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cw Injection Phase Locking in Homogeneously Broadened Media

I. Theory

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Abstract. A theory of cw injection phase locking in homogeneously broadened media is investigated using density matrix formalism to derive the interactions of three coherent fields with a two levels system. This powerful formalism leads to analytical expressions of the complex gain for each wave propagating inside the amplifier medium. Studies of the minimum signal intensity for single-frequency operation and power output are related.

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Many applications require high-power laser sources of high spectral purity or large frequency tunability range or even both. In the cw regime, such sources may be used for nonlinear process studies [1], interferometric detection of gravity waves [2], long-distance space communication or optical pumping of far or middle infrared lasers.

An injection phase locking technique, borrowed from microwave technology, seems able to solve this problem [3]. First used by Stover and Steier [4] for He–Ne laser, this technique has been studied by Buczek and Freiberg [5] for conventional CO_2 laser. More recently, Dunn et al. [6] described mode selection in He–Ne laser while Couillaud et al. [7] dealed with cw ring dye lasers and Man and Brillet [2] with the argon laser. The theory of injection phase locking has been established according to Lamb formalism [8–12] for both inhomogeneously and homogeneously broadened amplifier media.

In this paper, we describe an homogeneously broadened amplifier medium by a two levels system and we use a semi-classical interaction theory for both injected wave and waves oscillating inside the cavity. We find the analytical expressions for gain and mode pulling. The minimum injected intensity for single-frequency operation is also investigated. Then we deal with frequency detuning of the master oscillator with respect to the resonances of the slave oscillator as a function of the injected intensity and frequency offset from the center of the laser line.

1. Density Matrix Formalism

We consider a system shown on Fig. 1 in which upper states are population inverted by any incoherent processes. This system is enclosed inside a ring cavity and we assume that the laser field is oscillating in the

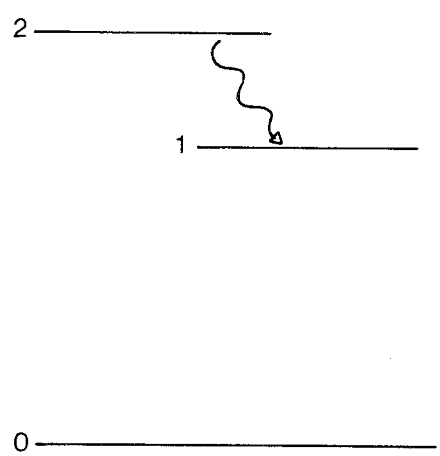

Fig. 1. Simplified two-levels system

cw regime. This laser field is formed by two counter propagating plane waves E_c^+ and E_c^- with a common frequency ω_c. Because of the homogeneous broadening, we also assume single-mode operation. An external field E_i at frequency ω_i is injected into the medium through one of the cavity mirror. These three fields connect the two upper levels of our system.

Let us consider the time-averaged density matrix, elements of which are given by

$$\text{(I)}\quad \begin{cases} \dot{\varrho}_{mm} = -\tau_{mm}^{-1}(\varrho_{mm}-\varrho_{mm}^{0}) - \dfrac{\mathrm{i}}{\hbar}[\varrho, H]_{mm}, & (1)\\[2ex] \dot{\varrho}_{mm} = -\tau_{mn}^{-1}\varrho_{mn} + \dfrac{\mathrm{i}}{\hbar}[\varrho, H]_{mn}, & (2)\end{cases}$$

where m, n are 1 or 2. We assume, that there is no applied field at frequencies ω_{01} and ω_{02}, and that spontaneous emission at these frequencies is negligible. The Hamiltonian matrix elements H_{mn} are given by the following relations

$$H_{mn} = E_m\delta_{mn} - \mu_{mn}E(t)$$

with

$$\delta_{mn} = 1 \quad \text{if} \quad n=m,$$

$$\delta_{nm} = 0 \quad \text{if} \quad n \neq m,$$

E_m being the eigenvalue, μ_{mn} the dipole moment matrix element, and $E(t)$ the applied field.

