Appl. Phys. B 47, 67–69 (1988)

Applied Physics B Photophysics and Laser Chemistry

Laser Induced Thermal Profiles in Thermally and Optically Thin Films

F. Bloisi and L. Vicari

Dipartimento di Fisica Nucleare, Struttura della Materia e Fisica Applicata, Università di Napoli, I-80125 Napoli, Italy

P. Cavaliere

Istituto di Fisica, Università di Palermo, I-90100 Palermo, Italy

S. Martellucci and J. Quartieri

Dipartimento di Ingegneria Meccanica, Università Tor Vergata, I-00100 Roma, Italy

P. Mormile and G. Pierattini

Istituto di Cibernetica C.N.R., Arco Felice, I-80072 Napoli, Italy

Received 5 November 1987/ Accepted 8 December 1987

Abstract. The temperature field generated by the weak absorption of a gaussian laser beam in an optically and thermally thin film bounded by two transparent plates is discussed. An analytical solution of the problem is presented together with an algorithm for the numerical integration. The influence of the finite thermal conductivity of the plates is shown in an example.

PACS: 42.65.Jx, 78.20.Nv, 78.65.Jd.

The temperature field generated in a weakly absorbing layer by a gaussian laser bem has been widely studied. The detections of the induced refractive index variations, by means of the self-focusing or defocusing of the pump or of a probe beam, allows one to determine the thermal properties of the sample. In many cases (liquids, liquid crystals, etc.) the sample must be bounded between two plates, usually of glass.

If the sample is thermally thick the plates may be considered perfectly conducting [1–4]. Such an approach gives good results also for the steady state. On the other hand, if the light pulse is very short the transient behaviour may be studied with the assumption of perfectly insulating plates [5–7]. These approximations do not give good results in determing the steady state temperature field induced in a thermally and optically thin sample: the conductivity of the plates has to be explicitely taken into account. In this paper we give an analytical solution for the temperature field in terms of a Hankel integral, then we introduce an algorithm for the numerical integration and show in an example the influence of the plates' conductivity.

1. Temperature Field

In a typical experimental apparatus (see Fig. 1) the pump beam intensity is

$$I(r)=I_0\exp(-2r^2/w_0^2). \quad (1)$$

The plates are assumed to be perfectly transparent and the sample to be weakly absorbing. The beam waist is assumed to be very small in comparison with the transverse dimension of the sample. The external surfaces of the plates (at $z=d_0$ and $z=d_3$) are kept at constant temperatures: convective heat exchanges in the sample are disregarded.

The steady state Fourier equation in the i^{th} layer is

$$\nabla^2 T_i(r,z) + Q_i/k_i = 0 \qquad i=1,2,3 \tag{2}$$

with the source term $Q_2 = \alpha I(r)$ $(Q_1 = Q_3 = 0)$ and with the boundary conditions

$$T_1(r,d_0) = T_3(r,d_3) = 0\,, \tag{3}$$

$$\left.\begin{aligned} &T_1(r,d_1) = T_2(r,d_1);\\ &T_2(r,d_2) = T_3(r,d_2)\,,\end{aligned}\right. \tag{4}$$

$$k_1 \left.\frac{\partial T}{\partial z}\right|_{z=d_1} = k_2 \left.\frac{\partial T}{\partial z}\right|_{z=d_1}, \tag{5}$$

$$k_2 \left.\frac{\partial T}{\partial z}\right|_{z=d_2} = k_3 \left.\frac{\partial T}{\partial z}\right|_{z=d_2}, \tag{6}$$

where $T_i(r,z)$ is the temperature difference with respect to the temperature at $z=d_0$ or $z=d_3$, k_i is the thermal conductivity and α is the optical absorption coefficient of the sample.

Let us consider the Hankel transform $\theta_i(\lambda)$ of $T_i(r)$ and $S_2(\lambda)$ of $Q_2(r)$. We have

$$T_i(r,z) = \int_0^\infty \lambda d\lambda\, \theta_i(\lambda,z) J_0(\lambda r)\,, \tag{7}$$

$$Q_2(r) = \int_0^\infty \lambda d\lambda\, S_2(\lambda) J_0(\lambda r)\,. \tag{8}$$

Substitution in (2) yields

$$\frac{\partial^2\theta}{\partial z^2} - \theta\lambda^2 + \frac{S_i}{k_i} = 0 \tag{9}$$

whose general solution is

$$\theta_i(\lambda,z) = A_i(\lambda)\exp(\lambda z) + B_i(\lambda)\exp(-\lambda z) + S_i/(k_i\lambda^2)\,. \tag{10}$$

