity is very long compared to the spatial extent of the pulse, there are no coherent effects and the pulse ratles inside and leaks out. The result is a constant energy since eventually all the rattling pulses will be measured. If however, the etalon is small as compared to the spatial extent of the pulse, the wave train can be folded over and interference effects take place. The etalon spacing then becomes important and can lead to oscillations of depth $\sqrt{2}R^2(1-T_c^2/2T_p^2)$ relative to the long etalon value. This result is the basis for pulse width measurement technique suggested in this paper.

Verification of the general behavior of the output energy ξ for various values of R has been done by numerical summation and integration of the equations and is shown in Fig. 1. The results show that stronger amplitude variationas are expected for values of R near unity. The calculations also show that the result in (10) is valid for $R<0.5$ and can be used if the experiment is properly designed.

Experimental Method

In this section we discuss how the etalon can be utilized to determine pulse widths. This method like autocorrelation techniques also must assume a pulse shape. For the sake of simplicity of calculation, the Gaussian shape will be utilized in discussing an experimental use for our results.

The result we will use is that of (10). The term of value is the $\sqrt{2}R^2\exp(-T_c/2T_p^2)\cos\omega_0T_c$; therefore we will define the dimensionless constant to be determined experimentally γ given by

$$\gamma=\frac{\xi_r-\xi_{nr}}{\xi_0}=\sqrt{2}R^2\exp(-T_c^2/2T_p^2)\,; \quad (12)$$

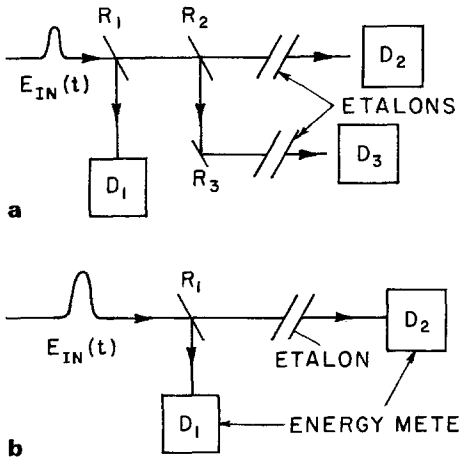

Fig. 2a and b. Experimental arrangements for pulse measurement

ξ_r and ξ_{nr} correspond to $\omega_0 T_c = 2M\pi$ and $\omega_0 T_c = M\pi$ (M odd), respectively. Using (12) we have that

$$T_p = \frac{(0.707)T_c}{\sqrt{\ln(R/\gamma)}} \tag{13}$$

for the case where we have $(T_c/T_p)^2 \ll 1$, the expression simplifies to

$$T_p \approx \frac{T_c R}{\sqrt{2R^2 - \sqrt{2\gamma}}}. \tag{14}$$

The results of (13) and (14) can be used in an experimental arrangement to determine pulse widths from total energy measurements. Using the arrangement shown in Fig. 2a we can determine the necessary variables from three energy meters D_1, D_2, and D_3. Calling the energy measured at each detector ξ_1, ξ_2, and ξ_3, respectively, and referring to the figure, we have that

$$\xi_0 = \frac{\xi_1(1-R_1^2)(1-R_2^2)(1-R^2)^2}{R_1^2}, \tag{15a}$$

$$\xi_{nr} = \frac{\xi_3(1-R_2^2)}{R_2^2 R_3^2}. \tag{15b}$$

Using these relations we can write as

$$\gamma = \frac{\xi_2 R_1^2 R_2^2 R_3^2 - \xi_3(1-R_2^2)R_1^2}{\xi_1 R_2^2 R_3^2 (1-R_1^2)(1-R_2^2)(1-R^2)^2}. \tag{16}$$

For $R_1^2 = R_2^2 = R_3^2 = \varrho$, we have

$$\gamma = \frac{\xi_2 \varrho^2 - \xi_3(1-\varrho)}{\xi_1[\varrho(1-\varrho)^2][1-R^2]^2}. \tag{17}$$

For the special case of $\varrho = 0.62$, (i.e. $\varrho^2 = 1-\varrho$) $R = 0.4$, we have

$$\gamma = \frac{4(\xi_2 - \xi_3)}{\xi_1}. \tag{17a}$$

An even simpler experiment can be designed which only uses two detectors and a resonant or non-resonant etalon. This experiment is shown in Fig. 2b. For this measurement we define ϕ

$$\phi = \frac{\xi_r - \xi_0}{\xi_0}. \tag{18}$$

In terms of ϕ, we have that the pulse width is given by

$$T_p = \frac{T_c}{2\sqrt{\ln\left(\frac{1.4R}{\phi}\right)}}. \tag{19}$$

In order to relate ϕ to the experimentally measured energies ξ_1 and ξ_2, we have the following

$$\xi_0 = \frac{\xi_1(1-R_1^2)(1-R^2)}{R_1^2} \tag{20}$$

and

$$\phi = \frac{\xi_2 R_1^2}{\xi_1(1-R_1^2)(1-R^2)} - 1. \tag{21}$$

For the case of $R_1^2 = 0.5$ and $R^2 = 0.5$, we have

$$\phi = \frac{4\xi_2}{\xi_1} \tag{22a}$$

and

$$(T_p)^2 = \frac{T_c^2}{4\ln\left(\frac{\xi_1}{4\xi_2 - \xi_1}\right)}. \tag{22b}$$

Conclusions

We have shown how a total energy transmission measurement through an etalon of suitable spacing can be utilized to measure pulse widths. Numerical solutions are given for arbitrary values of the reflectivity of the etalon R and an analytic approximation is presented for values of R less than 0.5. The results are given in terms of two possible measurement set-ups and interpreted.

This treatment has not considered the degree to which the shape assumed for the pulse affects the results. In addition, no account of non-lumped losses, dispersion and chirping have been included. These additional effects will be examined in a forthcoming publication which will also include experimental comprisons of this technique to TPF and SHG results. Finally it should be pointed out that this technique because of its simplicity and the availability of accurate energy meters throughout the spectrum may become a way of measuring pulse widths in spectral regions which have

been inaccessible. Furthermore, because current technology can produce very thin optical wafers, this method may pove to be the most straightforward way of measuring pulses in the 10-femptosecond regime.

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