We also assume that the oscillators have no permanent dipole moment; i.e.,

$$\mu_{mm} = 0.$$

Following these assumptions, the system (I) becomes

$$\text{(II)}\quad \begin{cases} \dot{\varrho}_{11} = -(\varrho_{11}-\varrho_{11}^{0})\cdot\tau_1^{-1} \\ \qquad -\dfrac{\mathrm{i}}{\hbar}\cdot\mu_{12}\cdot(\varrho_{12}-\varrho_{21})\cdot E(t), & (3)\\[2ex] \dot{\varrho}_{22} = -(\varrho_{22}-\varrho_{22}^{0})\tau_2^{-1} \\ \qquad +\dfrac{\mathrm{i}}{\hbar}\mu_{12}(\varrho_{12}-\varrho_{21})\cdot E(t), & (4)\\[2ex] \dot{\varrho}_{12} = -\varrho_{12}\tau^{-1} + \dfrac{\mathrm{i}}{\hbar}[\varrho_{12}(E_2-E_1) \\ \qquad + \mu_{12}(\varrho_{22}-\varrho_{11})E(t)], & (5)\end{cases}$$

where τ_1, τ_2, and τ are the lifetime of both the levels themselves and of the off-diagonal matrix element which represents coherence between the levels. The eigenfield of the cavity may be described as two counter propagating plane waves

$$E_c = E_c^+ + E_c^- = \tfrac{1}{2}[(B_c^+\cdot \mathrm{e}^{-\mathrm{i}k_cz} + B_c^-\cdot \mathrm{e}^{\mathrm{i}k_cz})\,\mathrm{e}^{\mathrm{i}\omega_ct} + \mathrm{cc}]$$

the propagating injected wave is also assumed to be plane

$$E_i = \tfrac{1}{2}(B_i\cdot \mathrm{e}^{-\mathrm{i}k_iz+\mathrm{i}\omega_it} + \mathrm{cc})$$

and thus the total field reads

$$E(t) = E_c + E_i = \tfrac{1}{2}\{A_c\mathrm{e}^{\mathrm{i}\omega_ct} + A_i\mathrm{e}^{\mathrm{i}\omega_it} + \mathrm{cc}\}$$

with

$$A_c = B_c^+\cdot \mathrm{e}^{-\mathrm{i}k_cz} + B_c^-\cdot \mathrm{e}^{\mathrm{i}k_cz}, \qquad A_i = B_i\cdot \mathrm{e}^{-\mathrm{i}k_iz}.$$

We assume a time-varying expression for the off-diagonal density matrix element of the form

$$\varrho_{12} = \Lambda_c\cdot \mathrm{e}^{\mathrm{i}\omega_ct} + \Lambda_i\cdot \mathrm{e}^{\mathrm{i}\omega_it} + \Lambda_c'\cdot \mathrm{e}^{-\mathrm{i}\omega_ct} + \Lambda_i'\cdot \mathrm{e}^{-\mathrm{i}\omega_it}$$

with regard with these assumptions, the term $(\varrho_{12}-\varrho_{21})E(t)$ in (3 and 4) of system (II) will generate oscillation of the populations density ϱ_{11} and ϱ_{22} at the frequencies $2\omega_c$, $2\omega_i$, $(\omega_c+\omega_i)$, and $(\omega_c-\omega_i)$. By neglecting the terms oscillating at high frequency (rotating waves approximation), only the terms oscillating at frequency $\pm(\omega_c-\omega_i)$ will be regarded. This population oscillation will induce a modulation of the polarization resulting in a coupling of the gains at frequency ω_c and ω_i.

Then, the population density may be expressed by

$$\varrho_{nn} = \varrho_{nn}^{(1)} + \varrho_{nn}^{(2)}\mathrm{e}^{\mathrm{i}\Omega t} + \mathrm{cc}, \qquad n=1,2,$$

with

$$\Omega = \omega_c - \omega_i.$$

Let Δ be the population difference between states 1 and 2:

$$\Delta = \varrho_{22} - \varrho_{11}$$

and Δ_0 the value of Δ in the absence of the applied fields. Δ will be also of the form

$$\Delta = \Delta^{(1)} + \Delta^{(2)}\cdot \mathrm{e}^{\mathrm{i}\Omega t} + \mathrm{cc}.$$