Imposing the boundary conditions (3–6) we have

$$\begin{aligned} A_1(\lambda) = M\{&2k_2k_3E_1^2E_2(E_2^2+E_3^2)\\ &-k_2E_1[k_2(E_1^2-E_2^2)(E_2^2-E_3^2)\\ &+k_3(E_1^2+E_2^2)(E_2^2+E_3^2)]\}\,,\end{aligned} \tag{11}$$

$$\begin{aligned} A_2(\lambda) = M\{&k_3E_2(E_2^2+E_3^2)[k_1(E_0^2+E_1^2)\\ &-k_2(E_0^2-E_1^2)]-k_1E_1(E_0^2+E_1^2)\\ &\times[k_2(E_2^2-E_3^2)+k_3(E_2^2+E_3^2)]\,,\end{aligned} \tag{12}$$

$$\begin{aligned} A_3(\lambda) = M\{&-2k_1k_2E_1E_2^2(E_0^2+E_1^2)\\ &-k_2E_2[k_1(E_0^2+E_1^2)(E_1^2-E_2^2)\\ &+k_2(E_0^2-E_1^2)(E_1^2-E_2^2)]\}\,,\end{aligned} \tag{13}$$

$$B_1(\lambda) = -E_0^2A_1(\lambda)\,, \tag{14}$$

$$\begin{aligned} B_2(\lambda) = M\{&-k_1E_1E_2^2(E_0^2+E_1^2)[k_2(E_2^2-E_3^2)\\ &-k_3(E_2^2+E_3^2)]-k_3E_1^2E_2(E_2^2+E_3^2)\\ &\times[k_1(E_0^2+E_1^2)+k_2(E_0^2-E_1^2)]\,,\end{aligned} \tag{15}$$

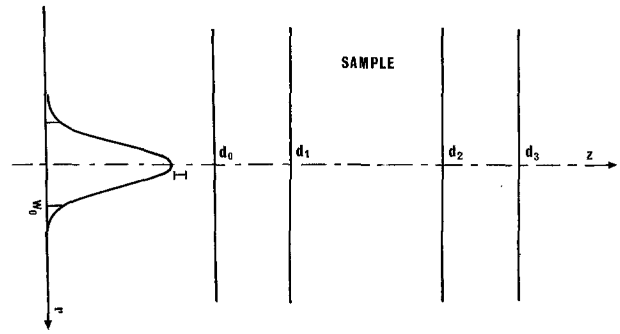

Fig. 1. Typical experimental set-up. The sample is bounded by two transparent plates; the external boundaries of the whole device (at $z=d_0$ and $z=d_3$) are held at a constant temperature. The laser beam is a TEM_{00} mode (gaussian intensity profile)

$$B_3(\lambda) = -E_3^2A_3(\lambda) \tag{16}$$

with

$$\begin{aligned} M = S_2/k_2\lambda^2\{&k_1(E_0^2+E_1^2)\\ &\times[k_2(E_1^2+E_2^2)(E_2^2-E_3^2)+k_3(E_1^2-E_2^2)(E_2^2+E_3^2)]\\ &+k_2(E_0^2-E_1^2)[k_2(E_1^2-E_2^2)(E_2^2-E_3^2)\\ &+k_3(E_1^2+E_2^2)(E_2^2-E_3^2)]\}^{-1}\,,\end{aligned} \tag{17}$$

$$E_i = \exp(\lambda d_i) \tag{18}$$

and, for a gaussian beam,

$$S_2(\lambda) = \alpha I_0(w_0/2)^2\exp(-w_0^2\lambda^2/8)\,. \tag{19}$$

2. Computer Implementation

Equation (7) allows the computation of the temperature as a function of r and z. Nevertheless for practical purposes the value of the temperature is usually required at several thousands of points. Direct integration of (7) could be quite cumbersome in these cases.

We observe, from (11–19), that, for a gaussian pump beam, $\theta(\lambda,z)$ goes zero with increasing λ. Let us take a real number R and an integer N so that

$$T(r,z) = 0 \quad \text{for} \quad r \geqq R \tag{20}$$

and

$$\theta(\lambda,z) = 0 \quad \text{for} \quad \lambda \geqq j_N/R\,, \tag{21}$$

where j_N is the N^{th} zero of the Bessel function of the zero$^{\text{th}}$ order J_0. Following the method presented in [8] we have

$$T(r_m,z) = \frac{2}{R^2}\sum_1^N{}_n \frac{J_0(j_nr_m/R)}{[J_1(j_n)]^2}\theta(j_n/R,z) \tag{22}$$