By substituting this expression in (5) and collecting terms that multiply $\exp(\pm\mathrm{i}\omega_ct)$ and $\exp(\pm\mathrm{i}\omega_it)$, we get

$$\Lambda_c = \frac{i\mu_{12}\tau}{\hbar}\cdot\frac{\Delta^{(1)}A_c + \Delta^{(2)}A_i}{1+\mathrm{i}\delta_c}$$

$$\text{with} \quad \delta_c = (\omega_c - \omega_0)\cdot\tau,$$

$$\Lambda_i = \frac{i\mu_{12}\tau}{\hbar}\cdot\frac{\Delta^{(1)}A_i + \Delta^{(2)*}A_c}{1+\mathrm{i}\delta_i}$$

$$\text{with} \quad \delta_i = (\omega_i - \omega_0)\cdot\tau.$$

The quantities Λ_c' and Λ_c' are negligible because they are proportional to $[\tau(\omega_c+\omega_i)]^{-1}$. By reporting the expression of ϱ_{12} in (3 and 4), we can compute the values of $\varrho_{nn}^{(1)}$ and $\varrho_{nn}^{(2)}(n=1,2)$ and then the values of $\Delta^{(1)}$ and $\Delta^{(2)}$.

Using the following definitions

$$E_S^0 = \frac{\hbar}{\mu_{12}}\left(\frac{1}{\tau \cdot T}\right)^{1/2} \quad \text{and} \quad T = \frac{\tau_1 + \tau_2}{2},$$

where E_S^0 is the saturating field at the line center

$$I_j = A_j \cdot A_j^*, \quad \mathcal{I}_j = \frac{I_j}{E_S^{02} \cdot (1+\delta_j^2)}, \quad j = c, i,$$

$$\delta = \tfrac{1}{2}(\delta_c - \delta_i),$$

$$X = (1+\delta^2)^{1/2}, \quad \exp(i\alpha) = \frac{1+i\delta}{X},$$

$$X_j = (1+\delta_j^2)^{1/2}, \quad \exp(i\alpha_j) = \frac{1+i\delta_j}{X_j}, \quad j = c, i,$$

$$X_R = \left(\frac{(1+\tau_1^2\Omega^2)(1+\tau_2^2\Omega^2)}{1+\dfrac{\tau_1^2 \cdot \tau_2^2}{T^2}\Omega^2}\right)^{1/2},$$

$$\exp(i\alpha_R) = \frac{(1+\mathrm{i}\tau_1\Omega)(1+\mathrm{i}\tau_2\Omega)}{\left(1+\mathrm{i}\dfrac{\tau_1\tau_2}{T}\Omega\right) \cdot X_R},$$

$$\eta = \alpha - \alpha_c + \alpha_i,$$

$$N_1 = \mathcal{I}_i \frac{X_i^2}{2X_c}\cos(2\eta + \alpha_c) + \mathcal{I}_c \cdot \frac{X_c^2}{2X_i}\cos(2\eta - \alpha_i) + X_R\cos(2\eta - \alpha_R),$$

$$N_2 = \mathcal{I}_i \frac{X_i^2}{2X_c} \cdot \exp[i(\eta + \alpha_c)] + \mathcal{I}_c \cdot \frac{X_c^2}{2X_i} \cdot \exp[i(\eta - \alpha_i)] + X_R \cdot \exp[i(\eta - \alpha_R)],$$

$$D = \left(\mathcal{I}_i \frac{X_i^2}{2X_c}\right)^2 + \left(\mathcal{I}_c \frac{X_c^2}{2X_i}\right)^2 \ldots + X_R^2 + \tfrac{1}{2}\mathcal{I}_i\mathcal{I}_c X_i X_c \cos(\alpha_i + \alpha_c) \ldots + X_R\left[\mathcal{I}_i \frac{X_i^2}{X_c}\cos(\alpha_R + \alpha_c) + \mathcal{I}_c \frac{X_c^2}{X_i}\cos(\alpha_R - \alpha_i)\right],$$

$$S = 1 + \mathcal{I}_i + \mathcal{I}_c, \quad A_j = \mathcal{I}_j^{1/2} \cdot X_j \cdot E_s^0 \mathrm{e}^{\mathrm{i}\varphi_j}, \quad j = c, i.$$

$\Delta^{(1)}$ and $\Delta^{(2)}$ may be expressed as

$$\Delta^{(1)} = \Delta_0\left(S - 2X^2 \cdot \mathcal{I}_i \cdot \mathcal{I}_c \cdot \frac{N_1}{D}\right)^{-1},$$

$$\Delta^{(2)} = -\Delta^{(1)} \cdot (\mathcal{I}_i\mathcal{I}_c)^{1/2} X \frac{N_2}{D}\exp[\mathrm{i}(\varphi_c - \varphi_i)].$$

In order to simplify further computation and considering that τ_2 must be larger than τ_1 for producing a high-efficiency and high-power laser, terms involving τ_1 will be neglected in the expression of X_R and α_R. Then, these expressions reduce to

$$X_R = (1 + R_a^2 \cdot \delta^2)^{1/2}, \quad \exp(\mathrm{i}\alpha_R) = \frac{1+\mathrm{i}R_a\delta}{X_R},$$

where

$$R_a = \frac{\tau_2}{\tau}.$$

We are now able to express ϱ_{12} and then the induced polarization given by

$$P = \mathrm{Tr}\{\mu\varrho\} = \mu_{12}(\varrho_{12} + \varrho_{21}).$$

This polarization includes terms oscillating at frequency ω_c and ω_i. As a result the complex susceptibilities relative to each wave are found to be

$$\chi_c = \mathrm{i}\frac{\mu_{12}^2\tau}{\hbar\varepsilon_0} \cdot \frac{\mathrm{e}^{-\mathrm{i}\alpha_c}}{X_c} \cdot \Delta^{(1)}\left(1 - X \cdot \frac{X_i}{X_c} \cdot \frac{N_2}{D} \cdot \mathcal{I}_i\right),$$

$$\chi_i = \mathrm{i}\frac{\mu_{12}^2\tau}{\hbar\varepsilon_0} \cdot \frac{\mathrm{e}^{-\mathrm{i}\alpha_i}}{X_i} \cdot \Delta^{(1)}\left(1 - X \cdot \frac{X_c}{X_i} \cdot \frac{N_2^*}{D} \cdot \mathcal{I}_c\right).$$

The amplitude gain reads

$$g = -\mathrm{i}\frac{k}{2}\chi.$$

Considering the small-signal gain at the line center

$$g_0 = k\frac{\mu_{12}^2\tau}{\hbar\varepsilon_0} \cdot \Delta_0,$$

where k is the wave number, we obtain

$$\text{(IV)} \quad \left| \begin{aligned} g_c^{\pm} &= \xi \cdot \frac{g_0}{2} \cdot \frac{\mathrm{e}^{-\mathrm{i}\alpha_c}}{X_c}\left(\frac{1}{S - 2\mathcal{I}_i\mathcal{I}_c X^2 \dfrac{N_1}{D}}\right) \times \left(1 - 2X\frac{X_i}{X_c} \cdot \frac{N_2}{D}\mathcal{I}_i\right), \\ g_i &= \frac{g_0}{2} \cdot \frac{\mathrm{e}^{-\mathrm{i}\alpha_i}}{X_i}\left(\frac{1}{S - 2\mathcal{I}_i\mathcal{I}_c X^2 \dfrac{N_1}{D}}\right) \times \left(1 - 2X\frac{X_c}{X_i} \cdot \frac{N_2^*}{D}\mathcal{I}_c\right), \end{aligned} \right.$$

where ξ is equal to + or -1 regarding to E_c^+ or E_c^-. We observe that g_c may be deduced from g_i by inverting the indexes c and i.

Figure 3 shows the saturation of the normalized intensity gains $|g_c^{\pm}/g_0|^2$ (dashed curves) and $|g_i/g_0|^2$ (solid curves) versus the normalized injected wave intensity $I_{\mathrm{inj}}/I_{\mathrm{sat}}$ inside the amplifier medium, for various values of the normalized self oscillating wave intensity I_c/I_{sat}. For these computations, the frequencies of the waves were assumed to be for the line center and set up in resonance with the slave cavity. We observe that when I_{inj} reaches a sufficient level, the gain for I_c becomes lower than the gain for I_i. As a result I_{inj} increases more rapidly than I_c. This process accelerates the enhancement of $|g_i|^2$ and the decrease of $|g_c|^2$ down to the round-trip gain that would prevail below oscillation threshold. When this condition is satisfied, the slave laser acts as a regenerative amplifier for the incoming light.