or, in matrix notation

$$\mathbf{T} = \mathbf{G}\cdot\theta\,, \tag{23}$$

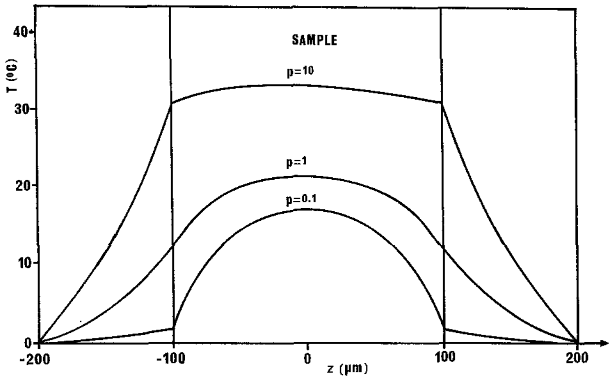

Fig. 2. Longitudinal temperature profile along the beam axis, $T(0,z)$, for different values of the ratio between sample and plate conductivities, $p=0.1$, 1, 10. In this example we assume a 5 W beam with $w_0=50\,\mu m$ and a sample with absorption coefficient $\alpha=10^{-4}/\mu m$

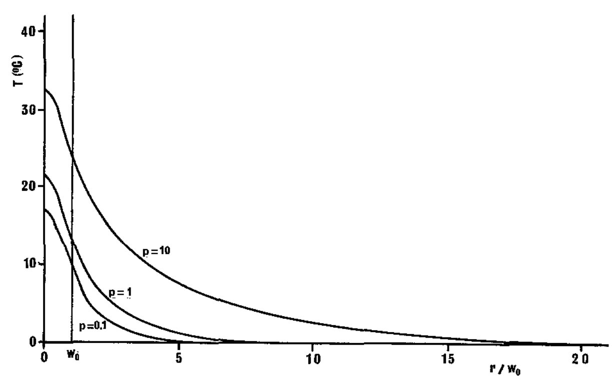

Fig. 3. Radial temperature profile at the middle of the sample, $T(r,0)$, for different values of the ratio between sample and plate conductivites, $p=0.1$, 1, 10. In this example we assume a 5 W beam with $w_0=50\,\mu m$ and a sample with absorption coefficient $\alpha=10^{-4}/\mu m$

where **T** is an array whose elements $t_m=T(r_m, z)$ are, for each z, the temperature at M arbitrary values of r, θ is an array of N values of θ: $\theta_n=\theta(j_n/R, z)$, and **G** is an $M\times N$ matrix whose elements are

$$g_{mn}=\frac{2}{R^2}\frac{J_0(j_n r_m/R)}{[J_1(j_n)]^2} \tag{24}$$

and do not depend on the pump beam intensity or beam waist, provided that conditions (20, 21) are satisfied. Hence the matrix **G** may be computed only once and then repetitively used for several different applications, and the computation of the temperature field reduces to a matrix product for each z value.

3. Conclusions

In Figs. 2 and 3 we show the results obtained for a 5 W beam with $w_0=50\,\mu m$ and a sample absorption coefficient $\alpha=10^{-4}/\mu m$; the ratio between sample and plates conductivities, p, is assumed to be 0.1, 1.0, and 10. In Fig. 2 is presented the longitudinal temperature profile along the optical axis $T(0,z)$, while in Fig. 3 we show the radial temperature profile in the middle of the sample $T(r,0)$. The influence of the finite conductivity of the plates is clearly shown, and hence the perfectly conducting or the perfectly insulating plate approximations would not be accurate in the case under consideration.

We conclude by observing that, despite the quite intricate expression involved, the computation of T in 30×200 points by means of the computer implementation of (23) requires only a few minutes on a IBM PC.

References

1. J.R. Lalanne, E. Sein, J. Buchert, S. Kielich: Appl. Phys. Lett.: **36**, 973–975 (1980)
2. P. Dorion, B. Pouligny, J.R. Lalanne: J. Physique Coll. C **6**, 197–201 (1983)
3. J.R. Lalanne, B. Pouligny, E. Sein: J. Phys. Chem. **87**, 696–707 (1983)
4. P. Dorion, J.R. Lalanne, B. Pouligny: IEEE J. QE-**22**, 1534–1538 (1986)
5. P. Calmettes, C. Laj: J. Physique Coll. C **1**, 125–129 (1972)
6. G. Koren: Phys. Rev. A **13**, 1177–1184 (1976)
7. J.P. Gordon, R.C.C. Leite, R.S. Moore, S.P.S. Porto, J.R. Whinnery: J. Appl. Phys. **36**, 3–8 (1985)
8. F. Bloisi, S. Martellucci, J. Quartieri, L. Vicari: Europhys. Lett. **4**, 905–908 (1987)