2. Description of the Ring

The ring cavity is shown in Fig. 2. The continuity conditions for the injected field upon mirror M read

$$A_i(0)e^{i\varphi_0}=t\cdot A_{inj}\,e^{i\varphi_{inj}}+\varepsilon r\cdot A_i(L)\,e^{i\varphi_L},$$

$$A_{out}\,e^{i\varphi_{out}}=-\varepsilon r A_{inj}\,e^{i\varphi_{inj}}+tA_i(L)\,e^{i\varphi_L}.$$

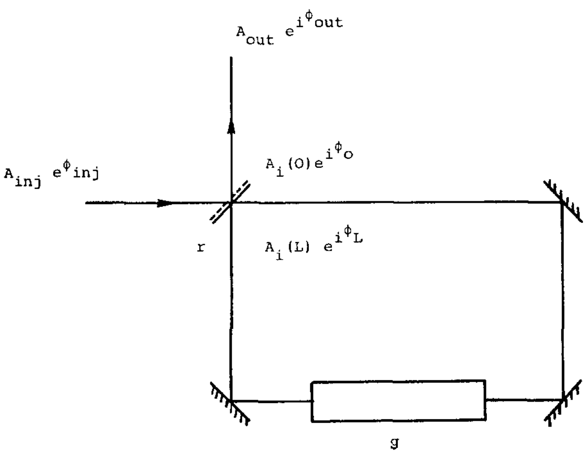

Fig. 2. Injected ring-laser cavity and relevant notations

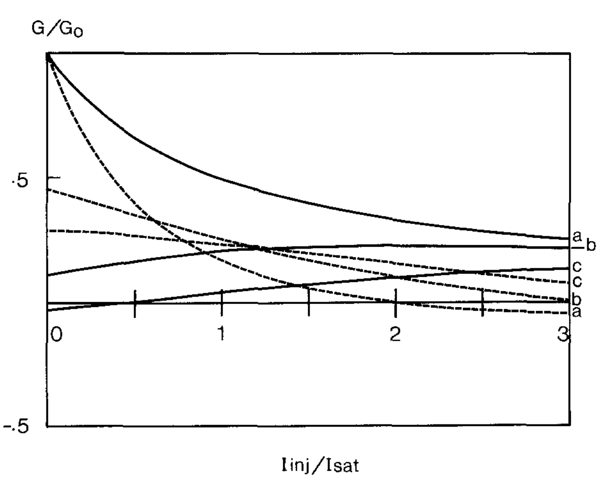

Fig. 3. Normalized intensity gains G_i/G_0 (solid line) and G_c/G_0 (dashed line) versus normalized intensity of the injected wave I_{inj}/I_{sat} for three values of the normalized laser intensity I_c/I_{sat} inside the amplifier medium. a $\frac{I_c}{I_{sat}}=0$, b $\frac{I_c}{I_{sat}}=1,2$, c $\frac{I_c}{I_{sat}}=2,4$

A is the time-independent amplitude of the wave and φ its phase. The subscripts "inj", "i", and "out" refer to injected wave, inside cavity wave and output wave, respectively. r and t are the amplitude reflection and transmission coefficient of mirror M, and $\varepsilon=\pm1$ according to the direction of the reflection on this mirror.

Let us set

$$\frac{A_i(L)}{A_i(0)}=g\,e^{i\Delta},\qquad \Delta=\varphi_L-\varphi_0,\qquad g\text{: real}.$$

Now, $A_i(0)$, $A_i(L)$, and A_{out} may be expressed as

$$A_i(0)\,e^{i\varphi_0}=\frac{t}{1-\varepsilon rg\,e^{i\Delta}}\cdot A_{inj}\cdot e^{i\varphi_{inj}},$$

$$A_i(L)\,e^{i\varphi_L}=\frac{gt\,e^{i\Delta}}{1-\varepsilon rg\,e^{i\Delta}}\cdot A_{inj}\cdot e^{i\varphi_{inj}},$$

$$A_{out}\,e^{i\varphi_{out}}=\frac{\varepsilon r+g\,e^{i\Delta}}{1-\varepsilon rg\,e^{i\Delta}}\cdot A_{inj}\,e^{i\varphi_{inj}},$$

and the corresponding intensities read

$$\text{(V)}\quad\left|\begin{array}{ll} I_i(0)=\dfrac{(1-r^2)}{1+r^2\cdot g^2-2\varepsilon rg\cos\Delta}I_{inj}, & (6)\\ I_i(L)=g^2I_i(0), & (7)\\ I_{out}=\dfrac{r^2+g^2-2\varepsilon rg\cos\Delta}{1+r^2g^2-2\varepsilon rg\cos\Delta}I_{inj}. & (8)\end{array}\right.$$

3. Minimum Intensity for Single-Frequency Operation

Let us consider now the expression of the gains derived in Sect. 1. When I_i reaches a sufficient high level, g_c is saturated. Then the threshold conditions necessary to maintain oscillation, E_c will not be satisfied. Let us regard the term of zero order with respect to I_c in the expressions of gains

$$g_c^{\pm}=\xi\cdot\frac{g_0}{2}\cdot\frac{e^{-i\alpha_c}}{X_c}\cdot\frac{1}{S}\left(1-2X\frac{X_i}{X_c}\cdot\frac{N_2}{D}\cdot\mathcal{I}_i\right),$$

$$g_i=\frac{g_0}{2}\cdot\frac{e^{-i\alpha_i}}{X_i}\cdot\frac{1}{S},$$

where

$$S=1+\mathcal{I}_i,$$

$$N_2=\mathcal{I}_i\frac{X_i^2}{2X_c}\exp[i(\eta+\alpha_c)]+X_R\exp[i(\eta-\alpha_R)],$$

$$D=\mathcal{I}_i\frac{X_i^2}{2X_c}\left[\mathcal{I}_i\frac{X_i^2}{2X_c}+X_R\cos(\alpha_R+\alpha_i)\right]+X_R^2.$$

Then, the evolution of the intensities $\mathscr{I}_j$ and the phase φ_j will be governed by

$$\frac{d\mathscr{I}_j}{\mathscr{I}_j}=(g_j+g_j^*)dz\,, \quad d\varphi_j=\frac{1}{2i}(g_j-g_j^*)dz\,, \quad j=i,c\,.$$

Integration along one round trip leads to

$$\text{(VII)}\quad \begin{cases} \ln G_c=\dfrac{X_i^2}{X_c^2}\cdot\ln G_i-2\dfrac{O_3}{X_c^2} \\ \qquad \times\ln\dfrac{M_L}{M_0}-2\dfrac{O_4}{X_c^2}(P_L-P_0)\,, & (8) \\ \varphi_c(L)-\varphi_c(0)=-\dfrac{1}{2}\delta_c\dfrac{X_i^2}{X_c^2}\ln G_i+\dfrac{O_4}{X_c^2}\ln\dfrac{M_L}{M_0} \\ \qquad -\dfrac{O_3}{X_c^2}(P_L-P_0)-k_cL_c\,, & (9) \\ \ln G_i+(G_i-1)\mathscr{I}_i(0)=\dfrac{g_0}{X_i^2}\cdot L_a\,, & (10) \\ \varphi_i(L)-\varphi_i(0)=-\tfrac{1}{2}\delta_i\ln G_i-k_iL_c\,, & (11) \end{cases}$$

where

$$O_1=1-R_a\cdot\delta\cdot\delta_c\,, \qquad O_2=R_a\cdot\delta+\delta_c\,,$$

$$O_3=1+\delta_c\cdot\delta_i+2\cdot\delta^2\,, \quad O_4=\delta\cdot(1-\delta_c\cdot\delta_i)\,,$$

$$P_L=\mathrm{tn}^{-1}\frac{2X_cO_2}{X_i^2\cdot G_i\cdot\mathscr{I}_i(0)+2X_cO_1}\,,$$

$$M_L^2=\left\{G_i\cdot\mathscr{I}_i(0)+2\frac{X_c}{X_i^2}O_1\right\}^2+\left\{2\frac{X_c}{X_i^2}\cdot O_2\right\}^2.$$

P_0 and M_0 are the quantities deduced from P_L and M_L by changing G_i into 1. $\varphi_c(L)-\varphi_c(0)$ and $\varphi_i(L)-\varphi_i(0)$ are the changes of phase that are undergone by the waves E_c^+ and E_i along a round trip and

$$G_i=g^2=\frac{\mathscr{I}_i(L)}{\mathscr{I}_i(0)}\,, \qquad G_c=\frac{\mathscr{I}_c(L)}{\mathscr{I}_c(0)}\,.$$

L_a and L_c are the amplifier and the cavity lengths, respectively.

The oscillation condition for the wave $I_c^\pm$ may be written as

$$\frac{\mathscr{I}_c^\pm(L)}{\mathscr{I}_c^\pm(0)}<\frac{1}{R^\xi}\,. \tag{12}$$

It is then possible to compute the minimum injected intensity required for single-frequency oscillation: Eqs. (6, 10, and 11) make it possible to compute $I_i(0)$ and G_i versus I_{inj} and the other parameters of the system. Reporting the so found values in (9), we can compute δ_c for the resonant field taking into account the dispersion induced by the energy stored in the cavity from the injected signal. Equation (8) will then be used to verify if the gain G_c is brought under oscillation threshold condition (12).

4. Single-Frequency Oscillation

If I_i is strong enough to verify the conditions defined in the former section, the field E_c does not oscillate. The normalized intensity supplied by such a system may then be computed by solving the system

$$\text{(IX)}\quad \begin{cases} \ln G_i+(G_i^2-1)\mathscr{I}_i(0)=\dfrac{g_0}{1+\delta_i^2}\cdot L_a\,, & (13) \\ \Delta=-\tfrac{1}{2}\delta_i\cdot\ln G_i-k_iL_c\,, & (14) \\ \mathscr{I}_i(0)=\dfrac{(1-r^2)}{1+r^2g^2-2\varepsilon rg\cos\Delta}\mathscr{I}_{\text{inj}}\,, & (15) \\ \mathscr{I}_{\text{out}}=\dfrac{r^2+g^2-2\varepsilon rg\cos\Delta}{1+r^2g^2-2\varepsilon rg\cos\Delta}\mathscr{I}_{\text{inj}}\,. & (16) \end{cases}$$

By combining the first three equations, it is possible to compute the round-trip amplitude gain g and the phase shift Δ corresponding to a given injected intensity. Inserting these values in the fourth equations leads to I_{out}.

5. Corresponding Equations for Waveguide Lasers

We consider now the case when the laser amplifier medium is located inside a waveguide. The relation which makes it possible to compute the round-trip gain, has to be modified in order to take into account the distributed loss due to guided propagation [13]. Neglecting the change of phase induced by the guided propagation and assuming a distributed loss per unit length α_e, Relations (VI) and (VIII) have to be replaced by

$$\frac{d\mathscr{I}_i}{\mathscr{I}_i}=(g_j+g_j^*-\alpha_e)dz\,,$$

$$d\varphi_j=\frac{1}{2i}(g_j-g_j^*)dz\,, \quad j=c,i\,.$$

If discrete losses are present inside the cavity (coupling loss, window, lens, etc.) it will be convenient to account for them into the round-trip gain acting in the cavity relationships (System V) by replacing g by $g\sqrt{1-l_c}$ where $1-l_c$ is the transmission of the cavity without distributed loss.

More indicated are the relations expressing G_c as a function of G_i and $\mathscr{I}_i(0)$ for the single-frequency operation threshold. Consideration similar to the one exposed in Sect. 3 leads to

$$
\text{(X)}\quad\left|\begin{aligned}
&\ln G_c = \frac{\beta_c-1}{\beta_i-1}\cdot\ln G_i - \left(\frac{\beta_c-\beta_i}{\beta_i-1}+2\beta_c k_2\right)\\
&\qquad\times\ln\frac{G_i\cdot\mathscr{I}_i(0)-\beta_i+1}{\mathscr{I}_i(0)-\beta_i+1}-2\beta_c k_2\\
&\qquad\times\ln\frac{M_L}{M_0}-k_2-2\beta_c k_1(P_L-P_0)\,, &&(17)\\
&\varphi_c(L)-\varphi_c(0) = -\frac{\beta_c\delta_c}{2(\beta_i-1)}\ln G_i+\beta_c\\
&\qquad\times\left[\left(\frac{\delta_c}{2(\beta_i-1)}-k_1\right)\right.\\
&\qquad\times\ln\frac{G_i\cdot\mathscr{I}_i(0)-\beta_i+1}{\mathscr{I}_i(0)-\beta_i+1}\\
&\qquad\left.+k_1\ln\frac{M_L}{M_0}-k_2(P_L-P_0)\right]\\
&\qquad-k_c L_c\,, &&(18)\\
&\ln G_i-\beta_i\ln\frac{G_i\cdot\mathscr{I}_i(0)-\beta_i+1}{\mathscr{I}_i(0)-\beta_i+1}=(\beta_i-1)\alpha_e L_a\,, &&(19)\\
&\varphi_i(L)-\varphi_i(0) = -\frac{\delta_i}{2}(\ln G_i+\alpha_e\cdot L_a)-k_i L_c\,, &&(20)
\end{aligned}\right.
$$

where

$$\beta_j=\frac{g_0}{\alpha_e\cdot X_j^2}\,,\qquad j=c,i\,,$$

$$O_5=X_i^2\cdot(\beta_i-1)+2X_c\cdot O_1\,,\qquad O_6=2X_c\cdot O_2\,,$$

$$k_1=\frac{O_3\cdot O_6+O_4\cdot O_5}{O_5^2+O_6^2}\,,\qquad k_2=\frac{O_3\cdot O_5-O_4\cdot O_6}{O_5^2+O_6^2}\,.$$

It is now possible to compute the threshold condition for single-frequency operation using the scheme described in Sect. 3 for non-guided propagation. We may also compute the output intensity using (15, 16, 19, 20) with the same method explained in Sect. 4. Examples of such computations will be exposed in [14] which will deal with experimental results.

6. Conclusions

Using a density-matrix formalism, we have derived exact equations describing the injection phase locking process for homogeneously broadened lasers. The expression of the gain and the dispersion experienced by both the injected wave and the slave-laser eigenwave are reported. The effect of saturation by the injected intensity has been investigated and we demonstrate that, since the injected intensity wave is increased, its gain is enhanced while the slave-oscillator eigenwave gain is depleted. Starting from these equations and the conditions imposed by the slave cavity, we have laid down coupled steady-state equations making it possible to determine single-frequency operation conditions and the output intensity when phase locking is achieved versus both intensity and frequency of the injected wave. We deduce that, when the injected light frequency is set in the neighbourhood of a slave cavity resonance, a large amount of the injected power is stored in the slave cavity. As a result, the gain of the slave-cavity eigenwave saturates. If the stored power is strong enough in order to bring this gain under oscillation threshold, the slave laser does not oscillate. It acts then as a multipass amplifier for the injected light. The number of passes will depend upon the injected-wave round-trip gain with regard to the slave cavity oscillation threshold. The theoretical formula have been laid down for both conventional and waveguide lasers.

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References

1. E. Freysz: Thésis, Université de Bordeaux I, F-33405 Talence, France 1983
2. C.N. Man, A. Brillet: Opt. Lett. **9**, 333–334 (1984)
3. R. Adler: Proc. IEEE **34**, 351–357 (1946)
4. H.L. Stover, W.H. Steier: Appl. Phys. Lett. **8**, 91–93 (1966)
5. C.J. Buczek, R.J. Freiberg: IEEE J. QE-**8**, 641–650 (1972)
6. R.W. Dunn, S.T. Hendow, W.W. Chow, J.G. Small: Opt. Lett. **8**, 319–321 (1983)
7. B. Couillaud, A. Ducasse, E. Freysz: IEEE J. QE-**20**, 310–318 (1984)
8. W.E. Lamb, Jr.: Phys. Rev. **134**, A1429–A1450 (1964)
9. M.P. Spencer, W.E. Lamb, Jr.: Phys. Rev. A**5**, 884 (1972)
10. M.H. Ibrahim: IEEE J. QE-**14**, 145–147 (1978)
11. W.W. Chow: Opt. Lett. **7**, 417–419 (1982)
12. W.W. Chow: IEEE J. QE-**19**, 243–249 (1983)
13. M.B. Klein, R.L. Abrams: IEEE J. QE-**11**, 609–615 (1975)
14. G.L. Bourdet, R.A. Muller, G.M. Mullot, J.Y. Vinet: Appl. Phys. B **43** (in press) (1987)